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Peer pressure in extortion game can resolve social dilemma

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Abstract. The extortion strategy is an important subset of the zero-determinant strategy, which ensures that the participant gets no less than its opponent's payoff, attracting the attention of many scholars. Peer pressure has proved to be an effective mechanism for maintaining cooperation between selfish individuals in evolutionary game dynamics. Therefore, in this paper, we use punishment to imitate peer pressure to study the influence of peer pressure on the evolution of cooperation in the extortion strategy. Peer pressure can be simulated with punishment and use α to control the punishment intensity. The simulation results show that the punishment of the extortioner plays a key role in the evolution of the cooperative strategy. When α is small, the punishment of the extortioner will make the system enter a three-state cycle which similar to that of rock–scissor–paper, greatly promoting cooperation. When α is large, the extortioner will dominant the entire population by punishing cooperator and defector. In addition, proper punishment will make the cooperator dominant the entire population when b is small.

1 Introduction

Cooperative behavior is ubiquitous in nature and complex and orderly human society. Whether it is cells, microorganisms, social insects, or social activities between people, there are cooperative and cooperative characteristics [1]. However, according to Darwin's theory of natural selection, individuals will become selfish when driven by self-interest during the evolution process, and there will be no altruistic cooperation between individuals [2]. This contradicts the phenomenon of cooperation existing in nature, so a suitable theory is needed to explain this widespread phenomenon of cooperation. Evolutionary game theory is an interesting method proposed on the background of this problem, which provides a powerful framework for solving this contradiction [3,54]. The prisoner's dilemma game (PDG) concisely describes the essential dilemma of cooperation, which has caused a lot of research. In the PDG, two participants choose to cooperate or defect at the same time. If they cooperate with each other, they will both get a reward R; if they defect each other, both players will get a punishment P; if one participant cooperates, and the other one defects, the cooperator gets a lower sucker's payoff S, and the defector will get a higher temptation benefit T. These four parameters satisfy the relationship $T > R > P > S$ and $2R > T + S$. When the individual is rational, defection is the best choice, but it will lead to a great loss of overall interests and form a social dilemma.

The core research question of evolutionary game theory is how cooperative behavior evolves. In recent years, more and more scholars have used this theory to study the mechanism of promoting the emergence of cooperation. Nowak et al. [4] summarized five mechanisms that promote the emergence of cooperation: kin selection [5], direct reciprocity [6], indirect reciprocity [7], network reciprocity [8] and group selection [9]. In addition, to explore the persistence and emergence of cooperative behavior, more mechanisms have been proposed. For example, reputation [10], voluntary participation [11], aspiration [12,13], asymmetry [14], rewards and punishments [15,16], environmental factors [17,37,55]. Besides social mechanism, network structure also attracted much attention [18] to explore its influence on the evolution of cooperation, such as small world networks [19], interdependent network [28], scale-free network [21], and so on. In addition, edge dynamics have also been studied in ref [22–25]. Edge dynamic takes into account other information related to social connections (such as geographic proximity, proximity of individual relationships), and the outcome of the interaction will not be solely determined by the individual's strategy.

Recently, Press and Dyson proposed a zero-determinant (ZD) strategy, which can unilaterally limit the

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returns of both parties to satisfy a certain linear relationship [26]. The extortion strategy is a subset of the *ZD* strategy, which ensures that one can get no lower payoffs than his opponent. Therefore, the extortion strategy has the potential to dominate any evolutionary opponent, which has attracted great attention. Adami and Hintze found that in a well-mixed population, the extortion strategy will be invaded by the defection strategy, and the evolution of the extortion strategy is unstable [27]. However, Hilbe et al. pointed out that extortion strategies can act as a catalyst in promoting the emergence of cooperation [20]. In addition, some literature indicates that extortion strategies can not only steadily evolve, but also promote the emergence of cooperation [51,52]. Extortion strategies provide a new perspective on the evolution of research cooperation.

Many theoretical [31–34,36] and experimental [38–42] studies have shown that punishment mechanism can effectively promote the maintenance and development of cooperative behavior. Punishment is to punish cooperators or defectors (usually defectors), the punished person needs to pay a fine, and the punisher also pays a certain price. Wang et al. studied the impact of social punishment [43] on cooperation and found that punishment strategies in the prisoner's dilemma game and public goods game models can well promote the evolution of cooperative behavior. In addition, Yang et al. proposed a symmetric punishment mechanism [44], in which each individual will punish neighbors who hold the opposite strategy, and the results show that appropriate punishment can enhance cooperation. Inspired by the above research, we introduce punishment into the extortion strategy to study its impact on individual behavior.

