

Improving the Accuracy of Marine Gravimeters

A. V. Sokolov^{a,b}, A. A. Krasnov^{a,b,*}, and L. K. Zheleznyak^c

^aConcern CSRI Elektropribor, JSC, St. Petersburg, Russia

^bITMO University, St. Petersburg, Russia

^cSchmidt Institute of Physics of the Earth (Russian Academy of Sciences), Moscow, Russia

*e-mail: anton-krasnov@mail.ru

Received April 8, 2019; revised May 21, 2019; accepted May 21, 2019

Abstract—Software- and hardware-based methods of compensation for dynamic errors of the marine gravimeters caused by inertial accelerations are considered. The error due to the fluid damping of the gravimeter sensing element is analyzed and taken into account for the first time. Some results of gravity measurements that confirm the increase in gravimeter accuracy are presented.

Keywords: gravity, gravimeter, gravity survey, data processing

DOI: 10.1134/S2075108719030088

INTRODUCTION

When measuring gravity from a mobile carrier, the gravimeter records the projection of the specific force on its axis of sensitivity. The main problem with such measurements is that the inertial forces generated by the carrier motion are several orders of magnitude greater than gravity anomalies. In marine conditions, the spectra of inertial and gravitational accelerations are separated in the frequency domain, and so digital filtering of data is effectively used to extract gravitational accelerations. In the case of airborne gravimetric measurements, low-frequency inertial accelerations are compensated for by using high-precision measurements of flight altitude variations obtained with the use of satellite navigation technologies.

Optimal methods for filtering marine gravimetric measurements are described in [1–7]. However, the data processing methods proposed in those publications did not take into account the design features of Chekan gravimeters, which are currently widely used for marine gravimetric measurements [8–14]. The main feature of Chekan gravimeters is the presence of a double quartz torsion-type system with fluid damping and a biaxial gyro platform with accelerometric control loop used to keep the sensitivity axis of the quartz system in the vertical direction [15]. The purpose of this work is to study methods for improving the accuracy of gravity measurements made with Chekan gravimeters by compensating for the errors that are characteristic of this type of instruments.

COMPENSATION FOR THE HARRISON EFFECT

The Harrison effect, well-known in gravimetry, arises due to the projection of horizontal accelerations on the gravimeter axis of sensitivity as a result of tilts caused by stabilization errors [16]. The error of the gravimeter readings δg_{HAR} due to the Harrison effect can be written as:

$$\delta g_{\text{HAR}} = W_X \alpha + W_Y \beta, \quad (1)$$

where W_X , W_Y are longitudinal and transverse horizontal accelerations, respectively; α , β are the tilts of the gravimeter sensitivity axis caused by stabilization errors.

The tilt of the gravimeter sensitivity axis is a function of the horizontal acceleration effect along the same stabilization axis. Then the phase shift between the horizontal accelerations and stabilization errors is defined by the frequency response of the accelerometric gyro control loop. In this regard, it makes sense to consider their combined effect on the gravimeter from the standpoint of the correlation theory of random functions. Since both terms in formula (1) have the same structure, we will consider only one of them. The systematic error $\Delta \tilde{g}_{\text{HAR}}$ of the gravimeter readings will be equal to the expected value of the product of $W_Y(t)$ and $\beta(t)$. Assume that the gravimeter carrier moves in a straight line, which corresponds to the state of the motion in the course of gravimetric surveys. In this case, the mathematical expectations of horizontal accelerations and the gravimeter tilts are zero. Then [17]:

$$\Delta \tilde{g}_{\text{HAR}} = k_{W_Y \beta} \sigma_{W_Y} \sigma_{\beta}, \quad (2)$$

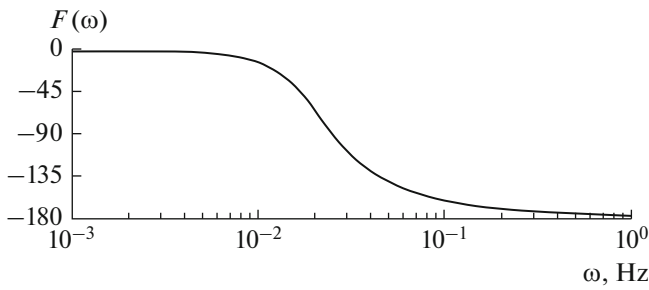


Fig. 1. Phase frequency response of the vertical control loop.

where σ_{W_Y} , σ_β are RMS values of horizontal accelerations $W_Y(t)$ and tilts of the sensitivity axis $\beta(t)$; $k_{W_Y\beta}$ is the correlation coefficient of $W_Y(t)$ and $\beta(t)$.

