ISSN 2075-1087, Gyroscopy and Navigation, 2016, Vol. 7, No. 2, pp. 189–196. © Pleiades Publishing, Ltd., 2016.
Published in Giroskopiya i Navigatsiya, 2015, No. 4, pp. 160–172.

Studying Some Algorithms for AUV Navigation Using a Single Beacon: The Results of Simulation and Sea Trials

F. S. Dubrovin[a] **and A. F. Scherbatyuk**[a, b]
[a] *Institute of Marine Technology Problems,*
Far East Branch of the Russian Academy of Sciences, Vladivostok
[b] *Far Easte Federal University, Vladivostok*
e-mail: alex_scherba@yandex.ru

Received June 28, 2015

Abstract—This paper is devoted to navigation of autonomous underwater vehicles (AUV) with a mobile hydroacoustic beacon transported by an autonomous surface vehicle (ASV). Two algorithms for AUV positioning using information on the distance to a single beacon and the data from the onboard autonomous navigation system have been studied. The first algorithm is based on the application of the extended Kalman filter, and the other one uses the particle filter. The simulation of the considered algorithms performance and the results of the sea trials using a marine autonomous robotic complex (MARC), including an AUV and an ASV, are discussed.

DOI: 10.1134/S2075108716020024

INTRODUCTION

One of the main objectives in the development of modern AUV is to provide high mobility and economic efficiency of the underwater robotic system as a whole, including the navigation support. The basis of an AUV navigation system is a dead reckoning system, which includes a Doppler log, AUV angular position sensors, and a depth sensor. The GNSS data are typically used only to determine the AUV position on the sea surface before the mission starts and after its completion.

The drawback of a dead reckoning (DR) system is gradual accumulation of errors included in the measurements of navigation sensors. Positioning errors may range between a few tens to hundreds of meters per hour, depending on the precision of the sensors, which is why the DR system is incapable of providing acceptable accuracy of AUV navigation on its own during lengthy missions that may last from a few tens of hours to several days. Furthermore, measurement of speed over ground with AUV Doppler logs is only possible near the bottom at a height of several tens of meters. In deep-sea missions, the AUV submerges for a rather long period of time until it reaches the height required for the operation of the Doppler log. In this period, the AUV horizontal drift due to currents can reach several hundred meters. This being so, it is necessary to refine the AUV position before the main mission starts.

In order to ensure the required accuracy of AUV navigation, which may range from 1 to 10 meters, the DR-coordinates of AUV must be periodically corrected during missions. AUV surfacing to update the GNSS position is too time- and power-consuming. The accumulated dead-reckoning errors can be reduced owing to long baseline hydroacoustic navigation systems (LBL HANS), which are of considerable current use [1, 2].

LBL HANS has several bottom transponder beacons equipped with transceivers. An AUV has an onboard transmitter of hydroacoustic signals, a receiver to record responses from transponders, and a computer to process incoming information and control the operation of the transmitter and receiver. In this system, the measured quantity is the time of the hydroacoustic signal propagation to the transponder beacon and back. The AUV position is determined based on the distances to the beacons. To obtain AUV coordinates unambiguously in a single step, it is necessary to have information from at least three of beacons.

Depending on the size of the AUV operation area, the LBL HANS operation may last from several seconds to several minutes. In this case the positioning error of a moving vehicle depends on the transponder coordinate accuracy, the knowledge of the signal effective velocity in water (to convert signal propagation time delays into ranges), and the accuracy of time recording (instants) when responses from beacons arrive. AUV positioning is significantly affected by multipath and false response of the HANS receiver to noise and pulse interference. In addition, responses may be lost because of obstructions between the AUV and the beacon. The error in determining the range between the beacon and the AUV may be from less than 0.01% of the measured range for precision LBL HANS to 0.1% for low-accuracy systems.

The application of an LBL HANS involves the deployment and calibration of a network of bottom beacons before the operation starts as well as their surfacing after the operation is over. These operations may take up to several days, and in so doing, there is a probability of losing some beacons. Moreover, the operation range of such a system is typically less than 10 km. In the surveys of extended areas, the system has to be deployed many times, which significantly increases the time and cost of the mission.

