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Game theoretic-based modelling of Krishna waters dispute: equilibrium solutions by hypergame analysis

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Abstract. This paper presents a hyper game analysis of the Krishna waters dispute and demonstrates its potential to yield an equilibrium solution even in the face of uncertainty that may be plausible in the intent behind the apparent position taken by contending parties. The approach can accommodate the real-world conflict situation in which players conceal their negotiating strategies and develop perceptions about apparent positions that may be misperceived. The hypergame model of the conflict formulated to resolve the water-sharing dispute amongst the riparian states of Maharashtra, Karnataka, Telangana and Andhra Pradesh, India. The paper demonstrates the potential of hypergame-based conflict resolution model to yield an equilibrium solution elegantly and which appears to have attributes of a "Fair and Equitable"" allocation. Hypergame is formulated by considering the perception of one player about the other 'players' game. All the possible perceptions are taken, and the stability analysis is carried out. The results of the stability analysis show that Fair and equitable allocation is part of the equilibrium solution. Our analysis demonstrates that the game-theoretic technique can be applied to solve any real-world conflict. Any allocation made based on "Fairness and Equity" undoubtedly lead to the equilibrium solution as seen in the present work.

1 Introduction

Conflicts have become an inevitable part of any treaty or sharing arrangement. Conflicts can develop when people interact and discern that their outcomes are opposing and that these interests cannot be resolved to fulfil all the parties involved [12]. The nursing of misperceptions and misunderstandings by the parties in any dispute is indeed a distinct possibility. Conflicts in the water-sharing arrangement are also not immune to these misperceptions. Water is synonymous with life. It has shaped the progress of humanity, and its scarcity has led to the end of great civilizations [2]. This is the main reason why there are numerous conflicts associated with river water sharing. India herself has seen many such examples of water conflicts. Peninsular India has many interstate rivers, and there is conflict associated with almost every river. India is one of the few countries to have a different law in the constitution to address the Interstate river dispute (Interstate water dispute act under article 262 of the Indian constitution). But even after having a separate statute, the conflicts have been rising. The problem is the unwillingness of the states to compromise when it comes to river water sharing. They tend to overestimate their contribution towards the river basin development and undermine the efforts of their co-riparian states.

This warrants the use of 'Fairness and Equity'in the water allocation. These two concepts have the potential to give a peaceful and permanent solution to any conflict. Rasinski [17], in his study, explained Fairness and Equity as "Proportionality and Egalitarianism", respectively. Proportionality means that the resource should be allocated in proportion to the deservedness. Those who contribute more should get more. Egalitarianism is the equal treatment of equals, which implies that those who are not equal cannot be treated simi-

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larly. For example, in the case of river water sharing, to create a level playing field for all the state virgin runoff should be considered instead of the regulated flow from the hydraulic structure because a poor riparian state will not have the resources and technology to build these infrastructures [18,19]. Fairness also implies that the procedure adopted for conflict resolution is the same for all the players. In a contemporary study, Baumol [3] classifies a distribution as fair if it involves no envy by any individual of any other. The present work tries to test and validate the efficacy of the 'air and Equity' with the help of the game-theoretic technique on the Krishna river water dispute.

The effectiveness of any conflict resolution method lies in the cooperation amongst the players. Alcalde et al. [1] has highlighted that if the players are not willing to cooperate, it becomes challenging to come to lasting arrangements. Perc [16] study highlights the need for cooperation in the social context and its significance in reaching the final agreement. Players would cooperate if they perceive the game (as in game theory) as fair and equitable. In game theory, there are two essential tools to solve any conflict as a game. One is the Metagame analysis, and the other one is hypergame analysis. Metagame analysis enables the game to be transparent as all the parties involved in the game know about the options tabled by other players. But when the resource under competition is limited and scarce, players tend to conceal their strategy, and this is where hypergame analysis comes in. In hypergame analysis, players can only form perceptions of the other 'players' game as they do not know their actual options. The different player can come together and form a coalition to serve their interest. Perc [16] has described the difference in the 'players' opinion as an individual and a group. Öztürk [13] has highlighted the importance of ethics in any water allocation scheme. The present work has thus included a key player in conflict resolution.

To address the "Fairness", we have derived the 'Fair and Equitable' allocation, and to address the ''Equity', we have used the hypergame analysis. 'Fair and Equitable' allocation [15] ensures that the states are getting the allocation in the proportion they deserve, and hypergame analysis ensures the transparency of the procedure. In the present study, the hypergame Analysis technique developed by Fraser and Hipel [6–9], has been applied to model possible misperceptions that are likely to arise in the sharing of Krishna river water amongst its riparian states. Hypergame technique is useful, as the study reveals, in problems that are made more complex by misunderstandings and misperceptions among the players/participants. These are typical features of most social and political conflicts, and hence hypergame analysis is expected to be a more realistic representation of a typical water resources conflict.

