

Mendel: Corroboration of the Idea of Binary Trait Coding by Methods of Statistical Physics

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Abstract—The paper by the Augustinian friar Gregor Johann Mendel “Experiments on Plant Hybridization” laid the foundation of the new field of knowledge, genetics, 150 years ago. It claimed that any characteristic is determined by two factors. On the one hand, the Mendelian idea of the binary coding of characteristics was inspired by Christian Doppler, with whose department Mendel had been in contact for seven years. On the other hand, the regularities discovered by Mendel confirmed the intuitive notion of the divine principle based on rational foundations of Pythagoras living in the 6th century AD, who was the first to point to the spiritual grounds of being: the world was created by the number, and the number is a nonmaterial and insensuous entity. All students of heredity before Mendel traced the fate of a characteristic in the succession of generations. Instead, in order to unveil the heredity mechanism, Mendel traced the fates of two invisible factors that determined the characteristic. Probably, the ideas of binary combinations and mathematical probabilistic variants arose from Mendel’s long meditation and an imaginary experiment. Experiments on pea crosses were undertaken in order to test this speculative idea of a set of invisible determinants. Methods borrowed from statistical physics allowed Mendel to decipher the process occurring in the experiments with the pea model: the fate of a characteristic is determined by the action of two invisible factors.

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It is an extremely rare case in the history of world science when a single paper gives rise to a whole new area of research and opens up a work program for many decades to come. Such was the paper of the Czech friar Johann Gregor Mendel (1822–1884), “Experiments on Plant Hybridization,” which 150 years ago, in 1865, laid the first stone in the knowledge of inheritance and determined the birth of a new science, genetics.

In 1822, in Moravia, in the village of Hynčice (Heinzendorf), a second child, a boy Johann, was born in a peasant family (the Mendels). Johann was used to peasant labor in childhood. The skills he acquired in childhood were of much use to him later.

Mendel was 11 years old when he was transferred from the village school to a four-year secondary school in the nearest town, and then to the gymnasium in the city of Opava. It was difficult for his parents to support their son and pay for his studies. In 1840, Johann graduated from the gymnasium and the school of candidate teachers at the same time. As he said himself, it provided him a modest existence.

Overcoming difficulties, Mendel continued his studies: first, in the philosophical school in the city of Olomouc, where in addition to philosophy he studied mathematics and physics, then in the Brunn Theological Institute in 1844–1848. At 21, taking his vows, he took on a new name, Gregor.

Studying at the University of Vienna, he traveled to Rome (where he was presented to the Pope) to participate in scientific congresses and floral tours.

In March 1853, being an extern at the University of Vienna, he learned to dye preparations in the laboratory of the famous cytologist Franz Unger, one of the first cytologists in the world. Making preparations was not the only activity in the lab. Along with the problems of the microscopic scale, Professor Unger was interested in questions about the driving forces of evolution, as he tried to delineate the path of life from primitive creatures to humans; as a result, he published his research in *Weiner Zeitung* (Vienna Newspaper) in the form of Seventeen Botanical Letters.

Over eight years (1856–1863), Mendel carried out his famous experiments on sowing peas (*Pisum sativum*) as a model object and formulated the law that states that any characteristic is determined by two factors which he called *elementen*—elements or anlages.

CAN WE ASSUME THAT MENDEL HAD PREDECESSORS?

Predecessors-Theorists

It seems that he did have predecessors. First of all, speculative hypotheses about the material basis of heredity, the idea of idioplasm put forward in 1860 by botanist Carl Nägeli from Munich; the special physio-

logical units of the English philosopher Herbert Spencer; the provisional hypothesis of pangenesis put forward by Charles Darwin, and then, as opposed to them, supported by German zoologist and evolutionary theory theorist August Weismann. For all the dissimilarities, and in some cases the fundamental incompatibility of these hypotheses, they were united by one important feature—the desire to find the *material* basis of heredity as the interaction of elementary biological units. Nägeli called them *micelles*; Spencer, *physiological units*; and Darwin, *gemmules*, *grains*.

With his ideas about idioplasm, C. Nägeli tried to justify his mechanical-Lamarckian theory of evolution. He believed that the transmission of hereditary characteristics is carried out by micelles (molecules of the crystalline form), whose totality is represented by the idioplasm contained in the sex cells. According to Nägeli, the plasma of somatic cells (trophoplasm) does not possess the ability of heredity; thus, the changes evoked therein under the influence of the external environment are nonhereditary (modifications). The idioplasm may hereditarily respond both as a result of external factors and spontaneously due to internal (autogenetic) reasons (Nägeli, 1866).

In fact H. Spencer also proposed a mechanical-Lamarckian hypothesis of the physiological units that are contained in both somatic and germ cells that undergo changes under the influence of the external environment (Spencer, 1899).

According to the ideas of Charles Darwin, special self-replicating corpuscles of the hereditary substance (*gemmules*), which separate from all cells of the body, form its hereditary basis, concentrating in the reproductive organs and being subjected to changes under the influence of the environment.

Predecessors-Experimenters

Speculation about the laws of heredity appeared in the 18th century in the first plant hybridizers. In the European literature, the hybridological effects were first described by German botanist Joseph Gottlieb Kölreuter in the experiments on the crossing of China pink and Sweet William, as well as different varieties of tobacco. Kölreuter observed the phenomena of uniformity of the characteristics in hybrids in the first generation and the emergence of parental forms in subsequent generations. He showed that in the crossing or cross-pollination of two different varieties of carnations the offspring of the first generation clearly acquired features of one of the parents, a terry flower. However, in the second generation, obtained from the self-pollinated hybrids, some of the plants revealed the characteristics of another initial variety, China pink. Kölreuter meticulously recorded this phenomenon as follows: the characteristics of the initial varieties do not disappear in the offspring, but for some reason they may either appear or not, as if competing with each other (Kölreuter, 1766; Kölreuter, 1940). How-

ever, he mistakenly interpreted this as a gradual return to the original parent species, which he considered to be invariable.