The transfer function of the Chekan gravimeter vertical control loop has the form [18]:

$$H_W^\beta(p) = \frac{2Tp + 1}{g(T^3 p^3 + 2T^2 p^2 + 2Tp + 1)}, \quad (3)$$

where T is the response time of the vertical control loop.

Figure 1 shows the phase frequency response of the vertical control loop corresponding to the transfer function (3) for $T = 50$ s.

It can be seen that the phase shift between the horizontal accelerations and the tilts at the basic rolling frequencies of 0.1–0.2 Hz is close to 180° , therefore the correlation coefficient $k_{W_Y\beta}$ in the calculations can be taken to be equal to 1 in absolute value.

In evaluating the magnitude of the Harrison effect, we limit ourselves to the effect of rolling acceleration as the one producing the most profound influence. Table 1 shows the RMS values of disturbing accelerations σ_{W_Y} used in the calculations [19].

The values of stabilization errors σ_β were obtained by mathematical simulation of the vertical control loop.

The estimated Harrison effect $\Delta\tilde{g}_{\text{HAR}}$ was 0.16 mGal for calm sea and 1.82 mGal for very rough sea, which confirms the need to compensate for this effect in the Chekan gravimeter readings, especially in connection with the increased relevance of surveys

Table 1. Evaluation of the Harrison effect

Parameter	Sea state code			
	0–1	2–3	4–5	6
σ_{W_Y} , m/s ²	0.08	0.17	0.21	0.26
σ_α , rad	2×10^{-5}	3×10^{-5}	5×10^{-5}	7×10^{-5}
$\Delta\tilde{g}_{\text{HAR}}$, mGal	0.16	0.51	1.05	1.82

under adverse weather conditions. It is proposed to calculate the correction using the formula similar to formula (1). In this case, the tilts of the gravimeter sensitivity axis caused by stabilization errors are formed by multiplying accelerometer signals by the transfer function of the gravimeter vertical control loop:

$$\Delta g_{\text{HAR}} = (W_X H_W^\beta(p))W_X + (W_Y H_W^\beta(p))W_Y, \quad (4)$$

where W_X , W_Y are the longitudinal and transverse horizontal accelerations recorded from the readings of the gravimeter accelerometers installed on a gyro stabilized platform; $H_W^\beta(p)$ is the transfer function of the gyro control loop (3).

COMPENSATION FOR THE CROSS-COUPLING EFFECT

The feature inherent in all gravimetric torsion-type systems is an orbital effect, known as a cross-coupling effect, on the gravimeter readings of horizontal and vertical accelerations of the base. This effect is due to the projection of the horizontal component of the specific force on the gravimeter axis of sensitivity:

$$\delta g_{\text{CC}} = W_x \varphi, \quad (5)$$

where φ is the angle of deflection of the gravimeter sensitivity axis from the vertical.

To describe the cross-coupling effect, we consider a gravimeter with a single elastic torsion-type system. The differential equation of pendulum motion in damping fluid can be written as [16]:

$$T_g \dot{\varphi} + \varphi = v(\Delta g + \ddot{z}), \quad (6)$$

where Δg —gravity increment, \ddot{z} is the vertical component of the inertial acceleration of the base, v is the transfer coefficient of the gravity sensor, and T_g is the gravity sensor response time. The cross-coupling effect is mainly created by the fast component of acceleration; therefore, neglecting the slowly varying part of the disturbing function, we can write:

$$T_g \dot{\varphi} + \varphi = v\ddot{z}. \quad (7)$$

Since the value of T_g is large enough (about 100 s for Chekan gravimeters), it can be written approximately as:

$$T_g \dot{\varphi} \approx v\ddot{z}. \quad (8)$$

After integrating, we derive:

$$\varphi \approx \frac{v}{T_g} \dot{z}. \quad (9)$$

It follows that the gravity sensor response to the acceleration is proportional to the speed of vertical motion, and if the motion is a harmonic oscillation, then the gravimeter pendulum lags 90° in phase with respect to acceleration. Thus, due to the phase shift of the gravity sensor response to the force acting on it, the

output signal of the gravimeter has a systematic error that leads to underestimation or overestimation its readings:

$$\Delta\tilde{g}_{CC} = \frac{v}{T_g} M\{W_x V_z\}, \quad (10)$$

where M is the mathematical expectation operator.

Since the mathematical expectations of horizontal and vertical accelerations at pitching are zero, then

$$\Delta\tilde{g}_{CC} = \frac{v}{T_g} k_{W_x V_z} \sigma_{W_x} \sigma_{V_z}, \quad (11)$$

where σ_{W_x} , σ_{V_z} are RMS values of the horizontal accelerations $W_x(t)$ and the vertical velocity $V_z(t)$; $k_{W_x V_z}$ is the correlation coefficient of $W_x(t)$ and $V_z(t)$.

In the case of a gravimeter with a double elastic torsion-type system with identical quartz systems, the errors due to the cross-coupling effect of each of the quartz systems

$$\begin{aligned} \Delta\tilde{g}_{CC1} &= \frac{v_1}{T_{g1}} k_{W_x V_z} \sigma_{W_x} \sigma_{V_z}, \\ \Delta\tilde{g}_{CC2} &= \frac{v_2}{T_{g2}} k_{W_x V_z} \sigma_{W_x} \sigma_{V_z}, \end{aligned} \quad (12)$$

where v_1, v_2, T_{g1}, T_{g2} are the transfer coefficients and response times of single systems, will be compensated for:

$$\Delta\tilde{g}_{CC} = \Delta\tilde{g}_{CC1} - \Delta\tilde{g}_{CC2} = 0,$$

for $v_1 = v_2, T_{g1} = T_{g2}$.

However, the elastic systems of real devices do not have identical parameters. In this case, the cross-coupling effect can be estimated by the following formula:

$$\Delta\tilde{g}_{CC} = \frac{v_1 T_{g2} - v_2 T_{g1}}{T_{g1} T_{g2}} k_{W_x V_z} \sigma_{W_x} \sigma_{V_z}. \quad (13)$$

Thus, the error due to the cross-coupling effect is characterized only by the conditions in which a gravimetric survey is performed, nonidentity of the pendulums of the double quartz elastic system, and it does not depend on stabilization errors.

To estimate the magnitude of the cross-coupling effect, we analyzed the nonidentity of the pendulums of a double quartz elastic system in transfer coefficients and response times for 40 Chekan gravimeters. According to the results, the pendulums were not identical in transfer coefficients, but the differences did not exceed 0.5%, and as for response time, the differences were no more than 2%. Given this, in the calculations, the transfer coefficient of single quartz systems was taken to be the same:

$$v_1 = v_2 = 0.17 \text{ rad/m/s}^2.$$

As with the evaluation of the Harrison effect, the calculation results given in Table 2 are divided into cat-

Table 2. Evaluation of the cross-coupling effect.

Parameter	Sea state code			
	0-1	2-3	4-5	6
σ_{W_x} , m/s ²	0.06	0.10	0.14	0.19
σ_{V_z} , m/s	0.2	0.3	0.4	0.5
$k_{W_x V_z}$	0.2	0.4	0.6	0.7
$\Delta\tilde{g}_{CC}$, mGal	0.01	0.12	0.32	0.50

egories depending on the conditions in which the gravity surveys were performed. The values of coefficients $k_{W_x V_z}$ were obtained from the statistical analysis of the data from marine gravimetric surveys.

As follows from the data obtained, it is necessary to compensate for the error $\Delta\tilde{g}_{CC}$ for the profiles of the gravimetric survey carried out at very rough sea.