2 Concepts of hypergame analysis

A hypergame is a model representation of a conflict situation where adversaries do not share a common understanding of the conflict on account of differences in perceptions. For example, players involved in a conflict may have incorrect understanding of each other's options, strategies or preference ordering of possible outcomes. Further, in many real-world conflict situations, it is not unusual to have indirect outside influences, in the form of individuals, institutions or interest groups, playing a defining, but unfathomable, role that determines the outcome of these disputes. Consider a simple conflict where there are two players, A and B and let '"'q' be a possible outcome in this game. Further, let $M_{\mathrm{A}}^{+}(q)$ be a set of outcomes preferred by player A to q while $M_{\mathrm{A}}^{-}(q)$ is the set of outcomes not preferred by player A (set includes q). Denote the set of outcomes accessible unilaterally to player A, from the current position '''q', as $m_{\mathrm{A}}(q)$ (set does not include q). The above situation considers that both the players involved are clear about the strategies chosen by them. But in reality, this will not happen as in actual conflicts; players tend to conceal their fundamental strategies.

A hypergame is a model representation of a conflict situation where adversaries do not share a common understanding of the conflict because of differences in perceptions. For example, players involved in a conflict may have an incorrect understanding of each 'other's options, strategies, or preferences ordering possible outcomes. There may be unspecified alliances between players in the conflict, and these alliances, understandably, can significantly influence the outcomes of these conflicts. Further, in many real-world conflict situations, it is not unusual to have indirect outside influences, in the form of individuals, institutions or interest groups, playing a definite but unfathomable role that determines the outcome of these disputes. Naturally, these externally directed attempts to influence outcomes of disputes may at times be in overt or covert alliance with one or more players directly involved in the dispute. A distinctive feature of water resources conflicts is that they have a powerful social and political dimension. In these disputes, the interests of one group compete with the interests of other groups and concealment of strategies in negotiations is often seen as a safeguard against possible disadvantageous outcomes. Naturally, therefore, a realistic model of such conflicts must have the capability to handle these attitudes and the consequent misperceptions that unfold at various levels as the negotiations progress.

Following the works of Bennett [4,5], significant contributions to the development of hypergame modelling theory were made by Takahashi et al. [20] and Fraser and Hipel [9]. The later authors developed a formal organizational framework to facilitate a systematic arrangement of 'players' misperceptions in the hypergame model. This approach was seen to permit efficient modelling of the conflict while, at the same time, providing a structure in which a variety of types of information about a conflict could be incorporated for an effective determination and communication of possible resolutions to the dispute.

The procedure developed by Takahashi et al. [20] and Fraser and Hipel [9] merges the numerous Metagame

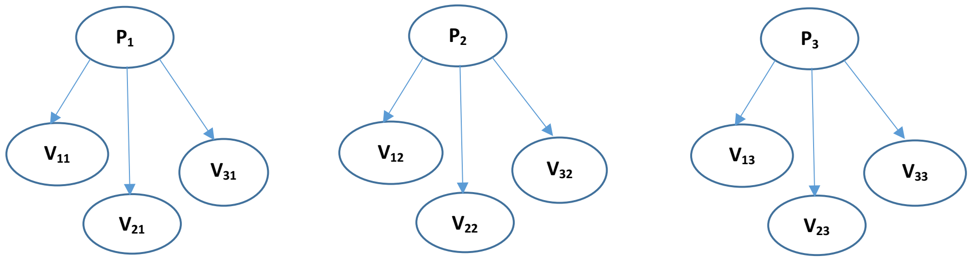

Fig. 1 1st level hypergame with three players (P_1, P_2 and P_3) in the dispute with their perceptions denoted as V_{ij} (ith: player forming perceptions regarding player j)

Analysis tables into one composite tableau that can incorporate the knowledge that a player has or believes he has about the other 'player's preferences. To facilitate a proper understanding of hypergame analysis, it is essential to understand the underlying differences between Metagame- and hypergame-based techniques. The most important feature of the MA approach is that the players see the same game, and the game proceeds with a characteristic openness and transparency.

Formally, a typical game is defined by the set of preference vectors of all players and, for a game with n players, maybe denoted symbolically by

$G = \{V_1, V_2, V_3, \ldots, V_n\}$; where V_i is the preference vector for player i.

A hypergame situation arises when one or more players carry misperceptions about other 'players' options and/or their preference orderings. Also, the existence of another player not directly involved in the conflict can influence the 'game's progress. It is not unusual for these peripheral parties to position themselves, in open or covert alliances with mainstream stakeholders, in such a manner as to influence the outcome of the game. Naturally, in such situations, different players perceive these games differently. Symbolically, player 'j's game may now be denoted as

$$G_j = \{V_{1j}, V_{2j}, V_{3j}, \ldots, V_{nj}\}, \tag{1}$$

where V_{ij} is the preference vector of player i as perceived by player j. and G_j represents the game played by player j (in a game consisting of total n players).

A hypergame H is then defined as a set of games as perceived by each player and is written as

$$H = \{G_1, G_2, G_3, \ldots, G_n\} \quad . \tag{2}$$

A 1st level hypergame is shown in matrix form in Table 1 where nth column represents the game as perceived by player n and denoted above as G_n. Graphically, a three-player, 1st level hypergame is depicted in Fig. 1, where the symbols P_1, P_2 and P_3 represent players 1, 2, and 3, respectively, with corresponding games represented as G_1, G_2, and G_3 (Games G_1, G_2 and G_3 are shown in table separately). In the figure, elements of G_1, namely V_{11}, V_{21}, and V_{31} represent player 1's misperceptions of itself (arguably true preference vector of player P_1), player 2 and player 3, respectively. Similarly, elements of G_2 and G_3, respectively, represent misperceptions of players P_2 and P_3 on other contenders.

