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Aspiration-driven strategy evolutionary dynamics under strong selection

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Abstract. Strategy update rules based on self-evaluation are very common in practice. Most of the previous studies on the update of aspiration-based self-evaluation strategies were based on the assumption that people's adjustment intensity was low. Whether the successful propagation of human behavioral traits falls within this parameter is unclear. Therefore, it will be necessary to derive analytical results applicable to any selected intensity. In this paper, we focus on the effect of selection intensity on the level of population cooperation, and mainly focus on strong selection. We derive the results of the analysis for any selection intensity. The results show that under the condition of strong selection intensity, the evolution of cooperative strategy is strongly driven by aspiration, and significantly increase the cooperative strategy proportion compared with the results under weak selection. In addition, there is a critical cost-benefit ratio, which makes the proportion of cooperative strategy decrease sharply. The critical cost-benefit ratio decreases as the value of aspiration increase. However, when the selection intensity was weak, the aspiration value has a little effect on the proportion of cooperative strategies. We also reveal, essentially, the cause of the effect of aspiration value on the proportion of cooperative strategies at stable equilibrium time is the effect of aspiration value on the probability of strategy update under different configurations. In addition, the theoretical results are verified by Monte Carlo numerical simulation and the results are qualitatively consistent for different system sizes and structures. The apparent difference in the level of cooperation between strong and weak selection will be crucial to our basic understanding of human behavior and may lead to new insights into human self-evaluation.

1 Introduction

Cooperative behavior is widely existed in real biological, social and economic systems [1-6]. Cooperative behavior plays a crucial role in the normal operation of these systems. In recent years, evolutionary game theory [7–11] has been introduced into the study of cooperative phenomena, and it is found that this theoretical framework is very effective in dealing with this problem. In evolutionary game theory, a cooperator (C) helps all individuals to whom it is connected. A defector (D)does not provide any help, but it can receive the benefit from neighboring cooperators. The following two typical game models are usually considered. The prisoner's dilemma game (PDG) [11–24] is used to describe a situation in which it is beneficial for all members of a group to cooperate, but it is always more beneficial for individuals to defect, no matter what others do. In contrast to the PDG, the optimal strategy in the snowdrift game (SG) [25-33] is no longer to defect, but to take the opposite strategy to the opponent. The classic evolutionary game theory is two players interact by each choosing a strategy of cooperation or defection. When their strategies agree, each player receives a payoff R (or P) for mutual cooperation (or defection). When the players' strategies disagree, the defector receives a payoff T while the cooperator receives S. In a primitive approach to evolutionary game theory, players copy the most suitable strategy, selecting different mechanisms based on kin selection, direct and indirect reciprocity, network reciprocity, and so on [11]. These models take an evolutionary perspective, using birth-death dynamics to simulate the replication of strategies.

Aspiration-driven strategy evolution has also been considered frequently in recent years [34–44]. Aspiration level can be thought of as an intrinsic property of the individual, and if the player's payoff does not meet the corresponding aspiration, the individual will change his strategy to achieve greater payoff. The idea behind the motivation of aspiration, self-evaluation or cognitive choice, is common in both the human and animal worlds.

Weak selection has a long history in population genetics and molecular evolution, and plays an impor-

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tant role in social learning and cultural evolution. A large number of previous studies have focused on the strategy update and evolution driven by aspiration under the condition of weak selection [40-44]. They analyzed the effect of neutral benchmarks on strategy and concluded that the intensity of human adjustment was low. Although weak selection approximations are widely used in research, it is unclear whether the successful propagation of human behavioral characteristics falls within this parameter range. Therefore, the case of strong selection intensity cannot be ignored. For example, if the player's payoff does not meet the corresponding aspiration, the individual will have a strong willingness to change strategies in order to achieve greater payoff. Since selection intensity can have a crucial influence on evolutionary outcomes, it will be necessary to derive an analysis that applies to any selection intensity.

In this paper, we focus on the effect of selection intensity on the level of population cooperation, and mainly focus on strong selection, but weak selection is also considered. We also derive the results of the analysis for any selection intensity. Compared with the results under weak selection condition, the new results under strong selection condition are highlighted. In addition, the theoretical results are verified by Monte Carlo numerical simulation and the results are qualitatively consistent for different system sizes and different system structures.

