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Model Based Control of a Quadrotor with Tiltable Rotors

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Abstract—Micro Aerial Vehicles (MAV) with vertical takeoff and landing capabilities such as quadrotors are often used as sensor platforms. The carried equipment like cameras or LASER range finders has to be aligned to some point of interest. In this article a modified type of a quadrotor will be presented: a quadrotor with tiltable rotors which in contrast to common quadrotors is able to perform independent velocity and attitude movements. This ability makes additional aligning equipment to move the payload redundant. After a system description, the used control algorithm based on Nonlinear Inverse Dynamics (NID) is explained. In this article an extension of this approach is presented. The pseudo control hedging method removes the influence of the actuator dynamics from the control loop. The extension and its integration into the control algorithm are explained and the influence on the quality of control is demonstrated by simulation results.

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1. INTRODUCTION

In order to support rescue forces and to monitor areas of catastrophic events Micro Aerial Vehicles (MAV) have become of increased interest during the past years. Therefore these vehicles should be able to perform the given tasks autonomously and without the aid of humans. An ongoing challenge is to navigate the vehicles in indoor as well as in outdoor environments. To meet the requirement of indoor operations often quadrotors are used because of their ability of hovering, vertical takeoff and landing. Many tasks to be performed require additional equipment to be carried along with the vehicle. Mostly this equipment consists of sensors to deliver additional information to the operator like cameras or LASER range finders. Therefore this equipment needs to be aligned in order to keep a point of interest in the focus. With common quadrotors which are coupled in velocity and attitude the alignment has to be done using additional equipment which comes with additional weight. A quadrotor with tiltable rotors solves this problem by design and furthermore adds new flexibility in the use of this class of vehicles. On the one hand the ability of tilting the rotors leads to a system where attitude and velocity can be controlled independently of each other. This ability does not only affect the alignment of sensors but as well the overall system performance. On the other hand, compared to a common quadrotor, additional weight is added which decreases the time of operation and the complexity of the hardware setup is increased. In the event of a disturbance like for example a sudden wind gust the quadrotor can keep performing its task without a change in attitude, velocity or height and the reaction is faster than with a common quadrotor. Next to this advantages the quadrotor with tiltable rotors holds the opportunity to land on uneven surfaces or even on a moving vehicle like for example an unmanned ground vehicle.

In order to control the vehicle a model based approach is used. The basic control algorithm is called Nonlinear Inverse Dynamics (NID) and this approach is extended to increase the controller performance. The extension used is Pseudo Control Hedging (PCH) which basically drives the system as close as possible to its limits and if necessary slows down the commanded dynamics.

1.1. Related Work

During the past few years quadrotors have become of interest for different kinds of application. Due to this fact the mechanical and software design has become an important area of research and development. By definition quadrotors are unstable systems. Therefore basic control algorithms had to be developed in order to keep the vehicle up in the air. Examples for the control of common quadrotors are given in [1, 2, 11, 12, 14]. The first two, S. Bouabdallah [2] and A. Nagaty [12] describe the basic control of a quadrotor system. M. Orsag [14] and K. Alexis [1] present more advanced approaches. Orsag describes a hybrid control approach where the linear controller is combined with a discrete automaton whereas Alexis uses model predictive control. The last one, M. Basri [11] describes a whole framework for the simulation of quadrotors and controlling them using a backstepping control design.

The design of M. Basri in combination with the work of K. Alexis leads to M. Ryll who discussed a new design of quadrotors: the quadrotor with tiltable rotors. It was discussed by Ryll in 2012 [15] and 2013 [16]. The same system setup was described by A. Nemati in 2014 [13], but in his approach he is using a linear controller. The control approach given by M. Ryll is quite similar to an approach called nonlinear inverse dynamics, which is used in this article. Nonlinear inverse dynamics were already discussed by A. Isidory [4], E. N. Johnson [5], N. Kim [6] and F. Holzapfel [3]. Johnson, Kim and Holzapfel used the NID approach to control airplanes or models of them. In our approach the control algorithm for a quadrotor with tiltable rotors is extended: pseudo control hedging is used which is a new approach in controlling quadrotors with tiltable rotors.