Table 1 1st level hypergame for n player's dispute in matrix form with their perceptions shown as V_{ij} and G_i representing the game from each players' perspective

Player being perceived	Game perceived by the player $123\ldots n$
1	$V_{11}V_{12}V_{13}\ldots V_{1n}$
2	$V_{21}V_{22}V_{23}\ldots V_{2n}\ldots$
.	$V_{n1}V_{n2}V_{n3}\ldots V_{nn}$
n	$G_1G_2G_3\ldots G_n$

Some of the important positions that the players would take in the subsequent section are explained below:

Unilateral improvement (UI) If a player, starting from his current position, can go to a more preferred position, and in doing so, does not affect the options of other players.

Rational (R) In this situation, the given player has no UI to move from a particular outcome implying, thereby, that the strategy already chosen is the best that can be taken.

Unstable (U) Here a given player, A, has at least one UI from where the other players can take no credible action that ultimately results in a less preferred outcome for player A.

Sequential stability (S) For all UIs available to the player, A, the outcome has sequential stability (S) if credible actions can be taken by other players that ultimately results in a less preferred outcome than the one from which player, A, is seeking improvement. The possibility that a worse outcome could result, for a given player changing his strategy, deters that player from unilaterally attempting an improvement in position and induces sequential stability and is labelled as 'S'.

Equilibrium (E) When an outcome is stable for all the players, it is equilibrium and is the probable resolution of the conflict. It is denoted by E.

Non-equilibrium (X) When an outcome is unstable for a single player, it is denoted by X, and it is not the final solution as the other players will not accept it as it has not reached equilibrium.

2.1 Stability analysis in hypergame

The stability of a hypergame is analyzed by treating each player's game separately. Intuitively, this corre-

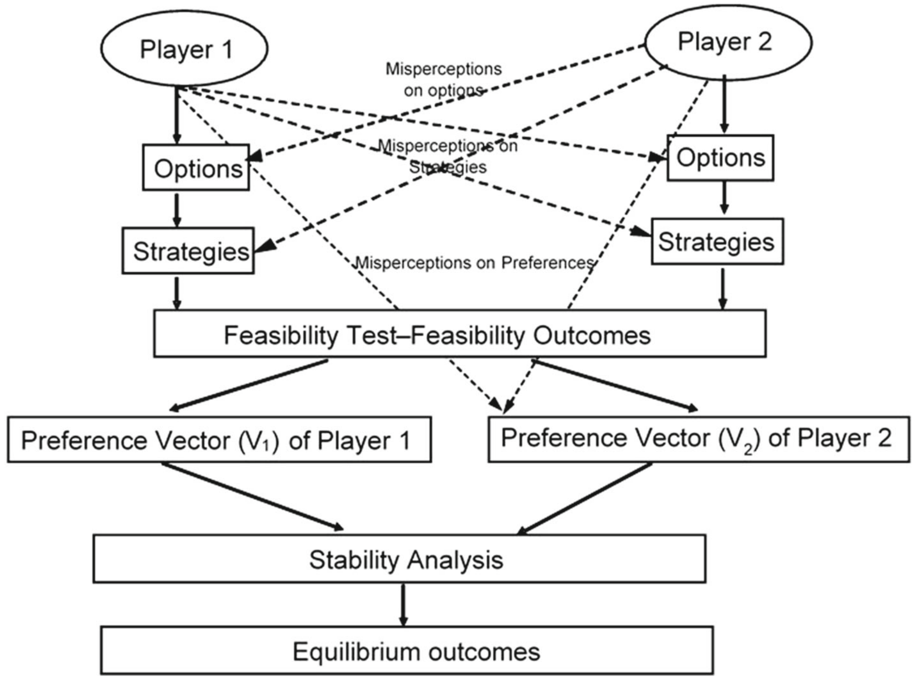

Fig. 2 A flowchart showing the sequence of the hypergame analysis between players 1 and 2 with misperceptions on account of options, strategies and preference vectors (V_1 and V_2)

sponds to analyzing player 'j's game as he perceives the conflict. It follows, therefore, the decision that player j makes, and thus the strategy that he chooses, depends only on how he interprets (or misinterprets) the situation. An outcome can be an equilibrium outcome of the hypergame if it is stable in all the preference vectors that the players perceive themselves. Thus, the outcome must be stable in all the preference vectors that appear along the main diagonal of the hypergame matrix of preference vectors in Table 1. Intuitively, the equilibrium of any hypergame depends only upon the stability of the outcomes of each player perceiving the game and, in a three-player game, it follows that only those outcomes are stable in the vectors V_{11}, V_{22} and V_{33} are equilibrium outcomes of the overall hypergame. The overall sequence in the hypergame is explained using the flowchart given in Fig. 2. Figure 2 shows that there can be misperceptions based on each 'player's options, strategies chosen, and the preferences order selected. Stability analysis is performed on all the games. Finally, an equilibrium outcome is obtained. It is essential that while forming perceptions, an individual player should respect the other 'players' options, strategies, and preferences. It will be challenging to obtain the final solution.