2 Model and Monte Carlo simulation results

In this paper, we focus on the SG, the general SG is R = b - c/2, S = b - c, T = b and P = 0, where the parameters b and c denote the benefit and the cost [26,27,30].

We consider stochastic evolutionary game dynamics with two strategies in a structured population of finite size N. A focal player can be of type C or D, and interact with all of its neighbors according to the underlying population structure. In the model, each individual occupies a vertex of a graph. Based on all the interactions, an average payoff of an individual is calculated based on the SG payoff matrix as follows

$$\begin{array}{ccc}
C & D \\
C & \left(\begin{array}{ccc}
1 - r/2 & 1 - r \\
D & \left(\begin{array}{ccc}
1 & 0
\end{array}\right),
\end{array}$$
(1)

whereas r = c/b is the ratio of costs to benefits. For example, a C player, which encounters k other individuals and i of those are type C, obtains payoff

$$\pi_{C,i} = \frac{(1-r/2)i + (1-r)(k-i)}{k}.$$
 (2)

A D player connected to i other C player and k-i players D obtains payoff

$$\pi_{D,i} = \frac{i}{k}.\tag{3}$$

We consider the simplest case of an entire population having a certain level of aspiration. Players need not see any particular payoffs but their own, which they compare with an aspired value. This level of aspiration, α , is a variable that influences the stochastic strategy updating. The probability of switching strategy is random when individuals' payoffs are close to the level of aspiration, reflecting the basic degree of uncertainty in the population. When payoffs exceed the aspiration, strategy switching is unlikely. At high values of aspiration compared with payoffs, switching probabilities are high.

The level of aspiration provides a global benchmark of tolerance or dissatisfaction in the population. In addition, when modeling human strategy updating, one typically introduces another global parameter, the intensity of selection (or sometimes called a noise parameter), ω , which provides a measure for how important individuals deem the impact of the actual game on their update [12, 13]. Irrespective of the aspiration level and the frequency dependent payoff distribution, vanishing values of ω refer to nearly random strategy updating. For large values of ω , individuals' deviations from their aspiration level have a strong impact on the dynamics.

Note that although the level of aspiration is a global variable and does not differ individually, owing to payoff inhomogeneity there can always be a part of the population that seeks to switch more often due to dissatisfaction with the payoff distribution. In our microscopic update process, at every time step, a player x is randomly selected from the population to update its strategy (a cooperator changes to defection and a defector changes to cooperation) with the following probability function

$$P(\alpha - \pi_x) = \frac{1}{1 + e^{-\omega(\alpha - \pi_x)}},\tag{4}$$

which π_x is the average payoff of the focal individual x, ω is the selection intensity (or sometimes called a noise parameter) [12, 13]. The wider the gap between aspiration and payoff, the higher the switching probability. Reversely, if payoffs exceed the level of aspiration individuals become less active with increasing payoffs. The aspiration level, α , provides the benchmark used to evaluate how "greedy" an individual is. Higher aspiration levels mean that individuals aspire to higher payoffs.

The selection intensity governs how strict individuals are in this respect. For $\omega = 0$, strategy switching is entirely random (neutral). Low values of ω lead to switching only slightly different from random but follow the impact of α . For increasing ω , the impact of the difference between payoffs and the aspiration becomes more important. In the case of $\omega \to \infty$, individuals



Fig. 1 The relationship between the equilibrium frequency of cooperation and the ratio of costs to benefits r for different values of α . Parameters are N = 10,000 and k = 4 for the square lattice networks. **a** Weak selection ($\omega = 0.1$); **b** strong selection ($\omega = 100$)

are strict in the sense that they either switch strategies with probability one if they are not satisfied, or stay with their current strategy if their aspiration level is met or overshot.

We first consider the evolutionary SG with players located on the square lattice networks with periodic boundary conditions, each player interacts with its four nearest neighbors (k = 4). Each player is initially designed as a cooperator or a defector with equal probability.