1.2. Goal of this Paper

This article presents a new design approach of a quadrotor. This approach requires a new method of control which is described in this article. The control approach is based on nonlinear inverse dynamics and is extended by pseudo control hedging. The basics of this approach are given and the application of the control design to the quadrotor is demonstrated and proven with the help of simulation results.

2. SYSTEM DESCRIPTION

In this section the basic setup will be described as well as the basic information needed to understand this article. Therefore a brief introduction to the used coordinate systems and to the navigation algorithm is given.

2.1. Mechanical Setup

The basic setup is defined as follows: the quadrotor consists of four lever arms which are arranged similar to the algebraic sign + (plus). Attached to each lever arm a servo-electrical motor is mounted which enables the tilting of the propulsion motor. The propulsion motor is a powerful brushless motor which delivers the needed force to create the thrust using the attached rotors. The rotors are made of plastic which is improved with carbon fiber. In the center of the vehicle are the motor controllers, the batteries and the navigation and computational board. In Fig. 1 the actually realized hardware setup is shown. The boxes at each end of the lever arms are the used servo-electrical devices to tilt the rotors.

The propulsion motors can be tilted around each lever arm with the tilting angle σ_i. The index i indicates the lever arm, on which the motor is mounted on where $i = 1$ defines the front lever arm. The other arms are defined using $i = 2...4$ in clockwise rotation from a top down view on the quadrotor. Each motor creates a lift force F_i and a torque M_i.

Fig. 1. Image of the actually built quadrocopter with tiltable rotors.

2.2. Coordinate Systems

For purposes of navigation the navigation coordinate system called n-frame (or *ned*-frame) is used. The origin of this frame is at the center of gravity of the vehicle. The first and second axes define a pane parallel to the surface of the earth. The first axis points towards north, the second one towards east and the third one completes the right handed coordinate system pointing downwards.

The body fixed coordinate system (b frame) is attached to the vehicle itself. The first axis of the body fixed coordinate system is aligned with the front lever arm of the vehicle. The second one is aligned with the right lever arm from a top down view. The last axis points downwards. Again the center of gravity of the vehicle defines the origin of this coordinate system.

Due to the fact that the rotors of the quadrotor can be tilted there is another body fixed coordinate system. In fact there are four different coordinate systems. They are located near each rotor of the vehicle and called rotor frames (r frame). The origin is the intersection of the lever arm axis with the axis around which each rotor is spinning. The first axis is defined as being aligned with the lever arm on which the rotor is mounted on. The third axis is defined to point downwards along the spinning axis of the rotor if it is in upright position and the second axis completes this right handed coordinate system lying parallel to the rotor pane.

The rotation from one coordinate system to another can be described using rotation matrices C. To rotate the resulting lift force F_L from the rotor measured in the rotor frame into the navigation frame the matrices C_r^b and C_b^n are used:

$$\vec{F}^n = C_b^n C_r^b \vec{F}^r = C_b^n \vec{F}^b. \quad (1)$$

The matrix C_b^n defines a rotation from the body frame (index b) to the navigation frame (index n). All other rotation matrices C are defined according to this definition.

2.3. System Model

In this section the main mechanical torques and forces influencing the system are described which are both connected to each other.

2.3.1. Sum of Torques

The overall torque M consists of additional parts and will be presented as a sum of all torques in the system:

$$M_{\text{total}} = M_{gT} + M_t + M_a + M_{\Delta V} + M_{Go} + M_{Bo} + M_L. \quad (2)$$

The torque M_{gT} results from the tilting angular velocity $\dot{\sigma}_i$ of the servo-electrical motors:

$$\vec{M}^b_{gT} = \sum_{i=1}^{4} -C^b_{r_i}\left(J^i_r \vec{\omega}^{r_i}_{r_i b} \times \dot{\vec{\sigma}}^{r_i}_{r_i b}\right), \quad (3)$$

with J_r being the inertia matrix of a rotor. The variable ω describes the rotational speed between the second and the first lower index given in coordinates indicated by the upper index. The time derivative is given by $\dot{\omega}$. The torque M_t results from the thrust of the rotors and is given by

