

Regular Article - Statistical and Nonlinear Physics

Effects of inequality on a spatial evolutionary public goods game

Jinzhuo Liu^{1,2,3,4}, Mao Peng^{1,2}, Yunchen Peng¹, Yong Li¹, Chen Chu^{4,5}, Xiaoyu Li^{6,a}, and Qing Liu^{1,2,b}

¹ School of Software, Yunnan University, Kunming 650504, Yunnan, People's Republic of China

² The Key Laboratory for Software Engineering of Yunnan Province, Kunming 650504, Yunnan, People's Republic of China

³ The Engineering Research Center of Cyberspace, Kunming 650504, Yunnan, People's Republic of China

- ⁴ School of Artificial Intelligence, Optics, and Electronics (iOPEN), Northwestern Polytechnical University, Xi'an 710072, People's Republic of China
- ⁵ School of Statistics and Mathematics, Yunnan University of Finance and Economics, Kunming 650221, Yunnan, People's Republic of China

⁶ School of Cybersecurity, Northwestern Polytechnical University, Xi'an 710072, Shaanxi, People's Republic of China

Received 2 June 2021 / Accepted 31 July 2021 / Published online 17 August 2021 \odot The Author(s), under exclusive licence to EDP Sciences, SIF and Springer-Verlag GmbH Germany, part of Springer Nature 2021

Abstract. Over the past decade, inequality has become one of the most complex and troubling challenges in the global economy. Many scientists are determined to eliminate inequality to achieve full cooperation. However, our research shows that not all inequalities hinder cooperation. In this article, we study the effects of inequality by introducing the disassortative mixing of the investment amount and enhancement factor assigned to certain individuals in the public goods game. Compared with the traditional version, we find that cooperation can be effectively promoted by aligned inequality, which means that individuals with the highest (lowest) investment capabilities contribute the greatest (lowest) investment amounts. The promotion of cooperation mainly depends on the heterogeneous contribution ability of players. Specifically, cooperators with high contribution ability can maximize collective benefits, causing cooperators with low contribution ability to form compact clusters and resist invasion by defectors. Our research indicates that the diversity of individual endowment and productivity may have a non-negligible influence on the evolution of cooperation among selfish individuals.

1 Introduction

Cooperation among selfish individuals, as a ubiquitous phenomenon in both biological and social systems, has always attracted substantial attention [1-3]. To clarify how cooperation emerges and is maintained, evolutionary game theory is considered a useful mathematical tool and provides a series of classic models, such as the prisoner's dilemma game (PDG) [4–6], the snowdrift game (SDG) [7,8] and the stag hunt game (SHG). These models are suitable for studying the cooperative behaviour between pairwise interacting individuals. However, in the case of group interaction, the public goods game (PGG) is the dominant paradigm used to study cooperation. In the traditional PGG, initially, each player in a group simultaneously chooses a strategy between cooperation and defection. Cooperators invest in the public pool, while defectors invest nothing. Finally, the total investment in the public pool is multiplied by an enhancement factor r and then equally divided among all players. Ideally, when all players invest, they can maximize collective interests. However, in reality, there are always players who choose to free ride for higher personal benefit, causing the system to inevitably fall into the social dilemma called the "tragedy of the commons" [9].

Therefore, numerous mechanisms have been proposed to avoid social dilemmas. Nowak reviewed five rules for the promotion of cooperation named kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection in 2006 [10]. Network reciprocity, as an effective mechanism to promote cooperation by forming cooperative clusters in a spatial network, has encouraged many scientists to study the evolution of cooperation in a network from the following three main directions: (1) studying the influence of network topology, including regular lattice [11–21], smallworld network [22-24], scale-free network [25-28], and interdependent network [29–31]; (2) exploring the influences of various evolution rules, including degree mixing [32], social diversity [33], personal reputation [34], and reward and punishment [35-41]; and (3) studying the coevolution of network topology and game dynamics [42-44].

^ae-mail: lixiaoyu@nwpu.edu.cn (corresponding author)

^be-mail: liuqing@ynu.edu.cn (corresponding author)

Inequality, as one of the most complex challenges in the global economy, has received widespread attention [45, 46] in the past decade. Many scientists are committed to eliminating inequality, which is generally believed to undermine cooperation and welfare [47–50]. Experimental research has indicated that inequality of individual endowments inhibits cooperation. [47, 48] and undermines the social structure of a population [50]. A seminal work by Oliver P. Hauser studied the influence of inequality on the evolution of cooperation under the direct reciprocity mechanism [51].