One way to improve the mobility of a navigation system is to use a network of sonar beacons moving on the sea surface and determining their locations with the use of GNSS data [3–6]. Another, more economical way to provide a mobile navigation system for the AUV is to design a HANS with a synthesized long base. The basic idea is to use a single mobile navigation beacon towed by a vessel or an ASV.

There are several approaches to the design of algorithms for solving navigation problems in a system with a synthesized long base, the most common being the application of the extended Kalman filter (EKF) [11–13]. However, its use is limited in the case of a large initial position error. In this situation, other methods are used, such as the method of maximum likelihood [7, 8], the sequential Monte Carlo method, also known as particle filter [12], and the algebraic method [14]. The geometric approach to this problem solution is described in [15], where the localization process consists in the constructing a domain that limits the possible positions of the AUV and executing some operations over this domain. Publications [16, 17] propose and study several algorithms for solution of this problem in the case of the arbitrary initial AUV position error.

This paper is devoted to navigation of an AUV with a single mobile hydroacoustic beacon. It is assumed that the navigation beacon is an ASV with a fixed hydroacoustic antenna. To determine the current location of a mobile beacon with the required accuracy, the ASV conveying it has a GNSS (GPS and GLONASS) receiver. This approach is based on using modem hydroacoustic communication, which allows the AUV and ASV to exchange data packets and determine the range between the vehicles, using the measured time of the acoustic signal propagation [7–10].

In their research, the authors implement the approach in which they determine the AUV initial position and estimate the effective velocity of sound signals in water at the preliminary stage [18]. Two algorithms for correcting AUV position using the data on the distance to the mobile hydroacoustic beacon have been studied. The first algorithm is based on the application of the EKF for estimation of the AUV position. The other algorithm consists in the application of the particle filter. A simulation system was used to study the performance of the proposed correction algorithms by the example of a mission, including long straight track lines. The simulation results make it possible to compare the accuracy of the AUV position correction algorithms under consideration. Some results of the proposed algorithms operation, as part of a marine autonomous robotic complex (MARC) including an AUV and ASV, obtained during the sea trials are discussed.

1. ALGORITHMS FOR AUV POSITION ESTIMATION

One of the approaches used by the authors to estimate the AUV position is based on the application of the EKF. The AUV coordinates and their estimation error are described by the state vector X and its covariance matrix P. Introduce the following notation: $\hat{X}_{k-1}$ is the estimate of the state vector obtained at the previous step, $\hat{P}_{k-1}$ is the covariance matrix for the estimated state vector, ΔX_k is the AUV motion vector, $\tilde{X}_k$ is the predicted state vector, and $\tilde{P}_k$ is its covariance matrix.

Since the depth of AUV submergence can be measured directly by a pressure sensor with high accuracy, in the subsequent discussion, we estimate only AUV coordinates in the horizontal plane and the state vector X consists of only two components.

Assume that the $(k-1)$-*th* navigation signal from a hydroacoustic beacon was received at time t_{k-1}. The result obtained due to the algorithm for correcting DR coordinates is the estimate of the AUV position in the horizontal plane $\hat{X}_{k-1} = [\hat{x}_{k-1}, \hat{y}_{k-1}]^T$.

In time intervals between the moments when navigation signals arrive from the beacon, AUV coordinates are reckoned by the DR system using the data from the Doppler log and the heading sensor. The trim and roll angles are assumed to be small and can be neglected. By time t_k, at which the AUV receives the k-th navigation signal, the AUV motion $\Delta X_k = [\Delta x_k, \Delta y_k]^T$ is determined by the following equations:

$$\Delta x_k = \int_{t_{k-1}}^{t_k} \upsilon(t)\cos\varphi(t)dt, \quad \Delta y_k = \int_{t_{k-1}}^{t_k} \upsilon(t)\sin\varphi(t)dt, \quad (1)$$

where $\upsilon(t)$ and $\varphi(t)$ are the AUV forward speed and heading, respectively