After Kölreuter, the dominance of the characteristics of one of the parents in the first generation of hybrids and the appearance of the characteristics of the other parent in the second and subsequent generations was discovered by the British breeder and grower Thomas Andrew Knight. Through the cross-breeding of different varieties of peas, Knight made an important observation. He found the indivisibility of small characteristics at various crossings. Proclaimed in ancient times, the discreteness of the hereditary material was first scientifically grounded in his works.

Before Mendel, in the middle of the 19th century, during the crossing of different varieties from the family of cucurbits, French botanists Augustin Sageret and Charles Victor Naudin found that all first-generation hybrids are alike. The main achievement of Sageret was the discovery of the phenomenon of dominance. When crossing varieties of vegetable crops, he often observed the suppression of a characteristic of one parent by the characteristic of another. To the greatest extent, this phenomenon manifested itself in the first generation after the crossing, and then the suppressed characteristics were again detected in some of the next-generation descendants. Thus, Sageret managed to confirm that elementary hereditary traits do not disappear in crossing.

This conclusion was reached by Charles Naudin, who, however, went even further, starting a quantitative study of the recombination of hereditary traits in crosses. However, along the way he was disappointed. An invalid methodical approach, the simultaneous study of a large number of characteristics, led to great confusion in the results, and he was forced to abandon his experiments. These were the flaws in Naudin's experiments that Mendel had taken into account before choosing peas (*Pisum sativum*) as a model object to confirm his thoughts.

In 1861, the Paris Academy of Sciences announced a contest titled "Study of Plant Hybrids in Terms of Their Fertility and the Permanence or Impermanence of Their Traits." The objective of the contest was to "do a number of exact studies" and, among others, to answer the following question: do the hybrids breeding by autogamy for several generations maintain the same traits ... or, on the contrary, do they always return to the forms of their ancestors? The work of Charles Naudin "New Research on Hybrid Plants" won this contest.

The 200-page paper of Naudin had fairly definite answers to the questions posed by the competition commission, namely, (1) the first generation of hybrids exhibits a similarity of all the descendants and their consistency; (2) starting from the second generation hybrid forms expand to the original parent types; and (3) the return to the parent forms and the emergence of new combinations are associated with the separation of the entities (hereditary anlagen).

Anyone familiar with the basics of genetics understands that the conclusions of Naudin generally correspond to the laws of inheritance of the characteristics established in the work of Mendel. It is no coincidence that Charles Darwin himself was in communication with Naudin and quoted him. In the first chapter of “On the Origin of Species ...,” considering the difficulty in distinguishing between species, Darwin writes: “The offspring from the first cross between two pure breeds is tolerably and sometimes (as I have found with pigeons) quite uniform in characteristics, and everything seems simple enough; but when these mongrels are crossed with one another for several generations, hardly two of them are alike, and then the difficulty of the task becomes manifest” (Darwin, 1859).

Thus, it is assumed that there were predecessors of the Mendelian experiments. However, it must be said that the work of Mendel could also be found by other researchers. Many publications contained references to it, including the *Encyclopedia Britannica* of 1881–1885 (article on hybridism).

At the end of December 1866, “Proceedings of the Society of Naturalists of Brunn” were published, with a summary of Mendel’s report. They were sent to 120 university libraries and societies of scientists in Vienna, Prague, Berlin, Munich, London, Paris, St. Petersburg, New York ..., and Mendel sent another 40 reprints of his paper to private addresses—close friends and researchers-botanists. However, only three copies were unwrapped, and only one answer was received—from the famous botanist, a professor at the University of Munich (Carl Wilhelm von Nägeli, 1817–1891), who perceived Mendel’s work with skepticism, without appreciating the simplicity and elegance of his experiments. In his letter to Mendel, he indicated that he would believe in this discovery only if Mendel was able to reproduce his experiments on hawkweed (*Hieracium*), on which Nägeli worked at the time himself, and get similar results. Following the advice of Nägeli, Mendel engaged in crossing hawkweed and got results that differed from those observed in peas. Neither Nägeli nor Mendel (nor anyone else at the time) knew that seeds in *Hieracium* are produced asexually. It turned out later that *Hieracium* has facultative-apomictic reproduction, which explains the splitting of the first and the absence of splitting in the second generation of hybrids. Thus, because of his professionalism and the highest authority, Nägeli put up a barrier that proved to be insurmountable for Mendel’s ideas.

In Russia, Mendel’s work was highly appreciated as early as 1874, when it was quoted in a thesis and included in his lectures by a professor at Moscow State University, Johannes Theodor Schmalhausen (the father of the author of the theory of stabilizing selection Ivan Ivanovich Schmalhausen, who headed the Chair of Darwinism at the Department of Biology of Moscow State University until the August session of VASKhNIL in 1948).