In view of the above situation, we are inspired to discuss the following question: how does inequality affect the evolution of cooperation in a spatial structure? In this paper, we focus on this issue and use the PGG model on a regular lattice. In a typical PGG, it is assumed that all players have the same endowments and productivity levels. Here, we introduce the disassortative mixing of unequal endowments and productivity of the players on the network. Through simulation, we find that aligned inequality promotes cooperation and that the level of cooperation is a monotonic function of the correlation coefficient.

2 Methods

We consider a multiplayer public goods game, wherein players receive the greatest personal benefit on a square lattice of size $L \times L$ with periodic boundary conditions when they all cooperate to maximize collective benefits. Players are distributed on grid points; thus, every player is adjacent to four neighbours and participates in five groups of investments at the same time.



Fig. 1 The frequency of cooperation ρ_c as a function of the enhancement factor r for different correlation coefficients. The traditional public goods game is introduced for comparison. For each value of *corr*, ρ_c increases with increasing r. The values of *corr* are equal to -1, -0.6, 0, 0.6 and 1. The above result is obtained for K = 0.5 and L = 100



Fig. 2 The frequency of cooperation ρ_c as a function of time step t for different correlation coefficients. For comparison, we introduce the traditional public goods game. For corr = -1, ρ_c gradually decreases and finally disappears. For both corr = 0 and corr = 1, the situation is similar to that of the traditional version. The difference is that ρ_c of corr = 0 ultimately fluctuates at 0.6, while ρ_c of corr = 1 reaches full cooperation. The result is obtained by setting K = 0.5, L = 100, $c \sim N(1,0.5)$ and $r \sim N(4,1)$



Fig. 3 ρ_c evolves with the enhancement factor r and the correlation coefficient *corr*. For a fixed r, a higher correlation coefficient can promote the cooperation level, and for a fixed *corr*, a higher enhancement factor r can promote the cooperation level. All the results are obtained for K = 0.5, L = 100

In the traditional PGG, the same investment amount c and enhancement factor r apply to all players. To introduce inequality, we set the fixed investment amount c and the enhancement factor r as random numbers assigned to individuals. An investment amount c_i and an enhancement factor r_i are randomly assigned for a certain player i. Without loss of generality, we set $c_i \sim N(c_o, 0.5)$ with $c_0 = 1$ and $r_i \sim N(r_0, 0.5)$



Fig. 4 Evolutionary snapshots of cooperation (red) and defection (blue) for the same prepared initial scenario. From the top to the bottom, the corresponding *corr* values are equal to -1, 0, and 1. From left to right, snapshots were taken under MCS = 0, 100, 300, and 20,000. Simultaneously, all the results are obtained for K = 0.5, L = 100, $c \sim N(1,0.5)$, and $r \sim N(4,1)$

1) with r_0 in the interval [3, 5.5] for all players. To introduce the disassortative mixing of unequal endowments and productivity, the correlation coefficient is used to change the dependence between investment amount c_i and enhancement factor r_i and is set in the interval [-1,1]. Alignment inequality (i.e., corr > 0) introduces the tendency for players with the higher (lower) enhancement factor to be assigned a greater (smaller) investment amount, while misalignment inequality (i.e., corr< 0) introduces the tendency for players with the lower (higher) enhancement factor amount to be assigned a greater (smaller) investment. In this way, the payoff of the player is asymmetric under the situation described above. Thus, the final payoff of player *i* can be calculated by the following formula:

$$P_{i} = \frac{1}{n} \sum_{j=1}^{n} r_{j} c_{j} + (1 - \mathbf{x}) c_{i}$$
(1)

On the left side of the equation is the payoff that player i receives from the common pool, and the right side of the equation represents the remaining endowment, of which x has only two values: 0 (cooperate) or 1 (defect). Moreover, j represents the community of neighbours of player i and itself.

Before the start of each simulation, each player chooses a strategy between cooperation and defection with equal probability. Each cooperator contributes his own product of r_i and c_i to the common pool, while defectors contribute nothing. Then, each player obtains a payoff according to formula (1). Subsequently, the game uses an asynchronous update method to iterate forward in time. After each full iteration, player *i* randomly selects one of his four neighbours and update his strategy with the following probability by comparing the respective payoffs, P_i and P_i :

$$W(s_i \leftarrow s_j) = \frac{1}{1 + \exp[(P_i - P_j)/K]},$$
 (2)

where K = 0.5 denotes the amplitude of noise or its inverse, the so-called intensity of selection [52, 53]. With different correlation coefficients, the results given below are obtained by averaging over ten full Monte Carlo steps (MCSs) on 100 × 100 lattices. Each full Monte Carlo step requires a total of 2×10^4 steps.