At the prediction stage of the Kalman filter operation, $\tilde{X}_k$ and $\tilde{P}_k$ are calculated with the use of estimates $\hat{X}_{k-1}$ and $\hat{P}_{k-1}$ obtained at the previous step, the AUV motion vector ΔX_k, determined from (1), and its covariance matrix N_k. Propagation is performed in accordance with the following equations:

$$\tilde{X}_k = \hat{X}_{k-1} + \Delta X_k, \quad (2)$$

$$\tilde{P}_k = \hat{P}_{k-1} + N_k. \quad (3)$$

At the correction stage of the discrete Kalman filter operation, we have

$$\hat{X}_k = \tilde{X}_k + K_k\left[D_k - d\left(\tilde{X}_k, z_k\right)\right], \tag{4}$$

$$\tilde{P}_k = \hat{P}_{k-1} + N_k, \tag{5}$$

where D_k and $d\left(\tilde{X}_k, z_k\right)$ are the measured and calculated (expected) ranges between the beacon and the AUV at the k-th cycle of the algorithm operation:

$$d(\tilde{X}_k, z_k) = \sqrt{(\tilde{x}_k - x_{bk})^2 + (\tilde{y}_k - y_{bk})^2 + (z_k - z_{bk})^2}, \tag{6}$$

x_{bk}, y_{bk}, z_{bk} are the beacon coordinates at the moment of its k-th response, z_k is the depth of the AUV submergence at the moment of the k-th response, K_k is the optimal Kalman gain matrix calculated by the equation:

$$K_k = \tilde{P}_k H_k^T\left[H_k \tilde{P}_k H_k^T + R_k\right]^{-1}, \tag{7}$$

where R_k is the error covariance matrix of range measurements; H_k is the matrix of partial derivatives for the measurement function at point $\tilde{X}_k$:

$$\begin{aligned} H_k &= \left[\frac{\partial d(X, z_k)}{\partial X}\right]_{X=\tilde{X}_k} \\ &= \frac{1}{\sqrt{(\tilde{x}_k - x_{bk})^2 + (\tilde{y}_k - y_{bk})^2 + (z_k - z_{bk})^2}} \\ &\quad \times \left[\tilde{x}_k - x_{bk} \;\; \tilde{y}_k - y_{bk}\right]. \end{aligned} \tag{8}$$

After two steps of the algorithm operation, the previous AUV position estimate $\hat{X}_{k-1}$, its covariance matrix $\hat{P}_{k-1}$ and the distance to the beacon D_k are used to generate the current posterior estimate of the AUV position in the horizontal plane $\hat{X}_k = \left[\hat{x}_k, \hat{y}_k\right]^T$ and the covariance matrix $\hat{P}_k$ for this error. The knowledge of the covariance matrix makes it possible to control the convergence and stability of the estimation process.

Another way of obtaining the AUV position is to use the particle filter [20, 21], which allows the probability distribution of the probable AUV position to be described by a set of points $\{\mathbf{X}_1, \ldots, \mathbf{X}_N\}$, so-called particles, each having its own weight ω_i.

The algorithm operation consists of several steps.

1. **Initialization.** The available prior information is used to generate a set of N particles with the same weights randomly placed on the plane over the area of the AUV probable location. The filter can be initialized in different ways, for example:

(a) particles with equal weights are distributed within a ring, the center of which is at the point of the beacon position, the ring average radius corresponds to the horizontal distance from the beacon to the AUV, and the width of the ring is chosen taking into account the accuracy of the range measurement by the HANS;

(b) particles with equal weights are distributed within a circle, the center of which is at the point of AUV submersion. The coordinates of this point are fixed by a GNSS receiver, and the radius of the circle is chosen taking into account the probable uncontrolled drift of the AUV caused by currents over the time of its dive. This method was used in our research.

2. **Propagation.** Using the information from the onboard DR system, a cloud of particles moves with the addition of random errors to the displacement vector.