$$\vec{M}^b_t = \sum_{i=1}^{4} \vec{l}^b_{r_i} \times \vec{f}^b_{ir_i} = \sum_{i=1}^{4} \vec{l}^b_{r_i} \times C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ -f^{r_i}_z \end{bmatrix}, \quad (4)$$

where l is the length of the side arm on which the motor is mounted on. The force $f^{r_i}_z$ is the force created from one propulsion motor in rotor frame coordinates (index r). The aerodynamic forces such as resulting drag force from the lift force are summed up and taken into account by the torque M_a:

$$\vec{M}_a = \lambda_c l \sum_{i=1}^{4} C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ f^{r_i}_z \end{bmatrix} (-1)^i, \quad (5)$$

where λ_c is a coefficient to calculate the resulting torque from a given thrust. The torque $M_{\Delta V}$ describes the influence of changes in the angular velocity to the resulting torque from the spinning rotors:

$$\vec{M}^b_{\Delta V} = C^b_{r_i} \vec{M}^{r_i}_{\Delta V} = C^b_{r_i} J_r \frac{\partial \vec{\omega}^{r_i}_{r_i}}{\partial t}. \quad (6)$$

According to equation (6), the influence of the inertia of the rotors must also be taken into account:

$$\vec{M}^b_{Go} = \vec{\omega}^b_{eb} \times C^b_{r_i} J_r \vec{\omega}^{r_i}_{rb}. \quad (7)$$

The inertia of the vehicle is considered by the torque M_{Bo}:

$$\vec{M}^b_{Bo} = \vec{\omega}^b_{ib} \times J^b_q \vec{\omega}^b_{ib}, \quad (8)$$

with J_q being the inertia matrix of the quadrotor. The above mentioned different parts of torque describe the torques resulting from the vehicle itself and its behavior. No external influences are taken into consideration. The torque M_L describes an induced torque with respect to a payload. In this case there is no payload and M_L can be neglected.

2.3.2 Sum of Forces

The forces applying in the system are given by

$$\vec{F}^n_{ib} = m \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + k_{adf} \vec{v}^n_{ib} [\vec{v}^n_{ib}] + \sum_{i=1}^{4} C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ -f^{r_i}_{\text{actuator}} \end{bmatrix}, \quad (9)$$

where m is the mass of the system, g is the gravity, k_{adf} is a coefficient for the aerodynamic drag force, $\vec{v}^n_{ib}$ is describing the velocity from body frame to the inertial frame given in navigation coordinates and $f^{r_i}_{\text{actuator}}$ is the resulting lift force of a single rotor.

2.4. Navigation and guidance

In order to control a system a command signal is needed as well as some information about the system response. In this article the term guidance indicates a generator for a command signal. The term navigation refers to an estimation of the system response. Both terms will be explained in the following two subsections starting with the guidance.

2.5. Guidance

As mentioned above the guidance is used to generate a command signal. The command values for a quadrotor with tiltable rotors have been defined to be the attitude expressed in Euler angles or better as quaternions to avoid ambiguities and a commanded height as well as a downwards velocity. So far this set of command values is similar to the command value set of a common quadrotor. Additionally for the control of the quadrotor with tiltable rotors velocity command values in northern and eastern (or body x and body y) direction have to be delivered.

This leads to a guidance system which is able to provide the control algorithm with these commands. The first implemented approach is a guidance based on waypoint navigation. An implementation for a common quadrotor is discussed in [10]. The new guidance design uses basically the same ideas like the common one: following a path represented by a list of different way points, waiting at a certain way point for a given time, turning around the yaw axis and aligning the body x axis to a point of interest while following the commanded path. Especially the latter task, the point of interest alignment, is only possible if the point of interest is at the same height as the vehicle itself. A new development is a point of interest which can be

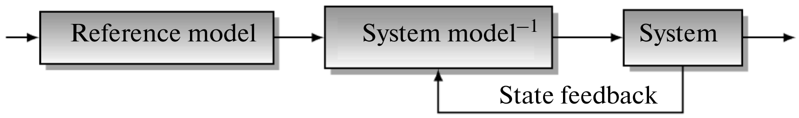

Fig. 2. Basic control strategy of nonlinear inverted dynamics.

observed from angles in height. So, like for example the quadrotor with tiltable rotors is able to align its body x axis to this point of interest while flying in circles around this point and constantly increasing its flight level.