In Chekan gravimeters, the angular position of the elastic system pendulums is read out by an optoelectronic converter. The output data of the gravity sensor have dimensionality of pixels, and the coefficient of conversion from pixels to the turn angle of the elastic system pendulum is $k = 3.15 \times 10^{-5}$ rad/pix.

It is proposed to perform compensation for the cross-coupling effect using the formula:

$$\Delta g_{CC} = k(m_2 - m_1)W_x, \quad (14)$$

where m_1, m_2 are the current readings of the first and second pendulums of the gravimeter elastic system.

COMPENSATION FOR THE FLUID DAMPING ERROR

When Chekan gravimeters were tested on the test bench of vertical displacements, there was an additional systematic measurement error [20].

The traditional equation for the operation of a highly damped gravimeter with a harmonic change in the input signal has the form:

$$T_g \dot{g} + g = z \cos(\omega t), \quad (15)$$

where T_g is the gravimeter response time, g is gravimeter readings, and z is the amplitude of vertical accelerations.

Formula (15) is written in the assumption that the motion of the proof mass does not cause any vortex flows in the fluid of the gravimeter elastic system. This assumption is true for low speeds of pendulum motion (small disturbing accelerations and high viscosity of the fluid). However, at supercritical speeds determined by the Reynolds number, a turbulent component is added, so that Equation (15) should be complemented:

$$k_g \dot{g}^2 + T_g \dot{g} + g = z \cos(\omega t), \quad (16)$$

Table 3. Test results of the gravimeters on the test bench of vertical displacements

Nos.	Response time T_g , s	Systematic error δg , mGal	\dot{g}^2 , mGal ² /s ²
1	31	2.6	1512900
2	33	2.6	1468944
3	36	1.5	1234321
4	39	1.0	1050625
5	40	1.5	1000000
6	44	1.5	917764
7	45	0.6	790321
8	57	0.5	491401
9	60	0.8	443556
10	65	0.9	379456
11	83	0.1	232324
12	88	0.6	207025
13	88	0.4	207025
14	100	0.1	160000
15	101	0.4	156816
16	123	0.1	105625
17	138	0.4	83521

data. To do this, we can use the data of the gravimeter tests on the test bench of vertical displacements.

The results of the Chekan gravimeter tests with different response time for vertical displacements with amplitude of 2 m and a period of 14 s are given in Table 3. For the motion with a specified period, the test bench sets harmonic acceleration with amplitude of 40 Gal. For each of the gravimeters, the amplitude of the change in its readings was calculated taking into account the value of the response time, as well as the rate of change in the readings and the squared rate. The calculated value of the squared rate \dot{g}^2 is also given in Table 3.

The curve for $\delta g = F(\dot{g}^2)$ approximated by the linear function using the least-squares method is shown in Fig. 2. Linear approximation has proved to be sufficient as will be shown in the discussion of the field-data processing results.

In accordance with the approximation, the sought coefficient was $k_{\dot{g}} = 1.5 \times 10^{-6} \text{ s}^2/\text{mGal}$ under the action of the harmonic signal. In real measurement conditions at sea, the rms value of the squared rate of change in the gravimeter readings is calculated. It is known that the rms value of the harmonic oscillation is $\sqrt{2}$ times smaller than that of the amplitude. Therefore, the required coefficient should be taken equal to $k_{\dot{g}} = 1.1 \times 10^{-6} \text{ s}^2/\text{mGal}$.

Thus, to compensate for the dynamic error caused by the effect of vertical accelerations, it is proposed to enter the correction Δg_{Wz} to the gravimeter readings:

$$\Delta g_{Wz} = k_{\dot{g}} \dot{g}^2. \tag{17}$$

where $k_{\dot{g}}$ is the drag coefficient determined by the geometric shapes of the pendulum and the fluid viscosity. The first term of Equation (16) causes an error that should be compensated for in order to increase the measurement accuracy. To calculate the appropriate correction, it is necessary to determine the drag coefficient $k_{\dot{g}}$ that can only be obtained from experimental