Even the three rediscoverers of Mendelian laws of heredity, the Dutch botanist and evolutionist Hugo Marie de Vries, who conducted experiments with poppy, evening primrose, and Jimson weed (he was the first); the German Associate Professor Carl Erich Correns, who studied the segregation of characteristics in maize; and the Austrian botanist Erich Tschermak von Seysenegg, who analyzed inheritance of traits in peas, knew Mendel’s work long before 1900 but did not understand and appreciate its deeper meaning immediately. All of them formulated their own *law of segregation and the law of independent assortment*. Why did it happen? Why was the idea of Mendel and his published work not understood and adopted? The Dutch historian of science O. Meijer admits that it was due to “reading without understanding” (Meijer, 1985). Why this was the case was more specifically expressed by philosopher-evolutionist Yu.V. Chaikovskii: “... Mendel’s laws could barely be observed, but they formed the basis of genetics. Similarly, Ohm’s law works perfectly everywhere, but try to discover it in TV, where it is disguised by lots of other laws” (Chaikovskii, 2008, pp. 510–511).

IS IT FAIR TO SAY THAT MENDEL’S LAWS WERE REDISCOVERED?

World Fame ... 35 Years after Discovery

It turned out that 1900 was taken to be the date of birth of genetics, when De Vries, Correns, and Tschermak independently of each other repeated Mendel’s work. Why was Mendel’s work in demand after the rediscovery? It is believed that in 1900 Hugo de Vries, who by the time was the famous discoverer of the mutation process, was the first to send his article to print. Correns and Tschermak sent their papers to press after him.

De Vries sent his article to a scientific journal without any reference to Mendel’s work; however, it turned out that his rival Correns was a reviewer in this journal. He could not admit that De Vries made the discovery before him, so he found a reference to the work of Gregor Mendel and contrasted it to De Vries’ work. In his memoirs, C. Correns wrote that he learned about Mendel’s work only after 1899, when he thought about his experiences on the crossings carried out in the period from 1896 to 1899. However, the study of Correns’ workbooks found a brief abstract of Mendel’s work, dated April 1896! This record drew attention to the ratio of 3 : 1 in second generation hybrids. Apparently, Correns understood the importance of the ratio of 3 : 1 only after reading Hugo de Vries’ work.

The American geneticist A. Sturtevant, referring to the rediscovery of Mendel’s work in 1900, rightly observes that only Mendel’s article itself was rediscovered, but none of the rediscoverers fully understood the meaning and depth of its laws (Sturtevant, 1965).

IF THERE WERE PREDECESSORS,
THEN WHY, DESPITE THE CONSIDERABLE
NUMBER OF OBSERVATIONS ON CROSSES,
IS JOHANN GREGOR MENDEL RIGHTLY
CONSIDERED TO BE THE FOUNDER
OF HYBRIDOLOGICAL ANALYSIS?

Why is it considered that Gregor Mendel, rather than the winner of the scientific competition of the Paris Academy of Sciences Charles Naudin, is the founder of genetics? The experimental work of Naudin is more solid than Mendel's work and the reported data for many species of plants, while Mendel had, it would seem, a special case, only in one species, peas. Moreover, Charles Naudin found many interesting and important facts and a number of laws before Mendel. This can be explained.

(1) Developing the problem of heredity, all researchers prior to Mendel traced the fate of a *characteristic* in a number of generations. At the same time, for understanding the mechanism of heredity, Mendel traced in a number of generations the fate not of a characteristic but *two invisible factors-determinants* defining the characteristic (1 : 2 : 1). Those "invisible factors" that later, in 1910, at the suggestion of Danish botanist Wilhelm Ludwig Johannsen, would be called genes, from the Latin "gennaō," to give birth.

(2) Using statistical calculations, Mendel showed that there must be two invisible factors-determinants, i.e., a characteristic is binary-coded.

In his lectures, Nikolay Vladimirovich Timofeev-Ressovsky pointed out that the main point in Mendel's discovery was an "brash hypothesis" that there are *two elements*, one of which is dominant (Konstantinov, 1993).

An analogy can be made with an example of the classification of periodic elements. There are well-known unsuccessful attempts to group elements by their chemical properties, which were made before Mendeleev, Mendel's namesake. The genius of the great chemist consisted in the fact that he based the system of elements on their atomic weight, rather than their mass. Mendeleev, just as Mendel, also had certain predecessors. On the one hand, they were practitioners who created classification systems, but along the way they were concentrating on special problems and did not follow through to the end; i.e., they did not see or substantiate the existence of the periodic law. The tables they composed were used in empirical research. On the other hand, there were theoreticians. Closest to the discovery of the periodic system were Alexander Emil Beguter de Chancourtois and John Alexander Reina Newlands. De Chancourtois even partially built a number of elements in order of increasing atomic mass.

CAN WE CONSIDER THE CHRISTIAN
THINKER ST. AUGUSTINE
THE IDEOLOGICAL FORERUNNER
OF MENDEL'S WORK ON INVISIBLE
DETERMINANTS OF HEREDITY?¹

The "mathematicity" of Mendel's work was not perceived by colleagues. He even earned the good-natured nickname, "our botanical mathematician." Some even said among themselves that Father Gregor seemed to be drawn to the mystical numbers of natural philosophers Oken and Schelling, and that these "hereditary anlagen" pretty much savor of "germ causes" (*rationes seminales*) from the writings of St. Augustine, the patron of the monastery in the courtyard of which Mendel cultivated his hybrids. Note that Gregor Mendel was born 14 centuries after Augustine (Volodin, 1969). This gives grounds to assume that the reflection of the Augustinian friar Gregor Mendel about the coding of a visible characteristic by invisible factors-determinants can be interpreted in a different way than it is presented in the literature.

Since Plato science had been dominated by views that English philosopher Karl Popper called *essentialism*: the world consists of a limited number of immutable essences (ideas, in Plato's terminology), and the variable manifestations of the visible world are only incomplete and inaccurate reflections of these entities. According to this view, a real change can occur only when a new entity emerges, either from an act of Creation or a spontaneous jump (mutation).