The above mentioned guidance is one possible use of the additional degrees of control which are delivered by the quadrotor with tiltable rotors in comparison to a common quadrotor. In order to perform the control task a navigation solution is needed which describes the actual state of the vehicle which is done in the next section.

2.6. Navigation

Different from guidance and control the navigation of a quadrotor with tiltable rotors remains mostly untouched compared to a common quadrotor. The basic navigation system using a Kalman filter is described in [10]. The Kalman filter is used to combine the inputs of different sensors in order to provide a stable navigation solution over time. Basically measurements from a Global Navigation Satellite System (GNSS) and an Inertial Measurement Unit (IMU) are fused. Additional sensors can be used to support this fusion. Examples given in [10] are a barometric height measurement and a magnetometer to estimate the vehicles orientation.

With the above described setup it is possible to design a controller in order to provide the needed information to the propulsion motors as well as to the servo-electrical motors.

3. CONTROL ALGORITHM

The control design is called nonlinear inverse dynamics and is based on a system model which has to be inverted. Therefore such a model must be provided and reasonable assumptions have to be made in order to keep the computational load on the hardware to a reasonable amount.

3.1. System Model of the Controller

The system model which is described in section 2.3 is too complex to be inverted for the controller design. Some simplifications have to be made. The system model for the controller is derived from the more complex models of the torques and forces given in equations (2) and (9).

The main influence to the rotational dynamics is given by the torques resulting from the thrust M_t in equation (4) and the aerodynamics M_a given in equation (5). The resulting torque $M_{c\text{Total}}$ has to match the inertia of the system in order to control the MAV and is given by

$$\vec{M}^b_{c\text{Total}} = J^b_q \dot{\vec{\omega}}^b_{ib} = \vec{M}^b_t + \vec{M}^b_a. \tag{10}$$

Rearranging equation (10) leads to

$$\dot{\vec{\omega}}^b_{ib} = J_q^{b^{-1}} \left(\sum_{i=1}^{4} \vec{l}^b_{r_i} \times C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ f_z^{r_i} \end{bmatrix} + \lambda_c C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ f_z^{r_i} \end{bmatrix} (-1)^i \right). \tag{11}$$

The analysis of the forces given in equation (9) leads to the following total control force $F_{c\text{Total}}$:

$$\vec{F}^n_{c\text{Total}} = m\ddot{\vec{p}}^n_{ib} = \vec{F}^n_g + \vec{F}^n_L. \tag{12}$$

Rearranging equation (12) leads to the acceleration $\ddot{\vec{p}}^n_{ib} = \vec{a}^n_{ib}$ given in navigation coordinates:

$$\ddot{\vec{p}}^n_{ib} = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \frac{1}{m} C^n_b \sum_{i=1}^{4} C^b_{r_i} \begin{bmatrix} 0 \\ 0 \\ -f_z^{r_i} \end{bmatrix}. \tag{13}$$

3.2. Nonlinear Inverse Dynamics

The nonlinear inverse dynamics approach is a model based one and was developed for the control of systems which contain nonlinear dynamics. The basic idea of NID control is to invert a model of the system in order to estimate the input to the system which is needed to achieve a desired system response. The application of NID control leads to a system with a linear input/output behavior. This rearranged system can be controlled using common methods from linear control theory. In order to keep the system model in a state as close as possible to the real system state a state feedback is used. The basic controller design is given in Fig. 2.