3. **Correction**. In measuring the distance to the beacon, the weights of particles are recalculated to take into account the previous weights and the degree of how close the agreement between the obtained range measurement and particle coordinates is.

4. **Resampling.** During long-term operation of the filter, only a small number of particles have weights significantly different from zero: most of the particles become degenerated (their weights decrease so that they can be neglected). In connection with this, after each step of correction, the effective number of particles is determined by the formula:

$$N_{\text{eff}} = \frac{1}{\sum_{i=1}^{N} \omega_i^2}. \tag{9}$$

If the effective number of particles is less than the specified threshold N_{th}, it is necessary to perform procedure of particle regeneration in order to remove degenerated particles and saturate the area of the most probable AUV position with new particles. Particles with small weights are removed and instead, new particles are generated, which are distributed around the remaining particles with large weights in proportion to their weights.

As an estimate of the AUV position, we use either the coordinates of the particle with a maximum weight, exceeding a predetermined threshold, or the weighted average of the coordinates of all particles:

$$\bar{\mathbf{X}} = \sum_{i=1}^{N} \mathbf{X}_i \omega_i. \tag{10}$$

The covariance matrix of the error ellipse is used as a measure of the estimation accuracy, which is calculated for the existing cloud of particles by the formula:

$$\begin{aligned} P = \begin{pmatrix} p_x & p_{xy} \\ p_{xy} & p_y \end{pmatrix} &= E\left[(\mathbf{X} - \mathbf{EX})(\mathbf{X} - \mathbf{EX})^T\right] \\ &= \sum_{i=1}^{N} \omega_i \left(\mathbf{X}_i - \bar{\mathbf{X}}\right)\left(\mathbf{X}_i - \bar{\mathbf{X}}\right)^T. \end{aligned} \tag{11}$$

2. SIMULATION SYSTEM

A simulation system was specially developed to analyze the operation of the proposed algorithms. The system is written in C++ with the use of the Boost and OpenCV open-source libraries for mathematical calculations and visualization. The simulation system is capable of generating any quantity of AUVs and bea-

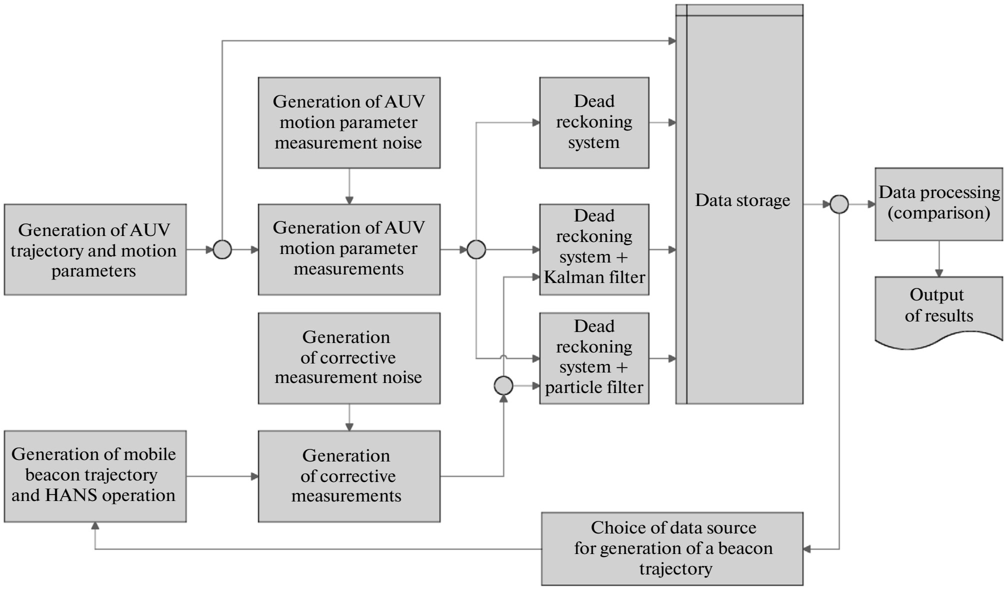

Fig. 1. A block diagram of the simulation system.