The Monte Carlo simulations are carried out for a population of size N = 10000, but the results are also valid for populations of other sizes. The simulation results are obtained by averaging over the last 10^7 Monte Carlo time steps of the total 5×10^7 .

Figure 1 shows the results of Monte Carlo numerical simulation, the equilibrium cooperation level f_C as a function of the ratio of costs to benefits r for different values of α . In the case of weak selection ($\omega = 0.1$), the fraction of cooperation at stable moment is basically maintained at 0.5, as see Fig. 1a inset, and its weakly decreases with the increase of the ratio r, see Fig. 1a. This result is consistent with previous results [41, 44].

However, in the case of strong selection ($\omega = 100$), see Fig. 1b, the aspiration drive will significantly increase the cooperative strategy proportion compared with the results under weak selection and the previous studied results [34,41,44]. In addition, as aspirations gradually increase, the fraction of cooperation at stable moments will begin to decline sharply when r exceeds the critical value under strong selection, as see Fig. 1b. The critical cost-benefit ratio decreases as the value of aspiration increase. These are significantly different results from weak selection. The non-monotonic phenomenon is explained at the end of the next section.

For other structures, such as random regular graphs [15] and small worlds [45], the results are qualitatively consistent, as see Fig. 2. The average connectivity $\langle k \rangle = 4$, which is guaranteed for each structure. Each player is initially designed as a cooperator or a defector with equal probability.

When $\alpha = 1$, no individual will change their strategy since the payoff for all individuals is less than the level of aspiration. When $\alpha = 0$, all individuals will



Fig. 2 The relationship between the equilibrium frequency of cooperation and the ratio of costs to benefits r for different values of α under strong selection ($\omega = 100$). **a** Random regular graphs; **b** small world networks. Parameters are N = 10,000 and k = 4

change their strategies since the payoff of all individuals is greater than the level of aspiration, that is, the cooperative strategy will become the defection strategy, the defection strategy will become the cooperative strategy, and so on. Overall, the proportion of cooperators in the population does not change when $\alpha = 0$ and $\alpha = 1$. The level of cooperation in the population is determined by the initial state.

3 Theoretical analysis and discussion

We now analyze the above results theoretically. The transition probability that the focal player will change its strategy to C from D is given by

$$P_{D \to C} = (1 - f_C) \sum_{i=0}^{k} {\binom{k}{i}} f_C^i (1 - f_C)^{k-i}$$
$$\frac{1}{1 + e^{-\omega(\alpha - \pi_{D,i})}},$$
(5)

whereas $f_C \in [0, 1]$ is the fraction of cooperators. While the transition probability that the focal player C will change its strategy to D is given by

$$P_{C \to D} = f_C \sum_{i=0}^{k} \binom{k}{i} f_C^i (1 - f_C)^{k-i} \frac{1}{1 + e^{-\omega(\alpha - \pi_{C,i})}}.$$
(6)

According to the master equation, the derivative of f_C with respect to time is

$$\dot{f}_C = (1 - f_C) P_{D \to C} - f_C P_{C \to D}.$$
 (7)

It indicates that there exists an interior stable fixed point f_C^* , satisfying

$$\dot{f}_C = 0 = (1 - f_C^*) P_{D \to C} - f_C^* P_{C \to D}.$$
 (8)



Fig. 3 a The relationship between the derivative of f_C with respect to time and the cooperation level f_C for different values of r. The aspiration value α is 0.5. b The interior stable fixed point f_C^* as a function of the ratio of costs to benefits r for different aspiration values. The selection intensity $\omega = 0.1$ is fixed. With the increase of the ratio r, f_C^* decreases very weakly. Parameter α has no effect on the internal stability point

We plot in Fig. 3a the relationship between the derivative of f_C with respect to time and the cooperation level f_C in the weak selection case ($\omega = 0.1$). It is obvious from Fig. 3a that the interior stable fixed point f_C^* corresponds to the value of $f_C^* = 0.5$. We also plot in Fig. 3b the interior stable fixed point f_C^* as a function of the ratio of costs to benefits r. With the increase of the ratio r, f_C^* decreases very weakly. Parameter α has no effect on the internal stability point. This is consistent with the corresponding simulation results, as shown in Fig. 1a.