3 Results

The results using various correlations (*corr*) are shown in the figures below.

We start by investigating the impact of inequality in the public goods game described above. Figure 1 shows how the frequency of cooperation evolves with the enhancement factor r for several different values of the correlation coefficient. For comparison, we introduce the traditional public goods game, whose cooperation emerges at approximately r = 3.75 and reaches full cooperation at approximately r = 5.5. In general, we can observe that the frequency of cooperation is considerably promoted as the correlation coefficient increases from 0 to 1. Particularly, when corr = 0, although the distributions of enhancement factor r and investment amount c have no correlation, they are still heterogeneous. Cooperation emerges around the value of r = 3.6 and eventually reaches full cooperation around the value of r = 4.7. The overall condition is obviously better than that of the traditional version. When corr = 1, the individual enhancement factor r and cost c are absolutely symmetrical, which means that individuals with the highest (lowest) investment capability have the greatest (lowest) investment amount. Evidently, this promotes cooperation to the greatest extent. When *corr* decreases, the frequency of cooperation also decreases, as shown when corr = -0.6and corr = -1. Particularly, when corr = -1, individuals with the highest (lowest) investment capabilities contribute the smallest (greatest) investment amount. In this case, cooperation does not emerge until r = 4.3, which is far larger than 3.75, the threshold at which cooperation emerges in the traditional version of the PGG.

Next, we further examine the time evolution of cooperation density for different values of the correlation coefficient. Figure 2 suggests how the frequency of cooperation evolves for different correlation coefficients. For comparison, we introduce the traditional public goods game, whose cooperation level decreases first at the beginning and then gradually increases to a steady state, hovering around the value of 0.5when r = 4. For corr = -1, the frequency of cooperation continuously decreases and finally disappears. We find that a negative *corr* makes the investment amount and ability assigned to all players very low, so the payoff of cooperation is always less than that of defection, and cooperation cannot be maintained. For both corr = 0 and corr = 1, the frequency of cooperation qualitatively evolves in the same way as for the traditional version. For corr = 0, the frequency of cooperation ultimately hovers around the value of 0.6, which is larger than that of the traditional version. For corr = 1, the fraction of cooperation increases faster and finally reaches full cooperation.

To study the joint effects of the enhancement factor r and the correlation coefficient *corr*, we present Fig. 3. For fixed *corr* = θ , when r < 3.6, defection occupies the whole network. When $r \geq 3.6$, cooperation emerges and coexists with defection; eventually, cooperation occupies the entire system as r increases. For a fixed r, cooperation gradually disappears as the inequality coefficient *corr* decreases and cannot reappear unless r is large enough. In contrast, as *corr* increases, cooperation emerges even at a smaller r. In brief, the frequency of cooperation increases as the enhancement factor r and the correlation coefficient *corr* increase together.

Figure 4 depicts the evolution snapshots of different strategy invasion processes for the same prepared initial scenario on the regular lattice. For all values of *corr*, we set the same initial strategy distribution scenario as in the first column. For corr = -1, cooperators are quickly invaded by defectors, cooperators (and cooperative clusters) gradually disappear, and eventually, defectors occupy most of the system. The reason for this is that the introduction of misaligned inequality makes cooperative clusters unsustainable. For corr = 0 and corr = 1, the cooperators on the boundary are dominant and can invade the defectors. Cooperators (and cooperative clusters) rapidly spread and then reach a steady state. At the end of the evolution process, we find that cooperation coexists with defection for corr = 0 and occupies the entire network for corr= 1. These results strongly prove our hypothesis once again.

Figure 5 displays the characteristic snapshots of strategy distributions for different types of players on the regular lattice. To further scrutinize the reasons for the evolution of cooperation, we separate players into four types by V_{RC} , which measures the contribution ability of an individual, that is, the value of individual investment amount c multiplied by enhancement factor r. When corr = -1, we can find that defectors occupy the entire network. When corr = 0 and corr = 1, we can observe that cooperators with high contribution ability become the centre of the cooperative clusters, while cooperators with low contribution ability surround the former and eventually form cooperative clusters. For corr = 0, cooperators coexist with defectors, and each occupies approximately half of the system. While corr = 1, cooperators succeed in continuing to expand until they occupy an overwhelming majority of the system.