The following content consists of three parts. In Sect. 2, we introduce game models and strategies. Section 3 will give the results of numerical simulation. Finally, the full text is summarized.

2 Model

We investigate the evolution of extortion strategies with punishment parameters on a square lattice of size $L \times L$ with periodic boundary conditions, where extortion was studied in the realm of the donation game [45]. Each individual has three choices: unconditional cooperation (C), unconditional defection (D) or extortion (E_χ) [46], which are described by

$$S_x = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \; or \; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{1}$$

respectively. In the donation game, cooperator will pay a cost c and provide a benefit b to its opponent; the defector will get benefit without any labor. The payoff matrix is

$$M = \begin{pmatrix} & C & D & E_\chi \\ C & b-c & -c & \frac{b^2-c^2}{b\chi+c} \\ D & b & 0 & 0 \\ E_\chi & \frac{(b^2-c^2)\chi}{b\chi+c} & 0 & 0 \end{pmatrix}, \tag{2}$$

where b $(1 < b \leq 2)$ is the temptation to defector and χ stands for the extortion factor. Since $\chi > 1$, there is a relationship between cooperative strategy and extortion strategy similar to snowdrift game: the optimal response of an individual is to choose the strategy opposite to his opponent. If y cooperates, x should choose extortion; if y chooses extortion, then x should cooperate. On the other hand, when the defection strategy meets the extortion strategy, they will get nothing, presenting a neutral drift relationship [47]. Following previous works [30,50], we set $\chi = 1.5$ and $b - c = 1$ so that there is only one parameters in the payment matrix that is b.

In our model, we consider the impact of peer pressure on player's payoff. Furthermore, peer pressure causes individuals to lose benefits as a punishment behavior, which is different from the punishment strategies in previous studies [15,48]. At each time step, an individual will punish the neighbors that hold different strategies. But the extortioners will not be punished. Therefore, the cumulative payoff of player x can be expressed as

$$P_x = \begin{cases} \sum\limits_{y \in \Omega_x} \left[S_x^T M S_y - \alpha \left(1 - S_x^T S_y\right)\right], S_x = C, D, \\ \sum\limits_{y \in \Omega_x} S_x^T M S_y, S_x = \mathrm{E}_\chi, \end{cases} \tag{3}$$

where Ω_x represents the sum of the nearest neighbors from player x, α is the punishment parameter. When $\alpha = 0$, it returns to the traditional situation, at this time, the extortion strategy cannot exist stably in the network, and the extortion strategy has no special effect on the evolution of cooperation [51,52]. So, the value of α is greater than 0 in this article.

The game iterates according to the Monte Carlo (MC) simulation program and adopts the asynchronous update process to update strategy. The specific steps are as follows: first, a randomly select player x accumulates payoff P_x by playing games with its four nearest neighbors. Then select a player y randomly from the four neighbors of x and accumulate the payoff P_y in the same way. Finally, the probability that player x choosing neighbor y's strategy S_y is given by Fermi function:

$$W\left(S_x \leftarrow S_y\right) = \frac{1}{1 + \exp\left[\left(P_x - P_y\right)/\mathrm{K}\right]}, \tag{4}$$

where K represents the noise level in different environments, which is set as 0.1 in this paper.

Initially, the proportion of cooperators, defectors and extortioners was the same in the population. Monte Carlo simulation results are obtained in a population of 800 × 800 individuals, and the stationary fraction of three strategies are determined by the average of the