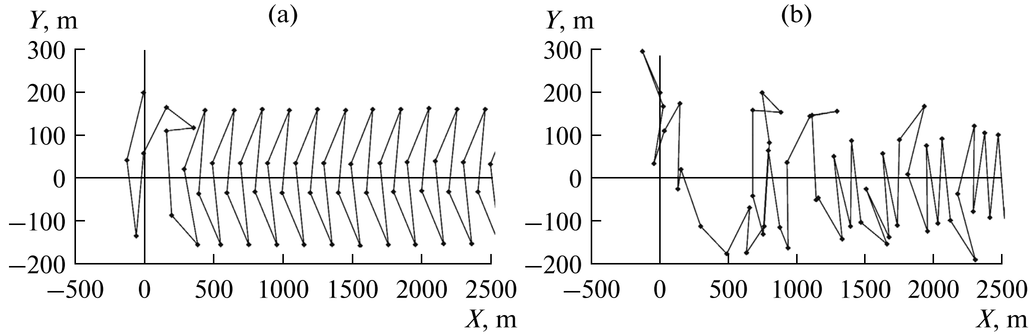

Fig. 2. A motion trajectory of a mobile beacon obtained with the use of the Kalman filter (a) and the particle filter (b).

cons. It is also possible to test several algorithms for estimation of AUV position simultaneously. A block diagram of the simulation system is given in Fig. 1.

This simulation system was used to study the operation of algorithms based on the EKF and the particle filter. These algorithms can be run in parallel with the same set of the simulation parameters, such as measurement noise parameters, AUV and beacon motion parameters, etc. In this case, each algorithm makes use of its own system that consists of an AUV and a beacon, the initial positions and parameters of which are the same.

The simulation system has a unit for generation of AUV motion trajectories, which makes it possible to generate trajectories of arbitrary shapes and control AUV motion parameters. The unit for generation of the beacon motion trajectories has several modes of operation. In the first mode, the beacon trajectory is rigid. It does not depend on external factors and is given as a set of coordinates versus time. In the second mode, depending on the result of the AUV position correction, the beacon chooses a target point at each step of the HANS operation, thereby forming its trajectory directly during the operation. In addition, the simulation system generates measurements of AUVs motion parameters and their measurement noise.

3. SOME SIMULATION RESULTS

We have conducted a series of simulation experiments with different parameters, their aim being to compare the accuracies of the algorithms for AUV position correction and the trajectories of the mobile beacon. Figures 2–4 show some plots of errors in the AUV position estimation in the case when the EKF and the particle filter are used.

The AUV moved straightforward at a constant depth of 500 m from the point with coordinates (0, 0), the heading $\phi = 0$, and at a constant veloc-

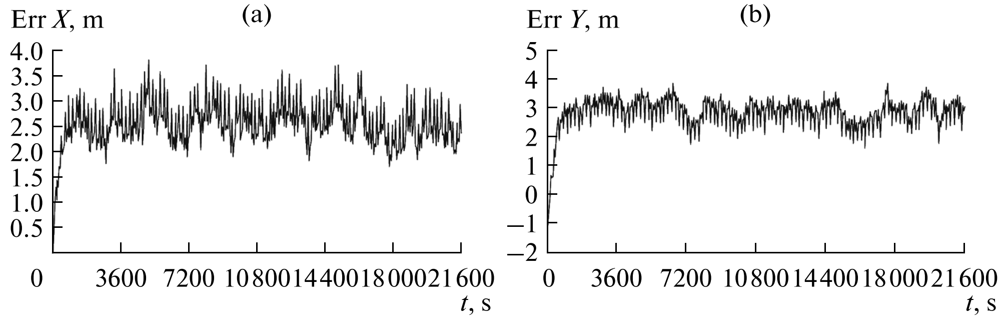

Fig. 3. Errors in the AUV position estimation with the use of the extended Kalman filter.