Combined with the observations of Figs. 1, 2, 3, 4 and 5, it is evident that the correlation coefficient significantly affects cooperation. To further explore the potential reason for this phenomenon, we present Figs. 6 and 7. Figure 6 depicts the distribution of V_{RC} of all players for different values of corr. We can observe that the distribution of V_{RC} has visible differences for different corr. Figure 7 depicts the relationship between the variance of V_{RC} and the correlation coefficient. As the correlation coefficient increases, the variance of V_{RC} keeps increasing, which means that the inequality of the contribution ability increases. Thus, it can be easily inferred that such inequality promotes cooperation, which is consistent with previous studies that have found that diversity promotes cooperation [33, 51, 54]. Specifically, individuals with high V_{RC} play a vital role in the evolution of cooperation, as they attract individ-



Fig. 5 Characteristic snapshots of strategy distributions for different types of players on the regular lattice. We separate players into four types by the product of individual investment amount c multiplied by enhancement factor r: cooperators with a high product (light red) and a low product (red), and defectors with a high product (light blue) and a low product (red), and defectors with a high product (light blue) and a low product (blue). From left to right, the values of *corr* are equal to -1, 0 and 1. The snapshots were taken at MCS = 20,000. All results are obtained for K = 0.5, L = 100, $c \sim N(1,0.5)$, and $r \sim N(4,1)$



Fig. 6 Distribution of different values of V_{RC} . From left to right, the values of corr are equal to -1, 0 and 1



Fig. 7 The relationship between the variance of V_{RC} and the correlation coefficient. The above two figures are obtained for K = 0.5, L = 100, $c \sim N(1,0.5)$, and $r \sim N(4,1)$

uals with low V_{RC} to be more cooperative. The greater the inequality, the more players there are with high V_{RC} . As a result, cooperation can dominate the system.

4 Conclusions

Most previous studies did not focus on the inequality of individual attributes. In most previous models of network reciprocity, individuals are sufficiently equal in all relevant aspects. Inspired by the fact that individuals are heterogeneous in reality, we have introduced and examined the effect of inequality on the evolution of cooperation in the public goods game. Specifically, we introduce the disassortative mixing of individual investment amount c and enhancement factor r. Through numerical simulation, we have found that individual inequality obviously affects cooperation, which can be promoted by a higher positive correlation coefficient and suppressed by a lower negative correlation coefficient. In detail, investment amount c and enhancement factor r together determine the individual contribution ability. Players with a high contribution ability can effectively drive cooperation by attracting players with low contribution ability to form compact clusters. The larger the correlation coefficient is, the more players with high V_{BC} and the higher the level of cooperation. Therefore, cooperative behaviour can be promoted to the greatest extent when corr = 1. Our research has shown that not all inequalities hinder cooperation. In contrast, aligned inequality can promote cooperation. We hope that this work can provide some insights into understanding the emergence of cooperative behaviour under inequality and a theoretical basis for how to manage inequality in the future.

Acknowledgements We appreciate the support from the National Natural Science Foundation of China (Grants No. 61866039, No. 62066045), the Natural Science Foundation of Yunnan Province (Grant No. 2019FB083), and the Open Foundation of the Key Lab in Software Engineering of Yunnan Province (Grant No. 2020SE201).

Author contributions

JL and QL conceived the idea; MP, YP and YL designed the computer code and analyzed most of the data; JL and MP wrote the paper; XL contributed to refining the ideas, carrying out additional analyses; all authors discussed the results and revised the manuscript.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: This article has no additional data.]

References

- J.M. Smith, Evolution and Theory of Games. Am. Sci. 64(1), 41–45 (1975)
- J.F. Hauer, D.J. Trudnowski, G. Rogers, B. Mittelstadt, J. Johnson, Evolutionary games and population dynamics. IEEE Comput. Appl. Power 10(4), 50–54 (1998)
- M.A. Nowak, A. Sasaki, C. Taylor, D. Fudenberg, Emergence of cooperation and evolutionary stability in finite populations. Nature 428(6983), 646 (2004)
- G.S. Wilkinson, Reciprocal food sharing in the vampire bat. Nature 308(5955), 181–184 (1984)