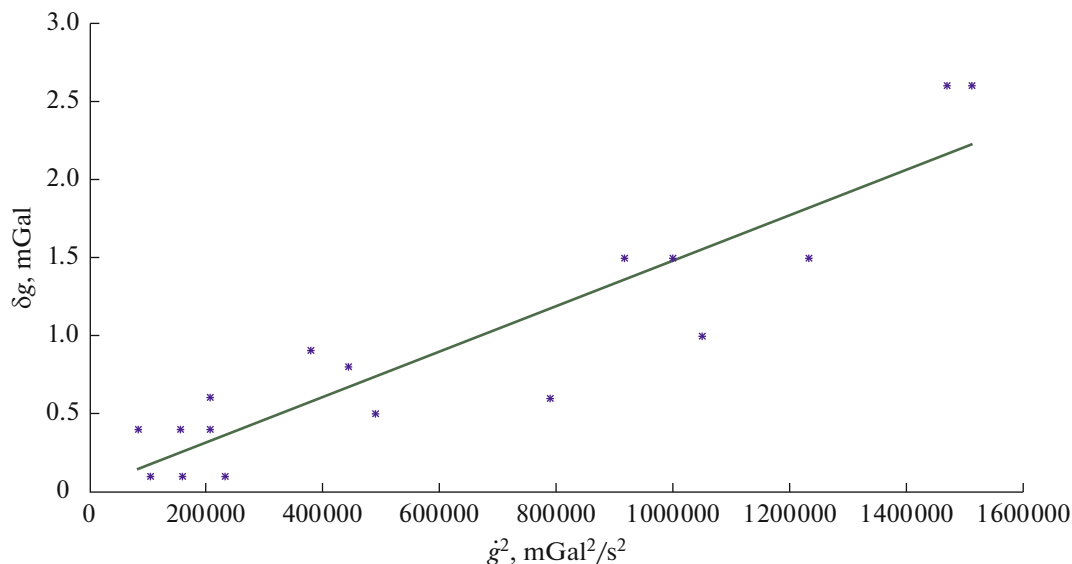


Fig. 2. Linear approximation of $\delta g = F(\dot{g}^2)$.

EXAMPLES OF MARINE GRAVIMETRIC MEASUREMENTS

Basically, the Harrison effect and the cross-coupling effect cause systematic changes in the gravimeter readings, whose values are determined by the dynamic conditions in which the measurements are performed. These effects can be taken into account by using methods of areal levelling of survey results [21].

Figure 3 shows an example of applying correction Δg_{W_z} for the effect of vertical accelerations to the gravimetric profile data obtained in substantially changing conditions at sea.

It is obvious that not only the systematic component of the dynamic error, but also its high-frequency component are compensated for, which also increases spatial resolution of measurements [22]. Besides, in the case of significant changes in sea swell on the profile, it is impossible to take into account the dynamic error using the areal levelling methods, but, as can be seen from Fig. 3, it is well compensated for by applying the correction Δg_{W_z} .

For experimental verification of the methods proposed for compensation of dynamic measurement errors caused by carrier acceleration, we performed data processing of repeated marine gravimetric profiles obtained in different weather conditions. First, the traditional corrections were entered to the data of all profiles: a correction for the gravimeter null-point drift, Eotvos correction, and a correction for the normal gravity value. The resulting gravity anomalies obtained from measurements on repeated profiles were processed by a low-pass filter whose parameters for each pair of profiles remained invariable. Next, we repeated data processing, wherein we entered, in addition to the above-listed corrections, the corrections for accelerations of motion: the Harrison correction, the corrections for damping fluid and the cross-coupling effect. The parameters of the low-pass filter for each pair of repeated profiles were consistent with those

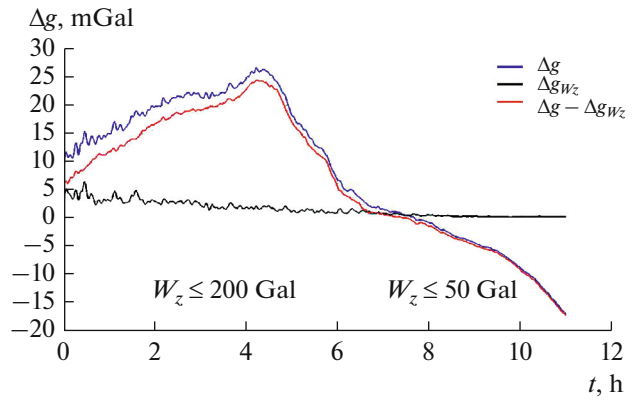


Fig. 3. An example of applying correction Δg_{W_z} to a gravimetric profile.

used at the first stage of data processing. The results of data processing are shown in Table 4.