Abiding the traditions of neo-Platonism (a pre-Christian way of philosophizing), St. Augustine believed that the Creator created this world not in the finished form but that he laid in it the appearance of new beings and objects: "the world, like a pregnant mother, is fraught with the causes of everything that is going to be born." He proposed a theory of germs (*rationes seminales*), according to which the things that gained their existence after the initial creation of the world by God were present from the very beginning in the form of invisible hidden features that are actualized only over time. These *rationes seminales* (he considered them as comprehensible) are laid down by God in matter, and it is from them all the living creatures appear.

The work of St. Augustine's mind was truly amazing. He divided cognition into two kinds: "scientia," rational cognition of the objective world, which allows people to use things, and "sapientia," cognition of the eternal divine affairs and religious objects. According to Augustine, they do not contradict each other. Sci-

¹ St. Augustine (Latin Augustinus Sanctus, full name Aurelius Augustine, 354–430), the most renowned Christian thinker of late antiquity who adhered to the traditions of Neoplatonism, a pre-Christian way of philosophizing, in which a metaphysical view of the world tolerates the presence of entities inaccessible to scientific investigation (Plato's ideas, Leibniz's monads, Hegel's spirit) (Goran, 1999; Golovko, 2007).

ence itself is necessary because a person is forced to live in the corporeal world. And since God created the corporeal world, the embryonic causes, and cognizant human beings, then humans learn the wisdom of God by recognizing the corporeal world and its driving forces (Guerrier, 1910).

A similar approach to the methodology of joint theological-scientific research is indicated by a specialist in the theory of the knowledge, philosophy, and methodology of science, Professor A.V. Yakovlev at Moscow State University, when referring to the research of the famous physicist Jan Barbour, who poses the following question: “At the same time, while science explains much of what exists in the world, there are some problems and issues that are beyond the capabilities of science in principle. For example, such a deeply metaphysical² question: *why does the world exist?*” (Barbour, 1990; Yakovlev, 2013).

Jan Barbour and another modern physicist, R.J. Russell, lean towards the need to form a common methodology of theological and scientific research, believing that science and religion are always about the coexistence of competing doctrines and discussions, and that each field has its own experts who evaluate the proposed innovations with respect to their claims to rationality and truth (Barbour, 1990; Russell, 2003).

It is assumed that Christianity has opened the possibility of a special approach for recognizing the created world, as illustrated by the fact that China—where multiple-charge rocket launchers were used as early as in the 13th century and the development of technology in general outpaced the Western technology until the 15th century—*did not draw Newton’s second law*. Newtonian astronomy was no better than Ptolemy’s astronomy, but the deeply religious Newton published a grand treatise “Mathematical Principles of Natural Philosophy” in 1687, which gave a different mathematical explanation of planetary motion and proved to be more successful for the next few centuries (Heisenberg, 1989).

In the same context, let us mention British mathematician and philosopher Roger Penrose in his attempt to create a mathematical model of the structure of the universe. In the present model, Penrose argues that “God-given” mathematical ideas exist as if outside of time and regardless of people (Yakovlev, 2013, p. 12). Thus, the specificity of modern mathematics lie in the fact that, at first glance, it studies artificially invented objects. After all, we do not observe in nature multidimensional spaces, groups, fields, and rings, whose properties are actively researched in mathematics. However, any rigorous mathematical theory sooner or later finds its application. Why were so many years spent over the proof of Poincaré’s conjecture? Poincaré’s statement is called “the formula of

the Universe” because of its importance in understanding the processes of the universe and of the fact that it answers the question of the shape of the universe. For example, which orbit must a spacecraft use to fly to Canes Venatici? What obstacles will it meet on its way?

Perhaps these are the reasons for the fact that in Russia the first analysis of Mendel’s work, which was made by botanist Ivan Parfenievich Borodin, one of the organizers of the Russian Botanical Society and Botanical Journal and an academician at the Imperial Academy of Sciences in St. Petersburg, appeared in 1903 not in the Botanical Journal but in the journal *Mir Bozhii* (God’s World) (Borodin, 1903a–c).

A POSSIBLE THOUGHT EXPERIMENT OF MENDEL IN THE IDEA OF BINARY CHARACTER ENCODING AND ITS CONFIRMATION BY METHODS DRAWN FROM STATISTICAL PHYSICS

It is assumed that thought experiments give us knowledge about the world. Where is this knowledge born? A look at the mental experimentation based on intuition refers to a form of Platonism through a priori knowledge of nature. The concept of mental models involves the manipulation of mental models instead of physical models. From a philosophical point of view, the problem of the thought experiment is that it connects us with the real empirical world. However, according to a very popular position, the source of new knowledge is a real experiment. According to Kant, it is empirical evidence (in particular, an experiment) that is the source of new knowledge. Is it possible to know reality only through thinking without the empirical data? There are only a few examples: Galileo’s thought experiments about the independence of the speed of falling bodies and their weight; Newton’s rotating bucket argument, which justifies the existence of absolute space; Einstein’s experience in the elevator, which simulates a frame of reference freefalling in the gravitational field; and Schrödinger’s cat in a dark room, which illustrates the incompleteness of quantum mechanics when going from subatomic to macroscopic systems.

These experiments should be attributed to the Platonic type because it is believed that the source of new knowledge resulting from them is the intuition of the Platonic type, which makes it possible to directly contemplate the abstract objects of mathematics and the laws of nature. However, the thought experiment requires one condition, which is plausibility (Kuhn, 1977, p. 242; Sorensen, 1992).