- M. Milinski, Tit for tat in sticklebacks and the evolution of cooperation. Nature **325**(6103), 433–435 (1987)
- M.A. Nowak, K. Sigmund, Tit for tat in heterogeneous populations. Nature 355(6357), 250–253 (1992)
- C.Y. Xia, J. Zhao, J. Wang, Y.L. Wang, H. Zhang, Influence of vertex weight on cooperative behavior in a spatial snowdrift game. Phys. Scr. 84(2), 25802(6)–25807 (2011)
- A. Szolnoki, M. Perc, Correlation of positive and negative reciprocity fails to confer an evolutionary advantage: Phase transitions to elementary strategies. Phys. Rev. X 3(4), 041021 (2013)
- G. Hardin, The Tragedy of the Commons. Science 162(3859), 1243–1248 (1968)
- M.A. Nowak, Five rules for the evolution of cooperation. Science 314(5805), 1560–1563 (2006)
- M. Perc, A. Szolnoki, G. Szabó, Restricted connections among distinguished players support cooperation. Phys. Rev. E 78(6), 066101 (2008)
- A. Szolnoki, M. Perc, G. Szabó, Phase diagrams for three-strategy evolutionary prisoner's dilemma games on regular graphs. Phys. Rev. E 80(5), 056104 (2009)
- H. Zhang, M. Small, H. Yang, B. Wang, Adjusting learning motivation to promote cooperation. Physica A 389(21), 4734–4739 (2010)
- A. Szolnoki, M. Perc, Impact of critical mass on the evolution of cooperation in spatial public goods games. Phys. Rev. E 81(5), 57101 (2010)
- Z. Wang, M. Perc, Aspiring to the fittest and promotion of cooperation in the prisoner's dilemma game. Phys. Rev. E 82(2), 021115 (2010)
- Z. Wang, Z. Wang, X. Zhu, J.J. Arenzon, Cooperation and age structure in spatial games. Phys. Rev. E 85(1), 011149 (2012)
- Z. Wang, A. Szolnoki, M. Perc, If players are sparse social dilemmas are too: Importance of percolation for evolution of cooperation. Sci. Rep. 2(1), 369 (2012)
- C. Shen, J. Lu, L. Shi, Does coevolution setup promote cooperation in spatial prisoner's dilemma game? Appl. Math. Comput. **290**, 201–207 (2016)
- J. Jin, C. Shen, C. Chu, L. Shi, Incorporating dominant environment into individual fitness promotes cooperation in the spatial prisoners' dilemma game. Chaos, Solitons Fractals 96, 70–75 (2017)
- H. Guo, C. Shen, D. Dai, M. Zhang, C. Chu, L. Shi, Environment promotes the evolution of cooperation in spatial voluntary prisoner's dilemma game. Appl. Math. Comput. **315**, 47–53 (2017)
- C. Shen, X. Li, L. Shi, Z. Deng, Asymmetric evaluation promotes cooperation in network population. Physica A 474, 391–397 (2017)
- M. Kupennan, G. Abramson, The Structure and Dynamics of Networks, in *Small World Effect in an Epidemiological Model*. (Princeton University Press, 2011), pp. 489–492
- Z.X. Wu, X.J. Xu, Y. Chen, Y.H. Wang, Spatial prisoner's dilemma game with volunteering in Newman-Watts small-world networks. Phys. Rev. E 71(3), 037103 (2005)
- 24. X. Chen, L. Wang, Promotion of cooperation induced by appropriate payoff aspirations in a small-world networked game. Phys. Rev. E **77**(2), 017103 (2008)