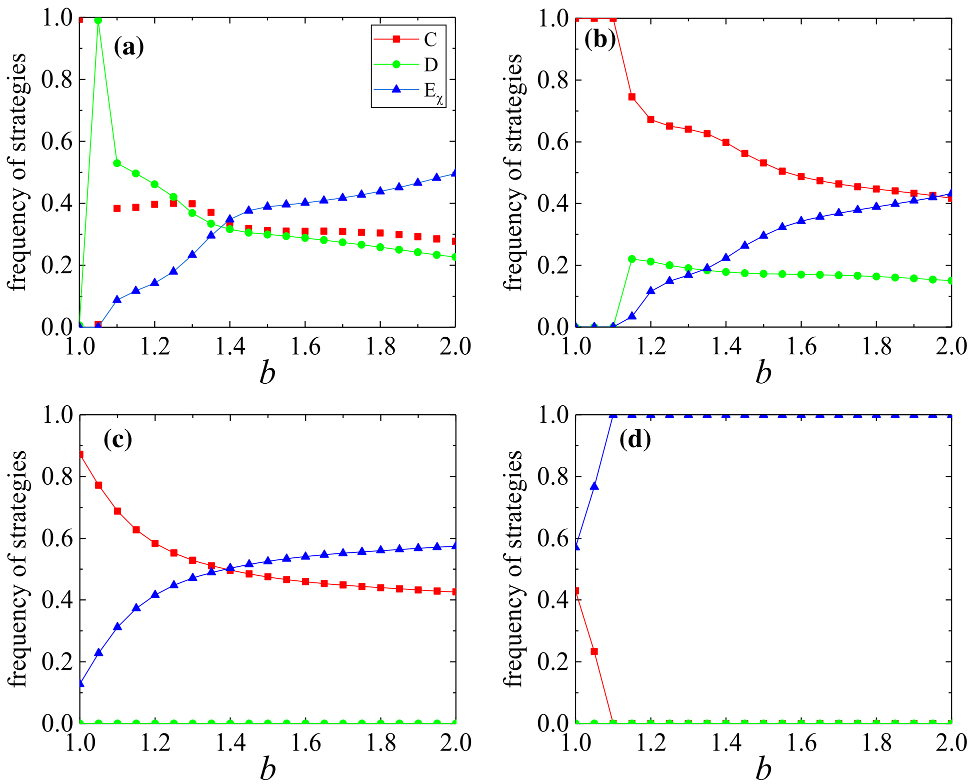

Fig. 1 Average frequency of cooperators (red), defectors (green), and extortioners (blue) as a function of b for several different values of α. From **a** to **d**, the values of α are 0.1, 0.3, 0.6 and 0.7. All the results are obtained for $K = 0.1$ and $L = 800$

last 3×10^3 results of 3×10^4 full MC steps. In addition, to avoid additional interference and ensure appropriate accuracy, the simulation results are obtained through an average of 20 independent operations.

3 Results

First, we consider the impact of peer pressure on the evolution of cooperation. Figure 1 shows the frequency of the three strategies with the temptation of defect b for different values of parameter α. As can be seen from Fig. 1a, when $\alpha = 0.1$, the frequency of cooperators decreases rapidly or even disappears when b is relatively small. But when the extortioner appears, the cooperator comes along with it. Even when $b = 2$, the cooperator can also survive. As the punishment intensity increases ($\alpha = 0.3$), it can be seen from Fig. 1b that the cooperator can completely dominate the system when b is small, the defector is suppressed, and the cooperator has an advantage in the entire system. When the punishment is further increased, the defector disappears, and the extortioner gradually increases because he is not punished (see Fig. 1c). When $\alpha = 0.7$, the cooperator in Fig. 1d disappears with the increase of b, and the extortioners completely dominates the system. Therefore, the introduction of punishment will promote the evolution of cooperation and solve social dilemmas, but the intensity of punishment should be maintained in a moderate range. Larger punishment intensity is not conducive to the emergence and maintenance of cooperation.

How does the peer pressure mechanism promote the evolution of cooperation? Here, we study the evolution of microcosmic cooperation. Figure 2 shows the time evolution of three different strategies with different α values. Figure 2a shows that when $\alpha = 0.1$, the initial stage cooperator will first reach the peak. This is because the extortioner punishes the defector, resulting in the decline of the defector, and the dominance of the cooperators causes the extortioner to become a cooperator. Later, as the number of extortioners decreased, the number of defectors quickly increased by exploiting cooperators. The increase of defectors will lead to the increase of extortioners, and the last three strategies will coexist in a stable state. In Fig. 2b, after the extortioner wiped out the defector, he was wiped out by the cooperator, and the cooperator ruled the entire population. In Fig. 2c, d, the extortioners rules the entire population by punishing cooperators and defectors. From this, we can see that the punishment of extortioners drives the evolution of cooperation.