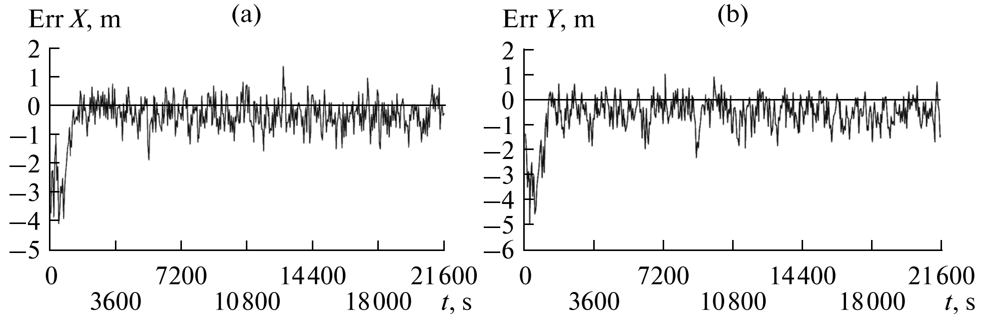

Fig. 4. Errors in the AUV position estimation with the use of the particle filter.

ity of $\upsilon = 1$ m/s. The beacon was located at a depth of 1 m and its maximum velocity did not exceed 4 m/s.

The operation cycle of the onboard DR system was 0.1 s and the HANS operation cycle was 50 s. The beacon trajectory was generated in accordance with the algorithm described in [18]. The mission lasted 6 hours. During the simulation with the particle filter, the cloud consisted of 1000 particles.

The measurement errors of the slant range between the AUV and the beacon were simulated by adding normally distributed random values with zero mean and standard deviation of 0.1% from the true distances. The AUV velocity measurement errors were simulated as a normally distributed random variable with a fixed bias of 0.01 m/s and standard deviation of 0.05 m/s. The AUV heading measurement errors were simulated as a normally distributed random variable with a fixed bias of 0.1° and standard deviation of 0.5°.

The above mentioned constant errors in velocity and heading measurements accumulated over 4 hours of operation cause the DR error of about 150 m.

From the results obtained it may be concluded that the errors in the AUV position estimation for the both algorithms are approximately the same, coming up to several meters.

4. SOME MARINE TRIAL RESULTS OF THE ALGORITHM PERFORMANCE

The planned marine trials of the MARC, including an AUV and an ASV, were performed in the Novik Bay of Russky Island from September 15 to October 26, 2014 (Fig. 5). The sea depth in the area of the field experiments did not exceed 15 meters and the sea wave height was within 0.5–1.25 m. In the main experiments the AUV moved at a depth from 2 to 5 m. The acoustic antenna of the AUV was at a depth of 1.5 m. The level of acoustic interference was such that the number of faulty cycles in the operation of the hydroacoustic navigation system was no more than 15% of the total number.

Figure 6 is a plot showing the sound velocity variations depending on the AUV depth in one of the experiments. The plot was constructed based on the data from the Citadel CTD sensor of water salinity, temperature and pressure (Teledyne RD Instruments, USA) installed onboard the AUV.

The main objectives of the sea trials were the following:

- adjustment and testing of the high-precision DGPS-aided system with a Trimble SPS 855 GNSS receiver and a Trimble GA810 GNSS antenna included in the base station and mobile (rover) stations operating in RTK mode;
- estimation of the calculated range accuracy by measuring the time of the hydroacoustic signal propa-

Fig. 5. Maneuvering of the ASV with an acoustical beacon and a DGPS-aided system in the vicinity of the AUV.