- F.C. Santos, M. Santos, J.M. Pacheco, Social diversity promotes the emergence of cooperation in public goods games. Nature 454(7201), 213 (2008)
- H.X. Yang, W.X. Wang, Z.X. Wu, Y.C. Lai, B.H. Wang, Diversity-optimized cooperation on complex networks. Phys. Rev. E 79(5), 56107 (2009)
- W.B. Du, X.B. Cao, M.B. Hu, W.X. Wang, Asymmetric cost in snowdrift game on scale-free networks. EPL (Europhysics Letters) 87(6), 60004 (2009)
- W.X. Wang, R. Yang, Y.C. Lai, Cascade of elimination and emergence of pure cooperation in coevolutionary games on networks. Phys. Rev. E 81(3), 035102 (2010)
- Z. Wang, A. Szolnoki, M. Perc, Evolution of public cooperation on interdependent networks: The impact of biased utility functions. EPL (Europhysics Letters) 97(4), 48001 (2012)
- M. Perc, P. Grigolini, Collective behavior and evolutionary games-an introduction. Chaos Solitions Fractals 56(0960–0779), 1–5 (2013)
- Z. Wang, L. Wang, A. Szolnoki, M. Perc, Evolutionary games on multilayer networks: a colloquium. Eur. Phys. J. B 88(5), 1–15 (2015)
- 32. Z. Wang, L. Wang, M. Perc, Degree mixing in multilayer networks impedes the evolution of cooperation. Phys. Rev. E 89(5), 052813 (2014)
- 33. M. Perc, A. Szolnoki, Social diversity and promotion of cooperation in the spatial prisoner's dilemma game. Phys. Rev. E 77(1), 011904 (2008)
- 34. C. Chu, Y. Zhai, C. Mu, D. Hu, T. Li, L. Shi, Reputation-based popularity promotes cooperation in the spatial prisoner's dilemma game. Appl. Math. Comput. 362, 124493 (2019)
- K. Sigmund, C. Hauert, M.A. Nowak, Reward and punishment. Proc. Natl. Acad. Sci. 98(19), 10757–10762 (2001)
- A. Szolnoki, M. Perc, Reward and cooperation in the spatial public goods game. EPL (Europhysics Letters) 92(3), 38003 (2010)
- A. Szolnoki, M. Perc, Evolutionary advantages of adaptive rewarding. New J. Phys. 14(9), 093016 (2012)
- C. Du, D. Jia, L. Jin, L. Shi, The impact of neutral reward on cooperation in public good game. Eur. Phys. J. B **91**(10), 1–6 (2018)
- H. Brandt, C. Hauert, K. Sigmund, Punishment and reputation in spatial public goods games. Proc. R. Soc. Lond. B 270(1519), 1099–1104 (2003)
- D. Helbing, A. Szolnoki, M. Perc, G. Szabó, Evolutionary establishment of moral and double moral standards through spatial interactions. PLoS Comput. Biol. 6(4), e1000758 (2010)
- J. Liu, H. Meng, W. Wei, L. Tong, Y. Yong, Synergy punishment promotes cooperation in spatial public good game. Chaos Solitons Fractals 109, 214–218 (2018)

- M. Perc, A. Szolnoki, Coevolutionary games–A mini review. Biosystems 99(2), 109–125 (2009)
- C. Chen, J. Liu, S. Chen, J. Jin, S. Lei, Coevolution of game strategy and link weight promotes cooperation in structured population. Chaos Solitons Fractals 104, 28–32 (2017)
- 44. C. Shen, C. Chu, L. Shi, M. Jusup, M. Perc, Z. Wang, Coevolutionary resolution of the public goods dilemma in interdependent structured populations. EPL (Europhysics Letters) **124**(4), 48003 (2018)
- T. Piketty, E. Saez, Inequality in the long run. Science 344(6186), 838–843 (2014)
- 46. M. Scheffer, B. van Bavel, I.A. van de Leemput, E.H. van Nes, Inequality in nature and society. Proc. Natl. Acad. Sci. 114(50), 13154–13157 (2017)
- 47. T.L. Cherry, S. Kroll, J.F. Shogren, The impact of endowment heterogeneity and origin on public good contributions: evidence from the lab. J. Econ. Behav. Organ. 57(3), 357–365 (2005)
- S. Heap, A. Ramalingam, B.V. Stoddard, Endowment inequality in public goods games: A re-examination. Econ. Lett. 146, 4–7 (2016)
- 49. O.P. Hauser, G.T. Kraft-Todd, D.G. Rand, M.A. Nowak, M.I. Norton, Invisible inequality leads to punishing the poor and rewarding the rich. Behav. Public Policy 5(3), 333–353 (2021)
- A. Nishi, H. Shirado, D.G. Rand, N.A. Christakis, Inequality and visibility of wealth in experimental social networks. Nature 526(7573), 426–429 (2015)
- O.P. Hauser, C. Hilbe, K. Chatterjee, M.A. Nowak, Social dilemmas among unequals. Nature 572(7770), 524–527 (2019)
- A. Szolnoki, M. Perc, G. Szabó, Topology-independent impact of noise on cooperation in spatial public goods games. Phys. Rev. E 80(5), 056109 (2009)
- 53. M. Perc, J. Gomez-Gardenes, A. Szolnoki, L.M. Floria, Y. Moreno, Evolutionary dynamics of group interactions on structured populations: a review. J. R. Soc. Interface 10(80), 20120997 (2013)
- A. Szolnoki, M. Perc, Leaders should not be conformists in evolutionary social dilemmas. Sci. Rep. 6(1), 1–8 (2016)