Analysis of the data in Table 4 shows that entry of dynamic corrections reduces the rms error in the measurement difference on repeated profiles by 1.5–4 times, depending on the magnitude of disturbing accelerations.

In addition, data processing of three marine gravimetric surveys in two presented variants was performed. The total number of cross points was more than 3500. It may be concluded that dynamic corrections make it possible to reduce the rms error of a survey by a factor of three.

CONCLUSIONS

The theoretical research and experimental survey have shown that the measurement accuracy of Chekan gravimeters can be improved both by increasing the response time to a reasonable limit and proper accounting for the effects of physical and design fac-

Table 4. Evaluation of measurement accuracy on repeated profiles.

Profile nos.	Length, km	RMS value of vertical accelerations, Gal	RMS error of measurement differences, mGal	
			without corrections	with corrections
1/2	65	60/15	2.20	0.85
3/4	60	50/20	1.42	0.36
5/6	80	47/21	0.95	0.45
7/8	75	46/16	1.75	0.83
9/10	70	45/10	0.63	0.40
11/12	60	42/8	0.66	0.34
13/14	20	40/15	1.11	0.36
15/16	45	38/23	0.69	0.43
17/18	65	37/22	1.36	0.47
19/20	70	30/20	1.08	0.51

tors in the software. The use of the proposed methods for compensation of dynamic errors makes it possible to increase the accuracy of measurements performed in adverse weather conditions by several times and, consequently, improve the efficiency and reduce the cost of geophysical surveys.

FUNDING

This work was supported by the Russian Science Foundation, project no. 18-19-00627.