3 Study area

The Krishna River water dispute has a history of more than 100 years. There had been many agreements for water sharing. The first one was between the Mysore (now Karnataka) and the Madras presidency (Andhra Pradesh and Telangana). India was under the colonial era at that time, and hence this treaty was mediated by the colonial masters. But soon after the Indian independence, dissension started regarding water sharing, and it exists till today. India reorganized after the independence, and as a result, the number of riparian states increased to three till 2014 and to four after that. The government of India has established Krishna Water Dispute Tribunal (KWDT—a constitutional body under the Indian constitution) to settle this dispute. KWDT

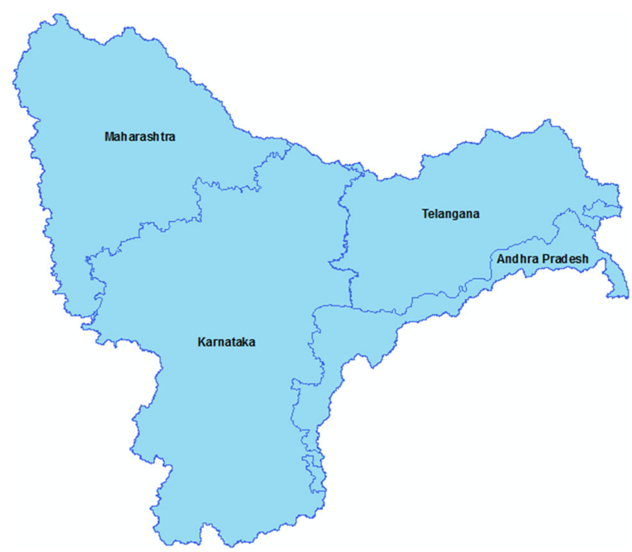

Fig. 3 Four riparian states (Maharashtra, Karnataka, Telangana and Andhra Pradesh) involved in the Krishna River water dispute

gave judgements-one in 1973, and one in 2007 but both these judgements did not satisfy the riparian states. Presently, there are four riparian states of the Krishna River water: Karnataka, Maharashtra, Andhra Pradesh, and Telangana. The erstwhile state of Andhra Pradesh reorganized into Andhra Pradesh and Telangana. Figure 3 shows the geographical setting of all the four riparian states. The relative area occupied by each riparian is as follows: Andhra Pradesh (20.11%), Telangana (9.25%) Maharashtra (26.36%) and Karnataka (43.8%). An essential feature of this river basin is the delta formation in the state of Andhra Pradesh rightly before the river falls into the Bay of Bengal. This delta is known for its rice cultivation, and Andhra Pradesh prioritizes water for its rice cultivation in this area. Karnataka is the uppermost riparian state and can regulate the downstream flow as it has a network of dams. Andhra Pradesh being the lowermost riparian state, always object to flow regulation by the upper riparian states.

4 Results

Section 4.1 deals with the derivation of 'Fair and Equitable' allocation based on the three factors which reflects the characteristics and the most critical use of the river basin. Since the resource under consideration is limited, there has to be compromise if required to have fair division. This addresses the fairness issue in the allocation. To address the equitable treatment of all the players, hypergame analysis is considered as part of a game-theoretic technique. Section 4.2 discusses the hypergame formulation for the two scenarios considered, one before the reorganization of Andhra Pradesh (which took place in 2014) and one after the reorganization of Andhra Pradesh. Scenario 2 considers only the newly reorganized states. Section 4.3 explains the entire sequence of the hypergame analysis between erstwhile Andhra Pradesh, Karnataka, Maharashtra and Environment. Hypergame is formulated keeping in mind the options presented by all the players in front of the Krishna water dispute tribunal (KWDT). Stability Analysis is carried out and it shows that 'Fair and Equitable' allocation is part of the equilibrium solution. Finally, Sect. 4.4 considers the scenario between the states of Telangana and Andhra Pradesh. Andhra Pradesh reorganized into two above states in 2014 and to address the fairness aspect of the game, there is a separate scenario. Stability analysis shows that 'Fair and Equitable' allocation is the only equilibrium outcome possible in this scenario.

4.1 Fair and equitable (F&E) allocations as an equilibrium outcome for the hypergame analysis

Fair and equitable allocation is derived using drainage area (F1), cultivable area (F2) and virgin runoff (F3) as the factors. The data is referred from the work of Panday [14,15] and the reports of KWDT-1, 2. Importantly instead of the actual runoff, virgin runoff is considered. The choice of virgin runoff strictly mandates the procedural justice as the developed states have made various projects within their part of the basin. They can demand prescriptive use based on this development. But it will not be a level playing field for the other states, especially Telangana which cannot claim any 'prescriptive use' as it is the latest player which has come into the existence after the reorganisation of the erstwhile Andhra Pradesh. Hence, these states will have a disadvantage if we consider the current runoff. Hence, virgin runoff is used in the present study which do not considers the intervention of the dams and the other hydraulic structures. The data is represented in the relative terms as required for the analysis and is shown in Table 2. Equal weightage is assigned to each factor to ensure fairness in allocation. States rank these factors differently, hence there cannot be unequal weightage to these factors. The final ranking of the factors by states is given in Table 2.

Now as can be seen from Table 2 if all the states go for their first ranked option, then the total allocation goes beyond 100% which is impossible. This calls for the negotiations in the allocation and is shown in Table 3. The round of negotiations will continue till the final allocation does not exceed 100%.