3.2.1. Basic Control Theory

At first a nonlinear system with multiple input and multiple output signals is considered. The state space representation is given by

$$\dot{\vec{x}} = \vec{f}(\vec{x}) + \vec{G}(\vec{x}) \cdot \vec{u}, \quad \vec{y} = \vec{h}(\vec{x}). \tag{14}$$

The input $\vec{u}$ to the system is given by the rotational speed of the four rotors and the tilting angle of each rotor $\sigma_1 \ldots \sigma_4$. The rotational speed results in a lift force according to equations (1), (9) and a torque according to equation (6). The tilting angle has an influence on the resulting forces and torque as well according to equations (3), (4), (5), (6), (7) and (9). The state vector $\vec{x}$ is given by the attitude of the vehicle (Φ, Θ, Ψ), the speed ($\vec{v}_{ned}$), the rotational speed ($\vec{\omega}$) and the position ($\vec{p}_{ned}$) of the vehicle where $\vec{v}_{ned}$ is the velocity of the vehicle measured in navigation coordinates. The output vector $\vec{y}$ is given by $\vec{y} = [\vec{p}_{ned}, \vec{v}_{ned}, \Phi, \Theta, \Psi]$.

The following derivations are displayed for a single input single output system which can be transferred into a multiple input multiple output system easily in order to simplify the equations. The functions $\vec{f}$, $\vec{G}$, and $\vec{h}$ are vector fields of nonlinear functions, they become single functions considering the single input, single output system.

As mentioned at the beginning of section 3 NID control is based on the inversion of a system model. Unfortunately the system model for most nonlinear systems cannot be inverted very easily. To be exact a Lie derivation of the system model is needed to invert the model. According to [8] the derivation is

$$\begin{aligned} \dot{y} = \frac{dy}{dt} = \frac{dh}{dx}\frac{dx}{dt} = \frac{dh}{dx}(f(x) + g(x)u) \\ = L_f h(\vec{x}) + L_g h(\vec{x})u. \end{aligned} \tag{15}$$

The degree of the derivation at which the input signal u is independent from the associated derivation of the system response y is called relative degree r. Considering the order of the system to be exactly the same size as the relative degree r the vector $\vec{z}$ is defined as

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_r \end{bmatrix} = \begin{bmatrix} L_f^0 h(\vec{x}) \\ L_f^1 h(\vec{x}) \\ \vdots \\ L_f^{r-1} h(\vec{x}) \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_n \end{bmatrix}. \tag{16}$$

The equation (16) holds the requirement of the input u being independent:

$$\dot{z}_r = b(z) + a(z)u. \tag{17}$$

If the order of the system is greater than the relative degree r, the remaining states must be transformed as well.

This is done according to [4] and called Byrnes-Isidori normal form:

$$\begin{bmatrix} z_1 \\ \vdots \\ z_r \\ z_{r+1} \\ \vdots \\ z_n \end{bmatrix} = \begin{bmatrix} \xi_1 \\ \vdots \\ \xi_r \\ \eta_1 \\ \vdots \\ \eta_{n-r} \end{bmatrix}. \tag{18}$$

The resulting vector z is composed of the external state ξ which is observable and the unobservable remaining part η. The unobservable part η leads to no difficulty if it is stable according to the Lyapunov stability theory. If the part is unstable the whole system cannot be proven stable. Defining the rth differentiation of the output y as:

$$y^{(r)} = v, \tag{19}$$

the state feedback as in [4]

$$u = \alpha(\vec{x}) + \beta(\vec{x})v, \tag{20}$$

and using the transformation given in equation (18), the state feedback loop of the system can be written as

$$y^{(r)} = b(\vec{x}) + a(\vec{x})u = b(\vec{x}) + a(\vec{x})[\alpha(\vec{x}) + \beta(\vec{x})v]. \tag{21}$$

To invert the system α and β must be chosen as

$$\beta(\vec{x}) = a^{-1}(\vec{x}), \quad \alpha(\vec{x}) = -a^{-1}(\vec{x})b(\vec{x}). \tag{22}$$