It is not surprising that the subject of scientific and philosophical research today is the following issue: after thousands of years why did the binary principle of notation turn out to be the most applicable in the theory of mathematical programming and why did the mathematicians who created the first computers chose as a basic principle the arithmetic archetype, the mathe-

² Metaphysics is an attempt to understand the world as a whole by means of thought (Bertrand Russell).

matics of ancient Egypt, based on the principle of duality? This system of mathematics arose before the Greek and Babylonian numeration, being radically different from them, and I note that modern computers basically use the most archaic ancient Egyptian binary principle.

Surprisingly, this mathematical archetype was chosen because of its simplicity and reliability. The binary number system quickly and easily performs all arithmetic operations and greatly simplifies all logic operations.

It can be assumed that Mendel's arguments were dominated by the idea of the archetype³ of duality (binarity) of the world order: the structure of the organization of life forms is fairly knowable when the invisible Platonic world dictates binary rules of organization to the visible world.

The idea of the duality of the world order was tragic at times. An example is the fate of this idea in the 1650s in the Russian state, when Patriarch Nikon, a man of cruelty and imperiousness, on the eve of the reunification of Ukraine and Russia decided to reform the Church: to change much in the church books and rituals, and as a symbol of the reforms to replace the binary archetype: a sign of cross made with two fingers for one made with three, which led to the withdrawal of one-third of the Russian population into Raskol (cleaving-apart), the Solovetsky Monastery Uprising that shook the Russian state, as well as the uprising of Stepan Razin and the Streltsy Uprising.

The laws discovered by Mendel confirmed the intuitive perceptions of the higher principle of nature postulated in the rational principles by Pythagoras who lived in the 6th century AD and was the first to point out the spiritual underpinnings of natural existence: the world is created by numbers and numbers represent intangible, imperceptible reality. It was the Pythagorean School that established the interrelation between religion and mathematics, which since that distant time has had a strong influence on the human thought (Heisenberg, 1989).

Nonetheless, the idea of the binary encoding of characteristics is most likely due to the influence of the school of the famous physicist Christian Doppler, whose department advised Mendel for seven years. We also cannot exclude the fact that he based his binary encoding of characteristics on Boolean algebra with the binary number system. In any case, there is reason

³ Archetypes, according to Carl Jung, "arche," are prototypes or primordial images, the first elements that have arisen in the past. In the cosmological teachings, they serve as a universal symbol of world harmony, reflecting a geometrically ordered picture of the world—a symbolic expression of the Cosmos that won over the primordial chaos. Jung sees in the concept of numbers an archetype associated with the desire of humans to order the chaos of the outside world. He wrote that "a number more than anything else that helps to put things in order in the chaos of visibility. It is a tool originally designed for either the creation of order or the comprehension of an already existing but still unknown order, organization, or orderliness" (*Archetip i Simvol*, 1991).

The principle of a prestored program represented in binary code makes it possible not only to perform calculations by sending a command to the controller and data to the arithmetic unit but also to convert the commands themselves, for example, depending on the results of calculations, using the command codes for formatting and operating with them as with the data.

The only disadvantage associated with the use of both the Egyptian nonpositional and the computer positional binary number system is the cumbersome record of values. Here is an example of how a decimal number is converted into the electronic binary number system:

$$45 = 22 \times 2 + 1$$

$$22 = 11 \times 2 + 0$$

$$11 = 5 \times 2 + 1$$

$$5 = 2 \times 2 + 1$$

$$2 = 1 \times 2 + 0$$

$$1 = 0 \times 2 + 1$$

Therefore, the electronic binary record of the number 45 is 101101. Here, as in Egyptian mathematics, the operation of multiplication is reduced to the repeated addition of the multiplicand, and division is reduced to subtraction. The use of a single mathematical principle by both the ancient Egyptians and the creators of the modern computer technology is obvious (Litovka, 2006, 2008).

to assume that Mendel acquired the idea of binary combinatorics (*AaBbCc* and *aabbcc*) and mathematical probabilistic variants (1 : 2 : 1) during the thought experiment in which the existence of ideal invisible objects in a possible world was imagined.

What type of organisms should one cross to check these thoughts? There is only one option: one must work only with self-pollinated plants, and Mendel selects experiments with peas carried out by the English breeder Thomas Andrew Knight. Evaluating Mendel's choice, Carl Correns later wrote that Mendel's success was due to the fact that he chose for his experiments that object, as pea flowers are pollinated almost exclusively by their own pollen. No other sex cells could intervene to violate the purity of the experiment.

Prior to the crossings, Mendel went over 34 varieties of peas and left for the experiments only seven pairs of varieties. Each pair differed from the other only in one characteristic. One variety of seeds was smooth, the other wrinkled; the stems of one variety were high (2 m), while the stems of the other variety were barely 60 cm; the coloring of the corolla in one variety was purple, and in the other variety it was white. Over three years, Mendel carefully watched the seeds and plants of all the seven pairs of varieties, to make sure that those seeds were clean from contamination with other seed varieties. Convinced that his varieties were free from impurities, Mendel began his intravarietal crosses.

We note that Mendel used the hybridization experiments with peas only for confirming his thoughts about the existence of a structure of the invisible binary determinants of heredity. The experiments on the crossing of peas in a small (35 × 7 m) garden under the windows of a monastery refectory were required only to check his purely speculative assumption. The existence of some two invisible factors that determine the manifestation of a visible trait was confirmed in the model experiment during the eight years of statistical calculations of thousands of peas. The object for the simulation was not only thought over but also selected to be ideal. It was only necessary to indirectly measure the probability variants (1 : 2 : 1) via statistics. These statistical methods were accumulated in a particular area of physics, statistical physics, which studies the properties and behavior of particles consisting of a large number of individual particles, molecules, atoms, and electrons.