This gives the 'Fair and Equitable' allocation as listed in Fig. 4. The total water available is 2275 TMC [10, 11,14]. Final allocation is given in both absolute and relative terms in Fig. 4. This result is offered as one of the potential solutions in Sects. 4.2 and 4.3.

4.2 Hypergame formulation for both the scenarios

The options for the hypergame analysis reflects the aspirations of the states as presented by them in front of the Krishna water dispute tribunal (KWDT-1,2) Options for both the scenarios are shown in Table 4. These options are presented by the states themselves before the KWDT. Options considered and the process followed for the conflict resolution is guided by the principles of a "Fair and Equitable" allocation. Author(s) have considered this to formulate the options and there is no bias from authors' perspective. KWDT-1 tribunal awarded its judgement in year 1973 (published in 1976) and it contains the options which the states (players) wanted to be considered as the basis of water allocation. Those factors have been used over here and in such a manner that maximize their share and optimize the hypergame analysis.

The total number of outcomes possible under scenario I (II) are $2^9 = 512$ ($2^5 = 32$). In the hypergame binary notation (0,1) is used to reject (0) and accept (1) a particular outcome respectively. It can be easily figured out that not all these outcomes are feasible as the total quantity of water available is limited and the demand to available water ratio is already much greater than 1. For example, outcomes of the type (- - -, 0 0) in the case of scenario 2 is not feasible as the second player i.e. Telangana is having (0,0) as it will never

Table 2 Ranking of the factors such that the share of each state is maximum and is based on their actual contribution to the river basin

State	Rank 1 Factor/percentage (%)	Rank 2 Factor/percentage (%)	Rank 3 Factor/percentage (%)
Maharashtra	F3/33	F1/26.79	F2/21
Karnataka	F2/48	F1/43.85	F3/39
Andhra Pradesh	F1/20.11	F2/12	F3/11
Telangana	F2/19	F3/17	F1/9.25

Table 3 Cooperative negotiations allocation to address the limited resource and keep the final allocation within the available resource, i.e., 100 %

Summation of rank 1 factors	$(33 + 48 + 20.11 + 19) = 120.11$	$> 100\%$	Allocation not possible
1st level of compromise; average (Rank1; Rank2)	$(26.895 + 45.925 + 16.055 + 18) = 106.875$	$> 100\%$	Allocation not possible
2nd level of compromise; average (Rank1; Rank2; Rank3)	$(26.93 + 43.61 + 14.371 + 15.08) = 100$	$= 100\%$	Allocation = available resource

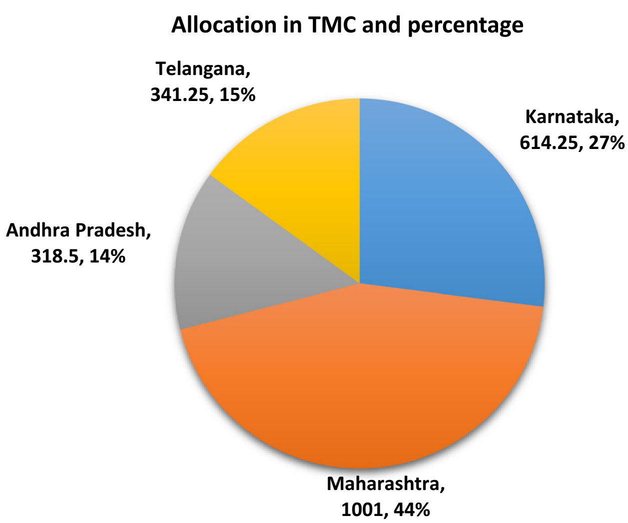

Fig. 4 'Fair and equitable' allocation based on the factors agreed by each state

select that option and the other player will also never perceive this as the option chosen by Telangana. Standard outcome removal methods [9] are used to eliminate the logically infeasible outcome. Also, to further represent the binary notation succinctly, decimal value representation is used. For example, (0, 1, 0, 1 0) can be represented as $0*2^0+1*2^1+0*2^2+1*2^3*0*2^4=10$.

4.3 Hypergame analysis for scenario 1 (Karnataka, Maharashtra, Andhra Pradesh and Environment)

The original preference vector of all the players and its decimal value is shown in Table 3. This preference vector is how the state will arrange all the nine options to maximize their allocation (Table 5).

Assumptions:

1. Since the environment has only two options and that can be taken care of in any of the flow regimes (lean or excess flow) so the perceptions of preference order of environment are the same as being seen by the other players and also environment has no misperceptions about the preference order of other players. This implies that $V_{14} = V_{24} = V_{34} = V_{44}$. and also, $V_{41} = V_{42} = V_{43}$.
2. Karnataka sees the preference order of the other players differently from what is seen by them. Since Karnataka gets its maximum share when it chooses option 469 and it is also benefitted from the F&E allocations, it perceives that Maharashtra will also go for F&E (though it can go for options 409 and 405 also). With erstwhile AP, Karnataka perceives that it will go for F&E as it also provides it for the Krishna Delta share.
3. Maharashtra predicts the game without any misperception about other players' preference vectors because it knows that as per the given options, erstwhile AP and Karnataka may select the options as listed in the original Metagame Analysis. AP sets of options are clearer because of its excess demand (Demand to F&E ratio is > 3) its options no.1 (Allocation of 1594 TMC of water annually to meet the irrigation requirements) is infeasible.
4. AP perceives the game differently as it thinks that another state recognizes the need for the importance of the protection of the Delta system and how productive it is for the overall economy of the nation.