We plot in Fig. 4a the relationship between the derivative of f_C with respect to time and the cooperation level f_C in the strong selection case ($\omega = 100$). It is can be seen from Fig. 4a that the interior stable fixed point f_C^* corresponds to the value of $f_C^* = 1.0$, when the ratio of costs to benefits r is small; the internal stability point becomes two: 0.5 and 1, when the ratio r is large. There's a bifurcation here.

We also plot in Fig. 4b the interior stable fixed point f_C^* as a function of the ratio of costs to benefits r. The results show that under the condition of high sensitivity (strong selection) to aspiration, the strategy evolvement is driven by aspiration, and there is a critical cost-benefit ratio, which makes the proportion of cooperative strategy decrease sharply at stable time. And the critical cost-benefit ratio decreases as the aspiration value increases. This is consistent with the corresponding simulation results, as shown in Figs. 1b and 2.

Figure 5 shows the relationship between selection intensity ω , the ratio of costs to benefits r and the interior stable fixed point f_C^* (the proportion of cooperation strategy at stable time). It can be clearly seen from the Fig. 5 that the internal stability point f_C^* almost does not change with the ratio of costs to benefits r under weak selection intensity and is stable at about 0.5. It can also be seen from the Fig. 5 that when the selection intensity is large and r is small, the proportion of cooperative strategies is the highest, and almost all the system is composed of cooperative strategies. However, when the selection intensity is high, the propor-



Fig. 4 a The relationship between the derivative of f_C with respect to time and the cooperation level f_C for different values of r. The aspiration value α is 0.5. b The interior stable fixed point f_C^* as a function of the ratio of costs to benefits r for different aspiration values. The selection intensity $\omega = 100$ is fixed



Fig. 5 The relationship between selection intensity ω , the ratio of costs to benefits r and the interior stable fixed point f_C^* (the proportion of stable time cooperation strategy). The aspiration value $\alpha = 0.5$ is fixed

tion of cooperative strategies decreases suddenly with the increase of r. For example, under strong selection intensity ($\omega = 100$), the internal stability point f_C^* suddenly changes from 1 to about 0.5 at the critical r = 0.5. Different aspiration values correspond to different critical r, and the critical r decreases as the aspiration value increases.

Now let's explore why does aspiration level affect this way? Generally speaking, the influence of aspiration level is mainly carried out with individual payoff. And the individual's benefit mainly depends on the level of cooperation among neighbors.

To explore the elaborate impact of the parameter α on the evolution of cooperation, we introduce (Str, nc)to characterize the configuration of individual strategies and cooperation levels in individual neighbors. Where the first parameter Str represents the strategy of the focal individual, and the second parameter ncrepresents the number of cooperative strategies in the surrounding neighbors of the individual. For example, (C, 2) represents a configuration where an individual's strategy is C and the individual has two cooperative strategy neighbors. (D, 0) represents a configuration where an individual's strategy is D and there are no



Fig. 6 The influence of aspiration value on the updating probability of individual strategy (given configuration) under weak selection intensity ($\omega = 0.1$). The aspiration values α are 0.1, 0.3, 0.5, 0.7 and 0.9 from top to bottom, respectively. (Left) The focal individual is the cooperator. (Right) The focal individual is the defector

cooperative strategy neighbors among the individual's neighbors.

Figure 6 shows the influence of aspiration value on the updating probability of individual strategy (given configuration) under weak selection intensity. When the focal individual strategy is C, with the increase of r or α , the probability of strategy update for different configurations of individuals also increases slowly. When the focal individual strategy is D, with the increase of r or α , basically keeping at about 0.5 with slight differences for different configurations. Therefore, in general, in the case of weak selection intensity, the aspiration



Fig. 7 The influence of aspiration value on the updating probability of individual strategy (given configuration) under strong selection intensity ($\omega = 100$). The aspiration values α are 0.1, 0.3, 0.5, 0.7 and 0.9 from top to bottom, respectively. (Left) The focal individual is the cooperator. (Right) The focal individual is the defector

value has no effect on the strategy update, while the strategy update probability increases weakly with the increase of cost-benefit ratio r.