To perform this rearrangement, $\alpha(\vec{x})$ must be invertible, which is ensured by definition of the relative degree r. According to [3], the input u of a multiple input, multiple output system can be written as

$$\vec{u} = A^{-1}(\vec{x})\vec{v} - A^{-1}(\vec{x})b(\vec{x}), \tag{23}$$

where A^{-1} is the inverted system matrix. Therefore the matrix A has to be a square matrix. Regarding the quadrotor with tiltable rotors this requirement is not met because number of freedom degrees of the system which is six does not match the number of actuator inputs which is eight. Therefore the system matrix is not a square matrix and the inverted system matrix A^{-1} must be replaced with a pseudo inverted system matrix $A^{\dagger}(\vec{x})$:

$$A^{\dagger}(\vec{x}) = A^T(AA^T)^{-1}. \tag{24}$$

The algebraic expression given in equation (24) is called orthogonal projection and is described in [7]. The orthogonal projection deals with two problems at a time: the over actuated system is transformed to a fully actuated system and the energy minimizing principle is established. The pseudo inverse matrix is given with the following definition taken from [7]:

Definition 1: Orthogonal projection

Given a $n \times m$ matrix $A = [a_1, a_2 \ldots a_m]$ consisting of linear independent column vectors a subspace of m^{th} dimension can be defined using the linear independent vectors a_i as base vectors:

$$\text{Image } A\text{: } \{y \in \mathbb{R}^n \,|\, \exists x \in \mathbb{R}^m, Ax = y\}. \tag{25}$$

A given vector $z \in \mathbb{R}^n$ can be separated into a part z_s lying in the subspace $\mathbb{R}^m$ and an orthogonal part z_0 which completes the vector z. The resulting transformation matrix is given in equation (24) and describes the projection of a vector from the $\mathbb{R}^n$ space to the $\mathbb{R}^m$ subspace.

Assuming an exact model of the system as input to the input/output linearization, a fully decoupled pseudo command signal can be achieved by using reference models for generation. Decoupled means that each pseudo control signal has a direct influence on the corresponding output y.

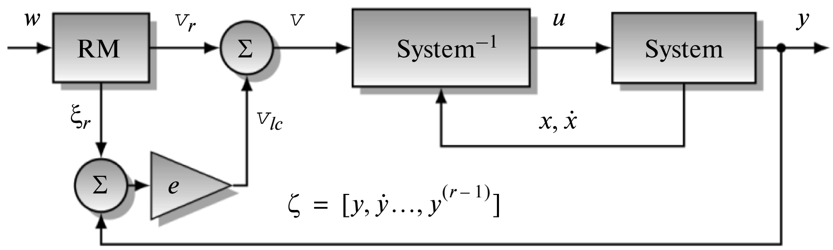

Fig. 3. Basic control loop with reference model and linear error controller: Reference model (RM), inverted system model dynamics (System^{-1}) and linear error controller (e). The input w into the reference model is a command signal generated by the MAVs guidance system. In case of the quadrotor with tiltable rotors the commanded signals are the desired attitude and velocity.

3.2.2. Reference Model

Considering the derivations which have to be done in order to obtain the inverted system dynamics the input to this inverted model must be generated. Obviously the output y of the system cannot be taken but an input generator is needed. This input generator is called reference model and provides the needed rth derivation of the output signal $y^{(r)}$ which is indicated in Fig. 2. According to [3, 5, 6] the reference model provides the wanted dynamics. In an ideal situation where the inversion of the system model is perfect the Laplace transfer function from the input to the inversion to the output of the system is $F_{\text{ideal}}(s) = 1$. This means that the reference model dynamics are mapped onto the system dynamics as long as the system is able to follow the reference model dynamics.

Dealing with a real system the reference model dynamics have to match the following criteria: first of all the reference model dynamics should be well known, they are not allowed to be faster than the system dynamics, they must provide input signals to the inverted system model at least up to the rth derivation and they should be stable. Considering the ability of providing input signals which are derived r times the whole control loop can be separated into a cascaded control loop if the system can be decoupled in an analogous way. According to [3] the cascaded system should have the fastest dynamics in the inner loop and the slowest dynamics in the outer loop. The control loop of the quadrotor with tiltable rotors requires a relative degree $r = 2$ for controlling the northern and eastern direction. The down direction and the attitude control require a relative degree $r = 3$.