Table 4 Four (two) players, namely Karnataka, Maharashtra, Andhra Pradesh and Environment and correspondingly, nine (five) options are proposed

Players	Options *under Scenario I*	Options *under Scenario II*
Karnataka	1. Fair and equitable allocation (1001 TMC) 2. Allocation of 1430 TMC of water annually to meet the irrigation requirements	
Maharashtra	1. Fair and equitable allocation (614.25 TMC) 2. Allocation of 828.8 TMC of water annually to meet the irrigation requirements	
Andhra Pradesh	1. Fair and equitable allocation (659.75 TMC) 2. Allocation of 1594 TMC of water annually to meet the irrigation requirements 3. Protection of annual utilization of 214 TMC based on Prior appropriation doctrine for the Krishna Delta system	1. Fair and equitable allocation (318.5 TMC) 2. Protection of annual utilization of 214 TMC based on Prior appropriation doctrine for the Krishna Delta system 3. Allocation of 1594 TMC of water annually to meet the irrigation requirements
Environment	1. F&$Eallocationsforallthestateswhenthe$ required E-flow is released 2. River water quality constraints for the protection of downstream flora and fauna	
Telangana	–	1. Fair and equitable allocation (341.5 TMC) 2. Allocation of 318.5 TMC of water annually to meet the irrigation requirements

Table 5 Preference vector of the players and its decimalized value for scenario I (players before the reorganization of Andhra Pradesh i.e., prior to 2014)

Karnataka		Maharashtra		Andhra Pradesh			Environment		Decimal value
1	0	1	0	1	0	1	1	1	469
1	0	1	0	1	0	0	1	1	405
1	0	0	1	1	0	0	1	1	409
1	0	0	1	0	0	1	1	1	457
0	1	1	0	0	0	1	1	1	454
0	1	0	1	0	0	1	1	1	458
1	0	1	0	0	0	1	1	1	453

Using all these assumptions and after removing the infeasible outcomes, the preference order for the hypergame analysis has been made.

4.3.1 Preference vectors according to the changed perceptions

Now each player knows only her preference vector correctly and can only perceive the order of the other players to the best of her knowledge. This is represented in Table 6.

4.3.2 Stability analysis for scenario I

$H = \{G_1, G_2, G_3, G_4\}$ where $G_1 = \{V_{11}, V_{21}, V_{31}, V_{41}\}$, $G_2 = \{V_{12}, V_{22}, V_{32}, V_{42}\}$, $G_3 = \{V_{13}, V_{23}, V_{33}, V_{43}\}$ and $G_4 = \{V_{14}, V_{24}, V_{34}, V_{44}\}$

Game G_1: Here the perspective of Karnataka is considered and how Karnataka sees the game from her point of view. Karnataka seeks to maximize her own share from the game G_1 but game G_1 should also not undermine the options, strategies and preferences of other players as there will be fewer chances of the equilibrium solution being obtained (Table 7).

Equilibrium outcomes: 469, 458 and 409

Similarly, stability analysis for the game G2, G3 and G4 is completed. These results are given separately in the supplementary section for the sake of brevity. The results in terms of final equilibrium outcomes are given below:

G2 (Game from the perspective of Maharashtra): equilibrium outcomes: 458, 469 and 409

G3 (Game from the perspective of Andhra Pradesh): equilibrium outcomes: 458 and 469

G4 (Game from the perspective of Environment): equilibrium outcomes: ALL

4.3.3 Stability analysis for the overall hypergame for scenario I: $H = (G_1, G_2, G_3, G_4)$

The hypergame H will comprise of all the four games. The final stability analysis is shown in Table 8. The out-

Table 6 Preference vectors according to the changed perception of each player towards the preferences of other players denoted as V_{ij}

V_{11}	V_{22}	V_{33}	V_{44}	V_{21}	V_{31}	V_{12}	V_{32}	V_{13}	V_{23}	V_{14}
454	457	469	469	458	469	458	469	458	458	469
458	409	405	405	457	405	45	405	454	457	405
469	458	409	453	409	409	453	409	453	409	453
405	469	453	409	405	457	405	457	457	453	409
453	405	457	457	453	454	469	454	405	454	457
409	453	454	454	469	458	409	458	469	405	454
457	454	458	458	454	453	457	453	409	469	458

Table 7 Stability analysis game G_1, i.e. game from the perspective of player 1, i.e. Karnataka

Row no.		1	2	3	4	5	6	7
Player	Overall stability	X	E	E	X	X	E	X
Karnataka	Stability	R	R	R	R	U	R	U
	Preference vector	454	458	469	405	453	409	457
	UI's					**454**		**458**
Maharashtra	Stability	R	R	R	U	U	R	U
	Preference vector	458	457	409	405	453	469	454
	UI's				**409**	**457**		**458**
Andhra Pradesh	Stability	R	U	R	U	R	R	U
	Preference vector	469	405	409	457	454	458	453
	UI's		**469**		**409**			**469**
								405
Environment	Stability	R	R	R	R	R	R	R
	Preference vector	469	405	453	409	457	454	458
	UI's							

UI's entries have been made bold as it represents the possible solutions of the game theory and these bold entries are used as input in the next step (stability analysis in Table 8)

Table 8 Stability analysis of the hypergame consisting of games (G_1, G_2, G_3 and G_4) from the perspective of all four players and its equilibrium solution denoted as E

Outcome	454	458	469	405	453	409	457
Karnataka	U	R	R	U	U	E	U
Maharashtra	U	R	R	U	U	E	U
Andhra Pradesh	U	R	R	U	U	U	U
Environment	R	R	R	R	R	R	R
Overall equilibrium	X	E	E	X	X	X	X

U unstable outcome, *R* rational outcome, *E* equilibrium outcome, *X* non equilibrium outcome

comes which are stable in all the four games will eventually be the equilibrium outcome as shown in table 8.