However, in the case of strong selection intensity, with the increase of r, the strategy update probabilities of individuals with different configurations have great differences. When the focal individual strategy is C, with the increase of r, there is a critical r that makes the individual strategy update probability suddenly increase from 0 to 1, as shown in Fig. 7, Left. When the focal individual strategy is D and the aspiration value $\alpha = 0.1$, the configuration is (D, 0), that is, the number of cooperative strategies in the individual's neighbors is 0, the update probability of the individual strategy is 1, no matter what the value of r is, as shown in Fig. 7b. The update probability of the individual strategy for other configurations is 0. As the aspiration value increases, the individual strategy update probability of more other configurations begins to change from 0 to 1. For example, when the aspiration value is 0.9, the individual strategy update probability of these configurations (D, 0), (D, 1), (D, 2) and (D, 3) are 1, and only the individual strategy update probability of configuration (D, 4) is 0, as shown in Fig. 7j.

The non-monotonic phenomena in the previous simulation, as shown in Fig. 1b or Fig. 2, are explained as follows. We focused our analysis on the case of $\alpha = 0.7$, and the other cases are similar. When r is less than 0.25, the probability that the focal individual is a cooperator changes its strategy is 0, as shown in Fig. 7, while the probability that a focal individual is a defector $\{(D,0), (D,1) \text{ and } (D,2)\}$ changes its strategy is 1. Therefore, the percentage of cooperators in the population will be higher than the initial moment after a period of evolution. As r increases, 0.25 < r < 0.35, the probability of changing its strategy when the focal individual is a cooperator starts to increase, 0 ;while the probability of changing its strategy when the focal individual is a defector is always 1. In general, the total probability of changing strategy for cooperating individuals at this time is smaller than the total probability of changing strategy for defecting individuals, thus the proportion of cooperators in the population will slowly increase. When r is greater than 0.35, the probability that a cooperative individual will change its strategy is greater than the probability that a defector will change its strategy, and therefore the proportion of cooperators in the population will begin to decline.

Therefore, in essence, the influence of aspiration values on the proportion of cooperative strategies in stable equilibrium is due to the influence of aspiration values on the probability of strategy update in different configurations.

4 Conclusions

In this paper, we have studied the effect of selection intensity on the level of population cooperation, focusing mainly on strong selection, but also considering weak selection. We also derive the results of the analysis of any selection intensity. The theoretical results have been verified by Monte Carlo simulation, and the results of different system sizes and different system structures are qualitatively consistent.

In the case of weak selection, both theoretical and numerical simulation results show that the cooperative ratio is basically maintained at 0.5 under stable equilibrium, and slightly decreases with the increase of costbenefit ratio. In addition, we also find that aspirationdrive dynamics can make the proportion of cooperation strategy unchanged. These results are consistent with previous results [41, 44].

However, under strong selected intensity, the aspiration-drive significantly increase the cooperative strategy proportion compared with the results under weak selection. In addition, under the condition of strong selection intensity, the evolution of cooperative strategy is driven by willingness, and there is a critical cost-benefit ratio, which makes the proportion of cooperative strategy decrease sharply. Different aspiration values correspond to different critical r, and the critical cost-benefit ratio r decreases as the aspiration value increases. These results are qualitatively consistent across different system sizes and structures. It is also found that under different configurations, the effect of aspiration value on strategy update probability is the essential reason that the proportion of cooperative strategies in the stable equilibrium is affected by aspiration value.

Although weak selection approximation has been widely used in self-evaluation studies in the past, it is not clear whether the successful propagation of human behavioral characteristics falls within this parameter. Since selection intensity can have a crucial influence on evolutionary results, it is necessary to derive the results of any selection intensity analysis. On the other hand, the apparent difference in the effect of strong and weak selection on the level of cooperation will be crucial to our basic understanding of human behavior and may guide new insights into human self-evaluation.

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Author contributions

B.-Q. Li discusses ideas with J.-Y. Guan. B.-Q. Li completes the numerical simulation and theoretical calculation. B.-Q. Li wrote the manuscript. The manuscript was revised by J.-Y. Guan.

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