3.2.3 Linear Error Controller and Conditions for Internal Dynamics

The control loop described in the previous sections needs an error feedback in order to keep track of the reference signal. Therefore the output y which represents the state vector ξ is compared to the equivalent reference model state. The resulting error signal is fed to a common linear controller and as a result the pseudo control signal v_{lc} is combined with the pseudo control signal from the reference model to achieve the main pseudo control signal $v = v_r + v_{lc}$. The setup is displayed in Fig. 3. Regarding equation (18) the linear controller only takes into account the state vector ξ which is the external part of the overall state vector z. The signal ξ is compared to the corresponding values ξ_r of the reference model. The remaining unobservable part η cannot be dealt with. Both parts of the state vector z represent dynamic systems. One requirement for the use of NID control is the stability of the internal dynamics represented by the state vector η according to the Lyapunov stability theory [3, 5, 6, 9]. The external part ξ defines certain dynamics, too. These dynamics can be controlled using the above mentioned common linear controller.

3.3. Pseudo Control Hedging

In the previous sections the basic idea of NID control is presented: the inversion of a system model in order to generate proper input signals to the system itself. In most cases there are systems to be controlled which contain nonlinearities which cannot be inverted or only be inverted at high costs. Examples for such nonlinearities are time delays and saturation. To use NID control with these nonlinearities pseudo control hedging (PCH) is used.

The basic idea of PCH given in [3, 5, 6] is to separate the system dynamics: the main system dynamics are modeled and inverted according to the previous sections but the actuator dynamics which contain the non-invertible nonlinearities are separated from the system dynamics. The separation can be done to any nonlinear system like a quadrotor. In this case the invertible system model is a multiple mass system with inertia like the quadrotor body. The non-invertible nonlinearities like saturation or time delay are part of the actuator dynamics, which is given in reality too. In order to control the system with NID control these actuator dynamics have to be modeled or the output of the actuators has to be measured. The former input signal u to the system is now separated into the commanded input signal u_c, the estimated input signal $\hat{u}$ and the actual input signal u to the system. The esti-

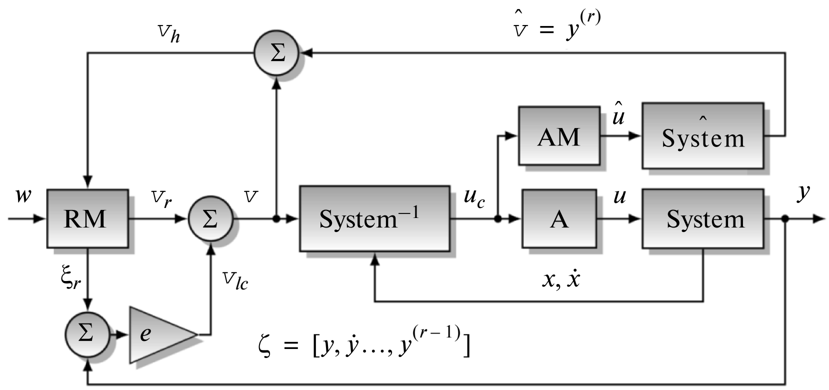

Fig. 4. Control loop as in Fig. 3, extended with pseudo control hedging: actuator dynamics (A), actuator model (AM) and forward system model ($\widehat{[\text{System}]}$).

mated or measured input signal $\hat{u}$ is fed into a system model. This system model is the same as the inverted system model which generates the commanded input signal u_c. Unlike the inverted system dynamics, the forward system model dynamics are combined with the nonlinear actuator dynamics. Therefore they represent realistic system responses instead of perfect or ideal ones. Using this system model an estimated output $\hat{y}$ can be achieved. This is necessary because the estimation of the actuator dynamics output signal $\hat{u}$ which is fed into the system model must be taken into account. It is important to provide appropriate signals which can be compared to the input signal ν of inverted system model dynamics. Due to the fact that the system model is known the output signal $\hat{y}$ can be derived multiple times until the rth derivation is reached. This derivation is comparable to the pseudo control signal ν which leads to the pseudo hedge signal ν_h. This is fed into the reference model in order to influence the reference model dynamics in a way that the commanded dynamics match the system dynamics:

$$\vec{\nu}_h = \vec{\nu} - \hat{\vec{\nu}}. \tag{26}$$

Using this method the actuator dynamics are removed from the control loop and the remaining system is treated with NID control. In addition to that the relative degree r is not increased but the system model is no longer fully inverted. Pseudo control hedging therefore provides a possibility to keep the internal error of control low because the reference model only provides commands in dynamics which can be achieved by the system. The architecture of the control loop including pseudo control hedging is shown in Fig. 4. In the reference model the hedge signal $\hat{\vec{\nu}}$ is used to influence the corresponding derivation of the input signal, which leads to the pseudo control signal ν.