Final solution $= 458, 469$

4.4 Hypergame analysis for scenario 2 (Andhra Pradesh and Telangana)

The options in the preference are taken from Table 4 as discussed in Sect. 4.2. Hypergame arises here because there is a difference in the preference vector ordering as perceived by AP about the preference vector of Telangana.

Consider the following 1st level hypergame:

$H = \{G_1, G_2\}$, where $G_1 = \{V_11, V_21\}$ and $G_2 = \{V_21, V_22\}$

V_ij: Perceptions carried by player j about the orderings of preference vector of player i.

As discussed in Sect. 4.2 in Table 2, the preference vector V_11 and V_22 are the original preferences by Andhra Pradesh and Telangana of their games respectively so it is same as discussed in Table 2. V_21 and V_12 represents the perceptions as perceived by Andhra Pradesh and Telangana of each other's game, respectively

$V_11 = \{21, 13, 17, 9, 20, 12\}$

Table 9 Preference vector ordering for both the player according to their perceptions about the other players' options, strategies and preferences

Andhra Pradesh ($V_1 1$)	Telangana ($V_2 2$)	Andhra Pradesh ($V_2 1$)	Telangana ($V_1 2$)
21	13	9	21
13	9	17	13
17	12	12	17
9	21	20	9
20	17	13	20
12	20	21	12

Table 10 Stability analysis of the hypergame consisting of games from the perspective of both the players (Andhra Pradesh and Telangana) and its equilibrium solution denoted as E

Outcome	21	13	17	9	20	12
Andhra Pradesh	U	R	U	U	U	U
Telangana	U	R	U	U	U	U
Overall equilibrium	X	E	X	X	X	X

U unstable outcome, *R* rational outcome, *E* equilibrium outcome, *X* non-equilibrium outcome

$V_2 1 = \{17, 20, 21, 13, 9, 12\}$
$V_1 2 = \{21, 13, 17, 9, 20, 12\}$
$V_2 2 = \{13, 9, 12, 17, 20, 21\}$

4.4.1 Preference vectors according to the changed perceptions

As discussed in scenario 4.1, each player can only perceive the preference vector to the best of her knowledge. All the possible perceptions are shown in Table 9.

4.4.2 Stability analysis for the overall hypergame for scenario II

The stability analysis is carried out in the same manner as in Sect. 4.1. The details are shown separately in the supplementary section for the sake of brevity. Then the hypergame analysis is done as shown in Table 10. The equilibrium outcome is the intersection of the stable outcomes of the two games G1(Game from the perspective of Andhra Pradesh) and G2 (Game from the perspective of Telangana).

$H = \{G_1, G_2\}$

The final solution is 13. This solution comes out to be a "Fair and Equitable" (F&E). This shows the efficacy of the F&E allocations.

5 Discussion

The present work explores the potential of hypergame analysis as a game theoretic tool to solve the dispute of the Krishna water. As contrary to the Metagame analysis [15] where each player has complete knowledge regarding the game of other players, players tend to conceal their actual strategies in the hypergames. Authors have first derived the 'Fair and Equitable' (F&E) allocation and offered it as a potential solution of the present conflict which is solved using hypergame analysis. F&E allocation is very important part of the work as without it being offered as a solution, there is no equilibrium solution as seen in the scenarios I and scenario II, respectively. Hence, the present analysis also validates the efficacy of the F&E solution. This is indeed true for any conflict that if players recognize the need of cooperation, then there can be lasting solution to the conflict. This methodology though applied here for the river water sharing, will be effective for any conflict scenario provided fairness and equity are used in the conflict resolution.

The fairness aspect is addressed by the derivation of fair and equitable allocation with the help of cooperated negotiations as seen in Table 3. Here the states are themselves selecting the factors (which is agreed by all the players) based on their contribution to the basin development and these chosen factors maximizes their share. Now as seen in Table 3 that if all the states select their first preference, the total allocation is slightly more than 120 %. But since the resources under consideration is cannot be replaced or fulfilled by any other means, states recognize the importance of cooperated negotiation. They negotiate and considers the average of their top two factors. But even this allocation exceeds the available resource. States finally come down to second level of compromise and here the allocation is equal to the resource available. After this, the hypergame analysis is carried out using two scenarios.

The first scenario considers the hypergame amongst the erstwhile parties of the game which were present before the reorganization of Andhra Pradesh (in 2014) but including the environment as the legitimate player of the game and the second scenario considers the hypergame between the newly reorganized states of Andhra Pradesh and Telangana. In hypergame analysis, the players do not know the preference vectors of the other players, they can only form perceptions of the preference vectors which may be correct or incorrect. This is the beauty of the hypergame analysis that it brings out varying perceptions in the game which may eventually turn out to be misperceptions. Hypergame is the set of many Metagames where each Metagame is due to the varying perceptions of the players. Hence, hypergame is modelled as a game of perceptions. The final equilibrium outcome is obtained from the stability analysis of these games of perception.