To further observe the effect of peer pressure on the overall evolutionary process, Fig. 3 shows the change of the proportion of each strategy under the corresponding b and α values through the phase graph. In Fig. 3a, we can see that punishment significantly contributes

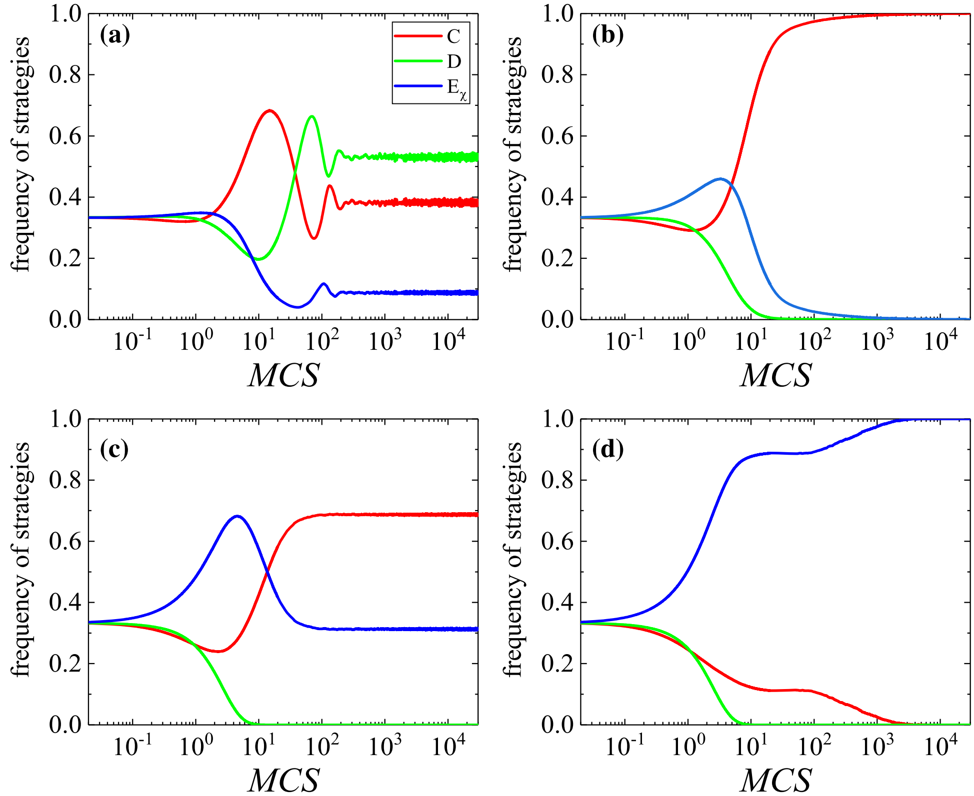

Fig. 2 Time evolution of cooperators (red), defectors (green), and extortioners (blue) on square lattices for $K = 0.1$ and $L = 800$ From (**a**) to (**d**), the values of α are equal to 0.1, 0.3, 0.6 and 0.7

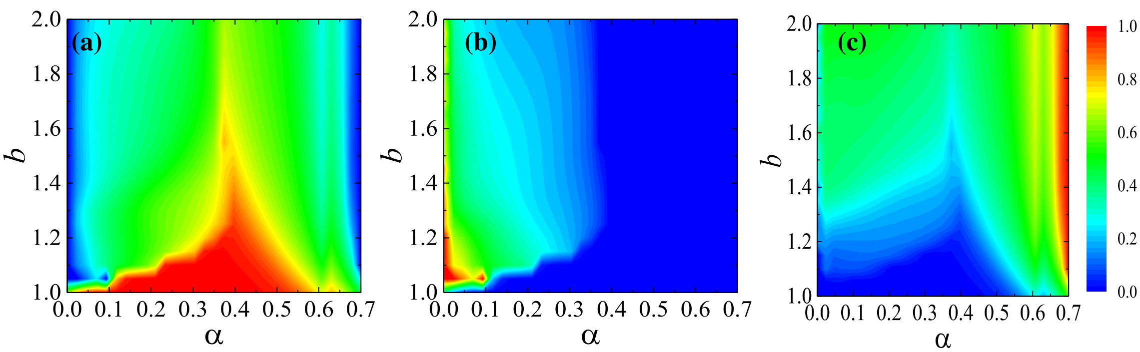

Fig. 3 The final density of three strategies evolves with the temptation to defect b and the penalty parameter α. From (**a**) to (**c**), they are cooperators, defectors and extortioners