There were no statistics in biology prior to Mendel. Many were skeptical about Mendel's statistical calculations. They were repeatedly studied and thoroughly examined. In most cases it was found that his observations completely coincide with the probabilistic expectation. The deviations are surprisingly small (Weiling and Hat, 1966).

PROBABILITY IDEAS ABOUT LIVING MATTER IN 1865 WERE ABSOLUTELY NOVEL, ESPECIALLY TO DESCRIBE THE BIOLOGICAL WORLD PICTURE

Mendel realized that counting characteristics on a selected model object should obey *one of the basic tenets of the theory of probability—the law of large numbers*, when the cumulative effect of a large number of counts and measurements leads to a fairly reliable statistical result that is almost independent of the element of chance. After all, the experimental verification of the calculations, as noted by Mendel himself, did not include all the seedlings. In one case, it was clearly stated that of the 539 grains he obtained 529 plants, in the other 639 out of 687, etc. The grains could have been damaged by birds, rodents, etc.

In separate families, i.e., small samples, Mendel observed considerable variation in the distribution of traits among the offspring. For example, he cited the data of the ten plants that had different ratios, including such as 28 : 6, i.e., 3.29 : 0.71, and 19 : 10, i.e., 2.72 : 1.38 (Rokitskii, 1978, p. 5).

Gregor Mendel not only provides a precise quantitative assessment of the phenomenon, but also for the first time applies the probability method to analyze biological processes. In his Experiments on Plant Hybridization (1865), he writes that according to the theory of probability on average each pollen shape A and a is connected by the same number of times to each shape of the germinal cell A and a ; thus, one of the pollen cells A meets at fertilization with the germinal cell A , and the other with the germinal cell a ; in the same way, one pollen cell a is connected to the germinal cell A , and the other to a . In con-

temporary language, the numerical ratios in the offspring reflected probabilities. Anyway, this method was never used in biology before Mendel.

The probability theory is regarded as a mathematical (abstract) science of the laws of massive random events, the science of chance. Randomness in the literal sense of the word rejects any patterns. Sometimes it is stated that a random event is an event that occurs with a certain probability (Sachkov, 2006).

The date of birth of the theory of probability is often referred to as 1654, when the religious thinker Blaise Pascal and the mathematician Pierre de Fermat, analyzing gambling, laid the foundations of the probability theory, independently pointing out the correct solution of the paradox of the division of the stakes. The very name “calculus of probabilities” is a paradox: probability is the opposite of veracity; probability is what we do not know and therefore, it would seem, we cannot calculate. This is a contradiction; at least it seems to be so. Analyzing this contradiction, the great Galileo solved one of the first tasks of combinatorics, an important tool for calculating probabilities. Later, Jacob Bernoulli showed that equal results for equal chances are more accurate the longer the series of events, thereby turning the random into the necessary, Bernoulli's law. When the account reaches billions of events, probabilistic predictions are accurate. Bernoulli's law served the basis for an important section of the natural sciences, statistical physics.

The problem of the correlation of necessity and chance, determinism and probability remains one of the most complicated in modern science; chance itself is subject to certain laws of necessity, without which there would be no theory of probability.

The idea of probability led to radical changes in the basic models of the universe and its cognition, in the transition from the Newtonian paradigm of the universe to the probabilistic. However, the revelation of the nature of probability remains a mystery in many ways. As noted by the failed father of the atomic bomb, the world famous physicist and philosopher Carl von Weizsäcker, probability is one of the most prominent examples of an “epistemological paradox,” when we can successfully apply our basic concepts without having a real understanding of them (Weizsäcker, 1973).

We can say that probability has confirmed the existence of genes as discrete units of heredity and made it possible to penetrate into the intimate mechanisms of the processes of inheritance⁴.

⁴ Apparently, it is no accident that in Novosibirsk Akademgorodok in 1960 geneticist D.K. Belyaev supported the initiative of mathematician A.A. Lyapunov, physiologist M.G. Kolpakov, and physicist by training V.A. Ratner on the establishment of the specialization in mathematical biology at the Novosibirsk State University. As a result, this course, led by academician N.A. Kolchanov, has now become one of the characteristic features of the profile of the Institute of Cytology and Genetics, Siberian Branch, Russian Academy of Sciences.

MENDEL'S WORK REVEALED AN AWARENESS OF THE LINK BETWEEN ERROR, METAPHYSICS, AND METHODOLOGY

Mendel was not only a mathematician, he was also a priest, so he deeply believed (as well as the deeply religious Newton) that the divine plan of the structure of the visible world can be comprehended by the human mind, one only must not make mistakes in the calculations of the cognition of this plan. Having passed through Doppler's physical school, Mendel was well aware of the problem of the experimental error. He needed to eliminate the possibility of errors in the calculations and take (in terms of the understanding of the problem) appropriate precautions, because any mistake, even minor, in the calculations could interfere in its statistical search for evidence of the invisible binary factors that determine the visible manifestation of characteristics in a number of generations.

Since accidents and errors, of course, had the right to exist in his search, it provides an answer to the question on why Mendel used such huge samples and was so scrupulous in setting up the experiment? It was necessary to test the hypothesis for noise immunity; thus, Mendel drew in disinterested helpers: Father Lindenthal and Father Winkelmayer, as well as gardener Maresh.