Scenario I considered includes erstwhile Andhra Pradesh, Maharashtra, Karnataka and environment as the players. The options of the players consist of their various demand and also includes the F&E allocation worked out for each player. Then the decimalized preference vector has been used based on these options. Decimalized value is a succinct representation of the vector. For example, vector (0, 0, 1, 0, 1) can be decimalized as: $0 \times 2^0 + 0 \times 2^1 + 1 \times 2^2 \times 0 \times 2^3 + 1 \times 2^4 = 20$. This simplifies the long vector representation. Now, the preferences order of one player may be perceived differently by other players. Hence, preference vector based on these perceptions has been formulated. This is denoted by V_{ij}. Then all these perceptions games are modelled. Only the 1st level hypergame has been analysed. G_1, G_2, G_3 and G_4 represent the game as perceived by player 1, 2 3 and 4 respectively. The solutions of these individual game are then used as input for the hypergame (H). Finally, the solution is obtained. Outcome which is stable for all the players in all the games will eventually be the equilibrium outcome. It is denoted as E in Tables 8 and 10 for scenarios I and II respectively. There may also be a case that due to the nature of the options chosen, there may be no equilibrium solution. Hypergame will be given the equilibrium solution if it exists. For the Scenario I, the equilibrium solution comes to be 458 and 469. These outcomes are rational for all the games denoted as R and hence is the equilibrium outcome. Similarly, outcome 469 is also rational in all the games and is also the second equilibrium outcome. That is why hypergame analysis is more complex as compared to the Metagame analysis. Outcome 469 corresponds to 'Fair and Equitable' allocations while 458 is the case when Andhra Pradesh gets only 'Prior appropriation doctrine' option while Maharashtra it is 'Fair and Equitable' share and Karnataka gets its maximum share. Outcome 469 once again shows that the 'Fair and Equitable' regime is an extremely potent tool and can act as a panacea to this conflict. Outcome 458 though may be unacceptable to Andhra Pradesh but this should make her realize that Andhra Pradesh has to lower its demand and make it consistent with the flows available.

Scenario II considers the hypergame between the newly reorganized states of Andhra Pradesh and Telangana (after 2014). As all the players have the right to fair procedure and to be an equal part of the game, this scenario is worked out. Hypergame arises here because there is a difference in the preference vector ordering as perceived by Andhra Pradesh about the preference vector of Telangana. Andhra Pradesh thinks that Telangana will not allow Andhra Pradesh to take both the 'Fair and Equitable' and prior appropriation doctrine' options because then it will not get the carryover from the excess flow from the upstream. Since the second option for Telangana is the demand of 306 TMC plus carryover. So, the V_{21} vector is formed accordingly. Stability analysis is carried out and outcome 13 ('Fair and Equitable' allocation). This concludes the hypergame analysis for both the scenarios.

The most important point to ponder here is that even though the players are forming perceptions about the other player's game but there has to be due respect given to this. If players will simply overestimate their game and undermine other players game, there can never be an equilibrium outcome. . Here even though the players are making perceptions (which may turn out to be misperceptions) regarding the other player's choices, but in doing so, it has been ensured that they respect another player's claim and see it as a legitimate one. Perceptions also reflect the fact that the resource under consideration is limited and the most important fact guiding the perceptions is that all the players want to maximize their share. Also in Sect. 4.3, game 4 under the perceptions of environment, all the outcomes are coming to be stable as if the required environmental flow is released, environment is always content and ready to accept the different scenarios. The results of the hypergame analysis show that the 'Fair and Equitable' allocation derived comes out to be the equilibrium outcomes in differing perception scenarios as well. This shows that the best solution for any dispute is when the parties themselves come and discuss on the table by having all the cards with them (either on the table {Metagame} or few cards in hand {Hypergame}).

6 Conclusion

The study has derived what can be called a 'Fair and Equitable allocation' and presented an elegant application of the game theoretic technique to solve a complex real-world dispute in the form of the hypergame analysis. 'Fair and Equitable' allocation ensures fairness while hypergame analysis considers the equity. Hypergame is formulated by choosing the options selected by each player. But since they are involved in a competition, they do not want to disclose their strategy. Hence, each player can only form a perception of the other players' game. Then the hypergame analysis is carried out which considers all the possible perceptions of all the players. Finally, the equilibrium solution is worked out. It is seen that Fair and equitable allocation comes out to be one of the solutions. It is imperative to note that hypergame analysis, like the Metagame Analysis, gives the equilibrium solution only if it exists. It cannot force a particular solution on the players. This is a very important aspect of any conflict resolution technique. Equilibrium outcome validates the efficacy of the "Fair and Equitable" allocation as the conflict resolution outcome. This concludes the present work.

The study has ignored the possibility of a coalition between two or more players in the game. As a conflict analysis methodology, coalition analysis maybe a recommended future addition that studies how a game would develop these coalition partners plan and execute joint strategies. The operating reservoir policy for each reservoir can be made so that the time and the quantity of water to be released can be found out. Water Trading can be further explored and implemented for any of the scenarios discussed in the present study.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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