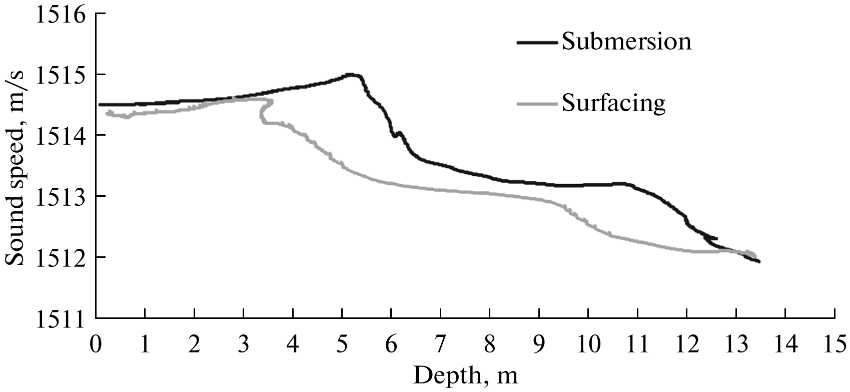

Fig. 6. A plot of sound speed variations against the AUV submersion depth in one of the missions.

gation with EvoLogics S2C R 18/34 hydroacoustic modems [19];

- execution of sea experiments on single-beacon navigation with the use of the MARC.

The feature of the precision DGPS-aided system with a Trimble SPS 855 GNSS receiver is its capability of receiving navigation data from different satellite navigation systems, including GPS, GLONASS, and the GALILEO.

The accuracy of determining the range between the ASV towing the hydroacoustic beacon and the AUV with the use of EvoLogics S2C R 18/34 modems was estimated by the following procedures. The AUV was fixed in a submerged position at the edge of the pier at the point the coordinates of which were measured with centimeter accuracy using the DGPS-aided system. Then the ASV, having the rover part of DGPS-aided system, performed various maneuvers, including stops at specified points. As a result, it was determine that the difference in determining the distance between the AUV and ASV obtained with the DGPS and the acoustic modems was no more than 10 cm for 1 km distances and less.

The accuracy of the LBL HANS operation was studied using the following procedures. The actual coordinates of the AUV start (submersion) and finish (surfacing) points were measured with the DGPS-aided system on the ASV. The obtained coordinates were compared with the results of the LBL HANS operation. In the sea experiments, slant distances between the AUV and the beacon did not exceed 150 m.

Figures 7 and 8 show the results obtained in one of the experiments performed on October 20, 2014. The estimates of the AUV position (relative to the start point) at the end of the mission were the following:

—based on the DR data: $X = 927.26$ m, $Y = -51.70$ m,

—in accordance with the EKF-based algorithm: $X = 934.27$ m, $Y = -9.07$ m,

—in accordance with the algorithm based on the particle filter: $X = 934.92$ m, $Y = -11.22$ m,

—based on the data from the ASV DGPS-aided system: $X = 934.57$ m, $Y = -11.26$ m.

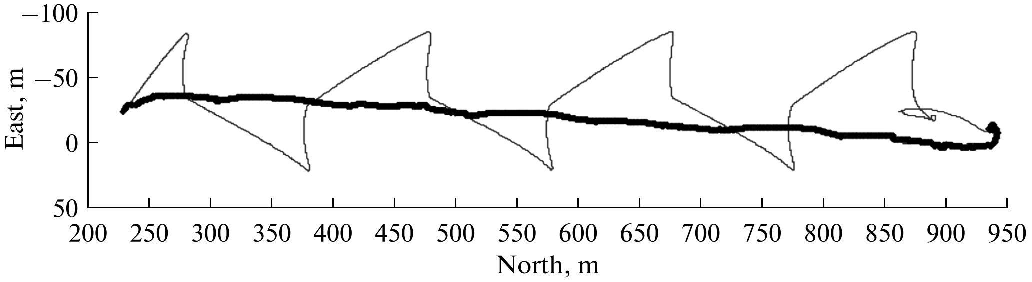

Fig. 7. The ASV trajectory from the DGPS-aided system (thin line) and the AUV trajectory obtained with the use of the particle filter (thick line) (Run no. 1, October 20, 2014).