MATHEMATIZED FORM OF WRITING NOT UNDERSTOOD AT FIRST BY EITHER BIOLOGISTS OR MATHEMATICIANS

Of Mendel's entire working archive there remains just a single calculation sheet, albeit, damaged by someone. However, it is known that Mendel was the first to apply mathematical symbols to interpret the coding patterns of a particular characteristic by two invisible factors. We do not know now at what stage of his work he came to realize the expediency of this particular method of solving the problem. It is not known how he handled these data in the beginning. However, he sought the principles of encoding characteristics, and the principal schemes are always abstract.

In this respect, he was likely to use the idea of the German idealist, philosopher, and mathematician Gottfried Wilhelm Leibniz (1646–1716), who dreamed in his work "Mathesis universalis" to extend algebraic symbolism to all areas of knowledge. Leibniz used letters to name the action of basic logic operations. For example, the record $a > b$ means that expression " a " is more than expression " b ". Leibniz's philosophy centers around two main ideas closely linked to each other—the idea of universal symbology and logical calculus. These two ideas formed the basis of modern mathematical analysis and modern symbolic logic.

Symbolization in science is the transition from a natural language as a means of expressing our thoughts to an artificial language (Lyusy, 2009). For example, Newton conveyed the following anagram to Leibniz: *aaaabbbeeeei*, etc., in which he simply wanted to say that he was able to convert (by the method of undetermined coefficients) the power series formally satisfying the proposed equation (Poincare, 1983). Moreover, Mendel first used the somewhat similar letter-algebraic symbology to notate the invisible hereditary factors (which at the suggestion of the Danish botanist B. Johannes were later called genes).

Not altered by mutation, normal hereditary factors shaping the standard phenotype or the norm were designated by a "plus" (+). Factors that have changed as a result of mutation were written in Latin letters a , b , c , etc. To indicate the dominance of one factor over another, Mendel borrowed from Leibniz's symbols $>$ (greater) and $<$ (smaller). Symbols communicate with each other according to specific rules, as in mathematics $+ > a$ means that the normal trait dominates over the mutant trait.

It should be noted that alphabetic symbols were first proposed in 1766 by Kölreuter in the description of characteristics in hybrids of China pink and Sweet William, as well as in different varieties of tobacco. However, Mendel gave it a completely different understanding. Mendel's introduction of binary letter symbology explained the nature of the inheritance of characteristics and the analysis of cleavage patterns. What did he have in mind when he wrote, for example, AA or Aa ? One hereditary factor came from the father, and the other from the mother. The alphabetical symbology served as the basis for the mathematized form of a biological record, which at first was not understood by either biologists or mathematicians. Later it will be interpreted as a transfer of genetic information in the chain of generations, algebraic in its nature (Petukhov, 2012, p. 86).

ON THE IMPERFECTION OF HUMAN LOGIC AND AN ATTEMPT TO CREATE A LEGEND ABOUT THE LUCKY AMATEUR FRIAR

The work of Mendel, who passed through Christian Doppler's physical school, was dominated by the logic of physical observation, and each of the conclusions in the Experiments on Plant Hybridization was formulated with the utmost perfection, such as: "In this generation, along with the dominant traits, we again observe recessive ones with all their features and, moreover, at the clearly expressed average ratio of 3 : 1." Each of these conclusions was preceded by careful statistical calculations.

Therefore, in 1936, Mendel's successor, the distinguished English mathematician and geneticist Ronald Fisher (a disciple of Francis Galton, a cousin of Charles Darwin), revising Mendelian statistics in his article *Has Mendel's Work Been Rediscovered?*, pointed

to the fact that Mendel got suspiciously good agreement in his results with those theoretically expected, while, according to the distribution χ^2 , the probability of this is too low. Fisher then expressed his suspicion that perhaps the gardeners of the abbey deceived Mendel? Could it be that they were rounding off the calculations to please Father Gregor? (Fisher, 1936).

The work of another British researcher, Gavin de Beer, who advocated the same views as Fisher, followed in 1965. In the following year, 1966, a work in the same tone appeared, authored by F. Weiling (Has J.G. Mendel Been “Too Accurate” in His Experiments? The χ^2 -Test and Its Significance for the Evaluation of Genetic Segregation) (Weiling and Hat, 1966).

In even more detail the confrontation of supporters of Mendel’s teaching and its critics is covered in the book of U.B. Provine “The Origin of Theoretical Population Genetics,” in the section “Darwinian Selection: Controversy of 1900–1918” published in 1970 by the publishing house of the University of Chicago (Provine, 1970).

Even in 2006 in the journal *Bulletin of the Russian Academy of Sciences*, Doctor of Psychology A.V. Yur’evich wrote: “One can assume that since there was a science there also formed a field of phenomena that can be attributed to shadow science, although the nature and severity of these phenomena changed over time. For example, A. Kohn in his book with the eloquent title *False Prophets: Fraud and Error in Science and Medicine* (Kohn, 1986) provides proof that even the founders of modern science, Newton, Kepler, Galileo, and others, regularly faked scientific data. The case of Mendel gained wide publicity after the mathematician Fisher “proved” [did not prove—O. Trapezov] that the quantitative data provided by the “great friar” to confirm the discovered laws of genetics could not be obtained in principle (Yur’evich, 2006).

Why do such opinions arise from time to time?

This attack on Mendel put and continues to place a specific purpose—to create a legend of the lucky amateur friar who just happened to be the father of genetics. Nevertheless, checks on the Mendel–Fisher case involving the computer processing of Mendelian experiments have shown that they are only slightly better than those carried out by the researchers who repeated his experiments, and therefore must be recognized as absolutely credible (Volodin, 1969).