REFERENCES

1. Pantelev, V.L. Dynamic synthesis of marine gravimeters, in *Morskije gravimetricheskie issledovaniya* (Marine Gravimetric Surveys), Moscow, 1975, no. 8, pp. 22–47.
2. Childers, V., Bell, R., and Brozena, J. Airborne gravimetry: An investigation of filtering, *Geophysics*, vol. 64, no. 1, 1999, pp. 61–69.
3. Bolotin, Yu.V. and Yurist, S.Sh., Suboptimal smoothing filter for the marine gravimeter GT-2M, *Proc. IAG Symposium on Terrestrial Gravimetry: Static and Mobile Measurements*, St. Petersburg, Russia, 2010, pp. 7–11.
4. Lianghui Guo, Xiaohong Meng, Zhaoxi Chen, Shuling Li, and Yuanman Zheng, Preferential filtering for gravity anomaly separation, *Computers & Geosciences*, vol. 51, 2013, pp. 247–254.
5. Stepanov, O.A. and Koshaev, D.A., Analysis of filtering and smoothing techniques as applied to aerogravimetry, *Gyroscopy and Navigation*, 2010, vol. 1, no. 1, pp. 19–25.
6. Stepanov, O. A., Koshaev, D. A., and Motorin, A. V., Designing Models for Signals and Errors of Sensors in Airborne Gravimetry Using Nonlinear Filtering Methods, *Institute of Navigation International Technical Meeting 2015*, ITM 2015, pp. 220–227.
7. Bolotin Y.V., Golovan A.A. Methods of inertial gravimetry // *Moscow University Mechanics Bulletin*, 2013, vol. 68. no. 5, pp. 117–125.
8. Forsberg, R., Olesen, A.V., Einarsson, I., Manandhar, N., and Shreshta, K., Geoid of Nepal from Airborne Gravity Survey. In: Rizos, C., Willis, P., (Eds.) *Earth on the Edge: Science for a Sustainable Planet. International Association of Geodesy Symposia*. vol. 139, 2014, Springer.
9. Barzaghi, R., Albertella, A., Carrion, D., Barthelmes, F., Petrovic, S., and Scheinert, M., Testing Airborne Gravity Data in the Large-Scale Area of Italy and Adjacent Seas. In: Jin, S., Barzaghi, R., (Eds.), *IGFS 2014* (International Association of Geodesy Symposia), Berlin, Heidelberg: Springer, 2015, pp. 39–44.
10. Kazanin G.S., Zayats, I.V., Ivanov, G.I., Makarov, E.S., and Vasil'ev, A.S., Geophysical surveys in the North Pole area, *Okeanologiya*, 2016, vol. 56, no. 2, pp. 333–335.
11. Koneshov, V.N., Nepoklonov, V.B., Pogorelov, V.V., Solov'ev, V.N., and Afanasyeva, L.V., Gravitational field of the Arctic: The state of knowledge and prospects for the future, *Fizika Zemli*, 2016, no.3, pp. 113–122.
12. Lu, B., Barthelmes, F., Petrovic, S., Forste, C., Flechtner, F., Luo, Z., He, K., and Li, M., Airborne gravimetry of GEOHALO mission: data processing and gravity field modeling, *J. of Geophysical Research: Solid Earth*, 122, 2017, pp. 10586–10604.
13. Forsberg, R., Olesen, A., Ferraccioli, F., Jordan, T., Matsuoka, K., Zakrajsek, A., Ghidella, M., and Greenbaum, J., Exploring the Recovery Lakes region and interior Dronning Maud Land, East Antarctica, with airborne gravity, magnetic and radar measurements, *Geological Society, London, Special Publications*, 461, 2017, pp. 23–34.
14. Peshekhonov, V.G., Sokolov, A.V., and Krasnov A.A., The current state and prospects for the development of marine gravimetry in Russia, *11th Russian Multiconference on Problems of Control* (11 Rossiiskaya mu'tikonferentsiya po problemam upravleniya), St.Petersburg: Elektropribor, 2018, pp. 6–16.
15. Peshekhonov, V.G., Sokolov, A.V., Elinson, L.S., and Krasnov, A.A., A new air-sea gravimeter: development and test results, *22nd St. Petersburg Int. Conf. on Integrated Navigation Systems*, St. Petersburg: Elektropribor, 2015, pp. 193–199.
16. Pantelev, V.L., *Osnovy morskoy gravimetrii* (Basics of Marine Gravimetry), Moscow: Nedra, 1983.
17. Stepanov, O.A., *Osnovy teorii otsenivaniya s prilozheniyami k zadacham obrabotki navigatsionnoi informatsii* (Fundamentals of the Estimation Theory with Applications to the Problems of Navigation Information Processing), Part 2, *Vvedenie v teoriyu fil'tratsii* (Introduction to the Filtering Theory), St. Petersburg: TsNII Elektropribor, 2012.
18. Krasnov, A.A., The results of bench and field tests of the airborne gravimeter gyrostabilizer, *IX konferentsiya molodykh uchenykh "Navigatsiya i upravlenie dvizheniem"* (9th Conference of Young Scientists "Navigation and Motion Control), 2007, pp. 26–33.
19. Kutepov, V.S., Operating conditions of marine gyrostabilized gravimeter, *Izvestiya TulGU. Tekhnicheskie nauki*, 2012, no. 2, pp. 260–267.
20. Krasnov, A.A., Nesenjuk, L.P., Sokolov, A.V., Stelkens-Kobsch, T.H., and Heyen, R., Test results of an airborne gravimeter, *Proc. IAG Symposium on Terrestrial Gravimetry: Static and Mobile Measurements*, St. Petersburg, Russia, 2008, pp. 73–77.
21. *Sovremennye metody i sredstva izmerenya parametrov gravitatsionnogo polya Zemli (Modern technologies and methods for measuring the Earth's gravity field parameters)*, Eds., V.G. Peshekhonov, O.A. Stepanov, St. Petersburg: Concern CSRI Elektropribor, 2017.
22. Krasnov, A.A. and Sokolov, A.V., A Modern Software System of a Mobile Chekan-AM Gravimeter, *Gyroscopy and Navigation*, 2015, vol. 6, no. 4, pp. 278–287.