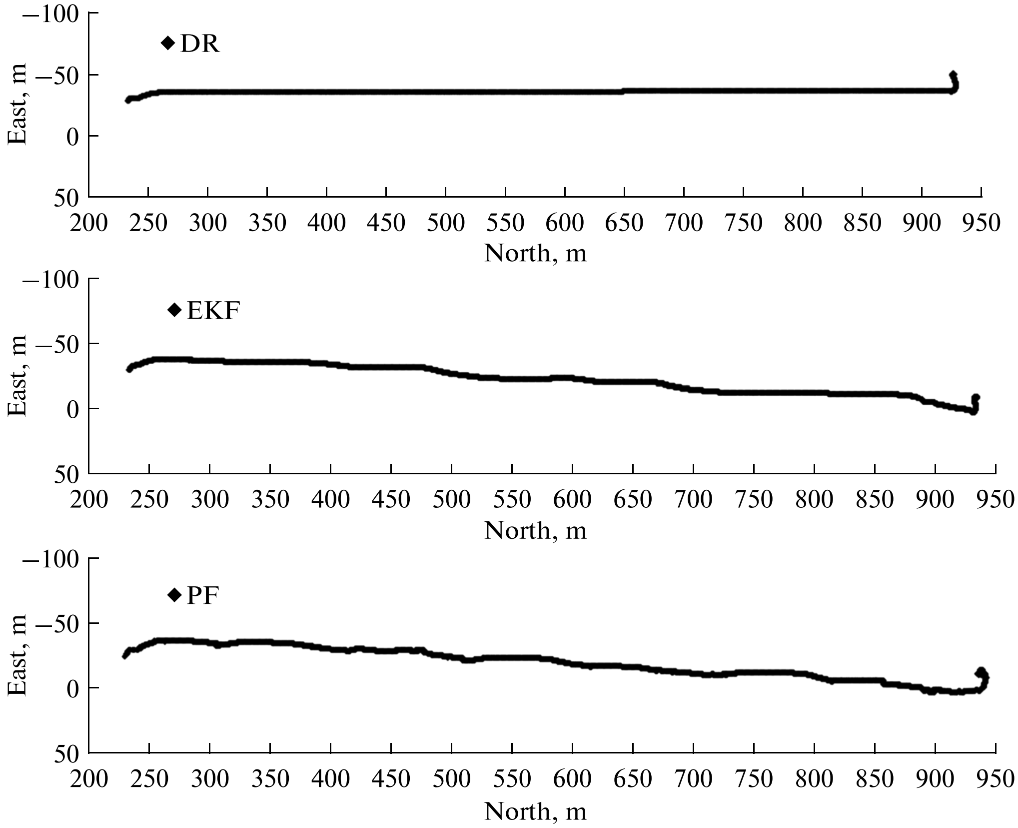

Fig. 8. AUV trajectories obtained with the use of the DR data, in accordance with the EKF-based algorithm, and the algorithm based on the particle filter (Run no. 1, October 20, 2014).

Thus, if the AUV coordinates obtained with the DGPS-aided system are considered true, the AUV positioning errors for the DR system, the EKF-based algorithm, and the algorithm based on the particle filter have the values shown in Table 1.

The errors in the AUV position estimation by the end of the mission

Dead reckoning	Extended Kalman filter	Particle filter
$\Delta X = 7.31$ m $\Delta Y = 40.44$ m	$\Delta X = 0.3$ m $\Delta Y = -2.19$ m	$\Delta X = -0.35$ m $\Delta Y = -0.04$ m

CONCLUSIONS

The results of the sea trials confirm the efficiency and high precision of the developed LBL HANS. The comparison of the navigation algorithms based on the EKF and the particle filter shows a higher accuracy of the algorithm based on the particle filter, as applied to our experiment.

ACKNOWLEDGMENTS

We are grateful to all the employees of the Institute for Marine Technology Problems (Vladivostok) and the Far Eastern Federal University who took part in the preparation and execution of the sea trials with the use of the MARC.

This work was supported by the Russian Foundation for Basic Research, project no. 130800967, Russian Scientific Foundation, project no. 145000034, and by a grant of the Far Eastern Federal University.

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