As a priest, Mendel was supposed to serve Mass. However, when he took off the monastic robe and put the prayer-book away, the canon turned into a scientist. Having passed through the school of Doppler and cytologist Unger, he knew that science lives under other laws: science is the realm of logic, experiments, and the repetition of experimental results by independent experimenters.

Mendel needed an experiment to decipher the process taking place in the “black box” using statistics on a large array of data. The patterns traced in the model

experiment on peas confirmed his thoughts: the fate of a characteristic is determined by two invisible factors. Eight-year statistical calculations confirmed that parents do not transmit to their children characteristics but something else, something that causes these characteristics. This “something” can be realized immediately or only after a certain time, being transmitted from generation to generation without showing itself. This something (information) does not disappear and does not occur again, just as matter does not disappear in vain and does not arise “out of nothing.”

After obtaining the experimental confirmation of his reflections, Mendel introduced a new concept to the idea of heredity—Elementen/Anlagen (elements/hereditary anlagen)—carriers of information about characteristics; information that gets involved in the process hidden from the researcher and processed therein.

This concept, Elementen (Anlagen), gives rise to genetics. A further line of Mendel’s thought is as follows. Each characteristic is connected to a material substrate, “hereditary anlage,” which is contained in the sex cells of the body. Each sex cell carries a full set of anlagen according to the number of characteristics of the future plant. When the male and female cells fuse into a zygote, the latter contains two anlagen for each characteristic. When a new creature that developed from this fertilized egg produces sex cells, the two anlagen will again disperse and the gamete, the egg cell or sperm, will have a single set. The hereditary substance is discrete, so combinations of invisible anlagen vary according to the laws of mathematical permutations, and the future plant will have new combinations of visible characteristics. Mendel predicted these combinations and the mathematic probability variants in his calculations and for experimental verification grew them in the garden under the windows of the monastery refectory.

Neither Mendel, nor his teacher, cytologist Unger, nor any of the scientists in the early 1860s knew that on the eve of cell division the cell nuclei reveal stainable corpuscles—chromosomes, which double in their number and diverge to the poles of the cell to form two nuclei of two new cells. Moreover, no one knew at the time that the sex cells (pollen, sperm and egg cells of animals, eggs of fish and amphibians, and eggs of reptiles and birds) pass through a special way of formation, in which the chromosomes are not doubled but only diverge to different poles of the progenitor cells. Furthermore, each of the two sex cells formed from it contain halved, or more precisely, single number of chromosomes. This set doubles only upon their fusion, upon fertilization of the egg. These processes were observed in the research programs of two American scientists: graduate student of Columbia University, William Sutton (1876–1916) and embryologist Thomas Hunt Morgan (1866–1945), who is often referred to as the author of the chromosome theory of heredity. In 1902, Sutton compared the Mendelian laws of heredity with the behavior of chromosomes and found a paral-

lelism between the inheritance of genes and chromosomes and formulated the chromosome theory: factors that determine heredity, i.e., genes, are located in chromosomes. For the creation of chromosomal theory Morgan was awarded the Nobel Prize in 1933.

At that time no one had ever uttered the word “gene,” which denotes the unit of hereditary material responsible for the basic difference. No one had identified the concept of “hereditary substance” with the word “DNA,” which denotes an amazing substance of chromosomes, deoxyribonucleic acid, in which combinations of nitrogen bases make up formulas of protein molecules, the order of their synthesis, and spatial packing (Volodin, 1969).

Gregor Mendel studied the black box (a term from physics and cybernetics). He knew which information is included in the box with convergent and divergent algebraic series of invisible hereditary anlagen *AaBBcC* and *aabbcc* and what happens after this information passes through a chain of events invisible to the eye. Mendel suggested that his readers should just accept the idea of the black box, which only resulted in the fact that no one was able to understand him. The 47 pages of “Experiments on Plant Hybridization” offered a special system of concepts that introduced the readers to the invisible world of the unknown, a place in which a particular language is spoken. Apparently, for this reason, in 117 libraries of the 120 to which the volume of works with Mendel’s article was sent, it stood on the shelves touched by nobody except perhaps the library’s mice. Only three of the 120 copies were opened. It was only later, 35 years after the publication of his work and 16 years after his death, that a new science began to develop and flourish. Mendel expected to find support. He believed that his results would be backed up by other studies.

In his book “Mendel,” B.G. Volodin wrote: “Mendel had not sent a single print of his work to the only person who would more than anyone understand it, Darwin, who was interested in the works on hybridization. In 1862, Mendel was in London. He did not know English and read the full text of Darwin’s “On the Origin of Species ...” only a year later, when it was published in German. However, he knew the content of the work because of the controversy that raged in the media. Leonard, the fourth and youngest son of Darwin, held a special investigation of whether Mendel had been in their house and found that he had not been there (Volodin, 1969).

Naturalists have learned the name of Gregor Mendel from Foke’s book “Pflanzenmischlingen” (Foke, 1881), in which the author meticulously and pedantically reviewed all the works on the problem of hybridization. Foke mentions Mendel’s name in the book 15 times. Due to Foke’s scrupulosity the “father of genetics” received his deserved glory 16 years after his death. After all, it is Foke’s book that introduced Mendel to both Correns and Tschermak, as well as the

author of the mutation theory, the Dutch botanist Hugo Marie de Vries.

As for the simultaneous rediscovery of Mendel’s work in 1900 by the three biologists, Hugo Marie de Vries, Carl Erich Correns, and Erich Tschermak von Seysenegg, the American geneticist Sturtevant rightly observed that only Mendel’s article itself was simultaneously rediscovered, but that none of the rediscoverers understood the meaning and depth of its laws all at once (Sturtevant, 1965).

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