to the evolution of cooperation. When the punishment parameters are moderate, the cooperator can rule the entire network with a small value of b. Even when $b = 2$, cooperators have a higher frequency. The defector disappeared in the population as the value of α increased. Interestingly, the extortioner plays a special role. The extortioner helps the cooperator to eliminate the defector when the α value is small, and then exploits the cooperator to occupy the entire population. Therefore, the punishment of the extortioner plays a key role in the evolution of the cooperative strategy.

To study the effect of this mechanism on the spatial distribution of strategies on a grid network, Fig. 4 shows a snapshot of the characteristics of three strategies over time at different values of α. In the process of evolution, we will fix the initial distribution of the entire population. The circle in the middle indicates a cooperator, the upper half indicates a defector, and the lower half indicates a extortioner. When $\alpha = 0.1$, from Fig. 4a1–e1, we find that the defector area is invaded by the extortioner and the cooperator area is invaded by the defector, which results in a three-state cycle similar to rock–scissor–paper. This is because punishment alters individual payoff, and individual payoff determines the course of strategy change. References [29,49,53] provide more explanation for the interaction of cycles. As α increases, a different situation occurs. When $\alpha = 0.3$ (Fig. 4a2–e2), after the defector is completely invaded

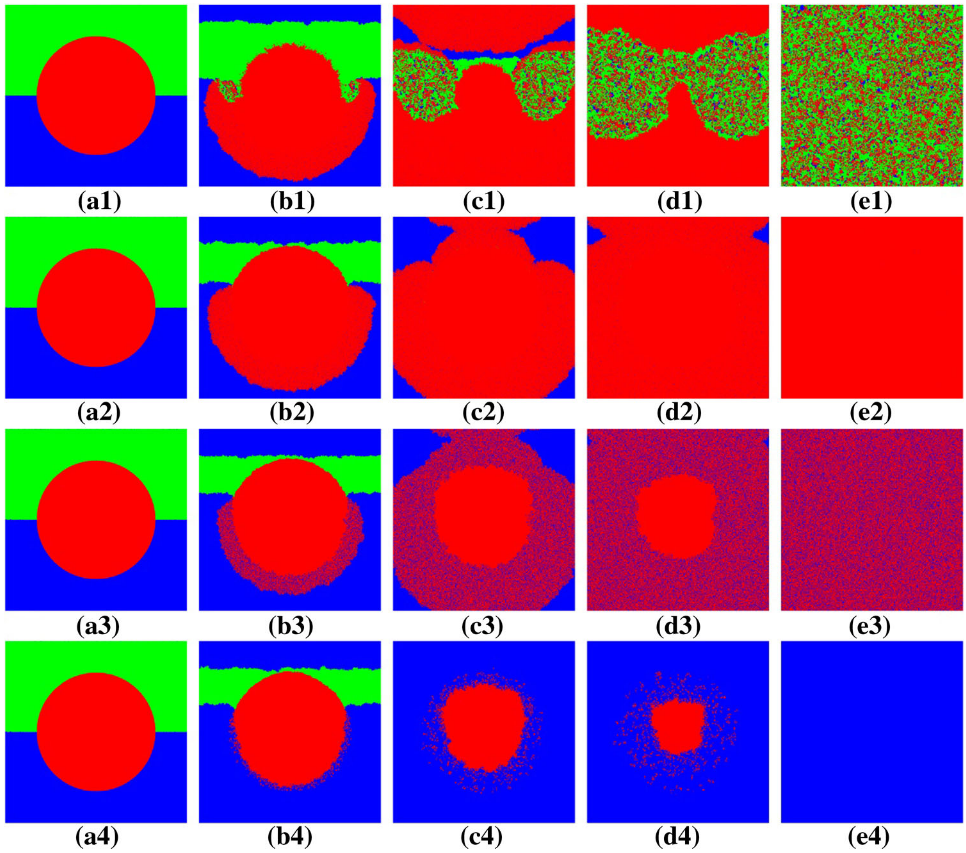

Fig. 4 Evolution snapshots of the clusters of cooperators (red), defectors (green) and extortioners (blue) on square lattice when $\alpha = 0.1, 0.3, 0.6, 0.7$ from top to bottom. From left to right, the time steps are 0, 200, 600, 1000 and 29999, respectively, for $K = 0.1$ and $b = 1.1$

by the extortioner, the cooperation strategy will prevail and the extortioner will be eliminated. When $\alpha = 0.6$ (Fig. 4a3–e3), the cooperators form a small cluster to resist the intrusion of extortioners, and finally the two coexist in the network. When $\alpha = 0.7$ (Fig. 4a4–e4), the cooperators are surrounded and finally invaded by the extortioner. It can be seen that appropriate punishment will promote cooperation, while larger punishment will reverse the invasion process of cooperators and extortioners, leading to the disappearance of collaborators.

The evolutionary pattern shows the distribution of individuals on the network, but the number of specific clusters and the number of individuals in the cluster are not available. Therefore, we use C clusters, D clusters and E_χ clusters to denote clusters composed of cooperators, defectors and extortioners. Then Fig. 5 shows the evolution of the number of pure strategy clusters (N_C) and the number of pure strategy players (L_C) corresponding to the largest pure strategy clusters as the b value changes. Figure 5b shows that the L_C of the cooperators and extortioners is very small, and the L_C of the cooperators is large before $b = 1.55$, which shows that the cooperators can exist as larger clusters when b is small. Even when the value of b is not greater than 1.1, $N_C = 0$ and $L_C = 0$ of the defector and the extortioner, while $N_C = 1$ and $L_C = 640000$ of the cooperators, at this time, cooperators gather into a strong cluster to occupy the entire network, this is the same as the results in Figs. 1b and 4. As b increases, the N_C of cooperators and extortioners increases, which means that the cooperators and extortioners use more clusters to resist intrusion. The extortioner's N_C peaks around $b = 1.5$, and L_C has been increasing, indicating that the extortioner has an advantage when b is large.

4 Conclusions

Based on previous research, we introduced peer pressure into the extortion strategy. A large number of simulation results show that punishment can effectively promote the evolution of cooperation, and extortioners plays a key role in the evolution of cooperation strategies. Specifically, when α is small, the punishment of the extortioner will cause the system to enter a three-state cycle similar to rock–scissor–paper, greatly promoting cooperation. Appropriate punishment will make the cooperator dominant the entire population when b is small, even if b is large, the cooperator has an abso-

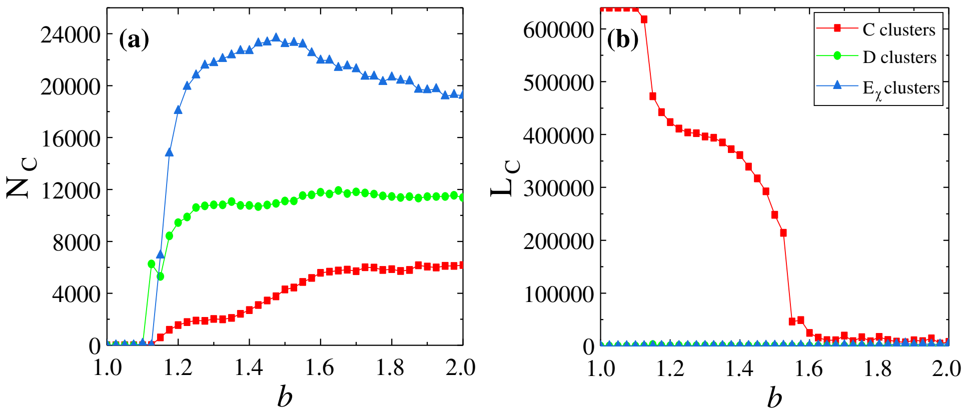

Fig. 5 Number of clusters of pure strategist N_C and number of the largest cluster of pure strategist L_C dependent on b on the regular lattice. All the results are obtained for $\alpha = 0.3$, $K = 0.1$ and $L = 800$

lute advantage. However, a larger value of α will cause the extortioner to dominant the entire population by punishing cooperator and defector. Our work provides a new situation for the extortioners' bleak prospects under the imitation update rules. The extortioners can exist stably in the network and can promote the evolution of cooperation. We hope this work will inspire more researches to solve the problem of social dilemma.

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