4. RESULTS

In this section some simulation results are given. The results will demonstrate that the basic control strategy is able to control the quadrotor with tiltable rotors. Further simulation results will proof the increased performance according to the use of pseudo control hedging. The used system model is represented with a state vector $\vec{x} = [\vec{p}_{ned}, \vec{\nu}_{ned}, \Phi, \Theta, \Psi, \vec{\omega}]$, where $[\Phi, \Theta, \Psi]$ defines the attitude expressed as EULER angles, $\vec{p}_{ned}$ is the position of the vehicle, $\vec{\nu}_{ned}$ is the velocity and $\vec{\omega}$ is the angular rate. The input vector $\vec{u}$ is defined according to section 3.2.1 $\vec{u} = [\omega_{r,1\dots4}, \sigma_{1\dots4}]$ and the output vector $\vec{y}$ is given by $\vec{y} = [\vec{p}_{ned}, \vec{\nu}_{ned}, \Phi, \Theta, \Psi]$.

The simulation environment and the simulated quadrotor model are based on the actually realized hardware setup. Therefore the propulsion and servo-electrical motors have been identified and modeled. The system model used in the control loop is kept less complex in order to lower the required computational load for the embedded computer on the actual quadrotor. This leads to a simulation environment which allows testing the controller along with the setup in order to ensure safe flight tests.

4.1. Basic Control Strategy

The first results given in Fig. 5 demonstrate the ability of the control algorithm to control the attitude of the vehicle. The oscillations result from the coupled system: while the attitude is changed, the velocity of the vehicle is changed as well which is demonstrated in Fig. 6. In this figure the change of the velocity and height are commanded. The system responds in a stable way with some issues at the beginning, which are a result of an unsettling initialization. The system responses are in all cases (attitude, velocity in horizontal pane and height) highly damped and slow. At the beginning of each diagram the quadrotor is stand-

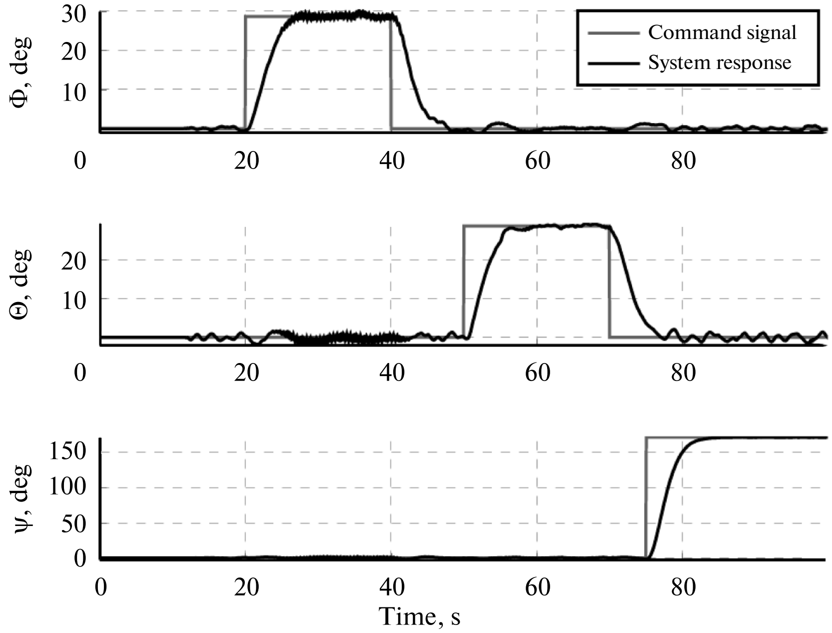

Fig. 5. Simulation results of the basic control algorithm concerning the attitude control. The system follows the commanded signal, but the speed of the movement is quite low.

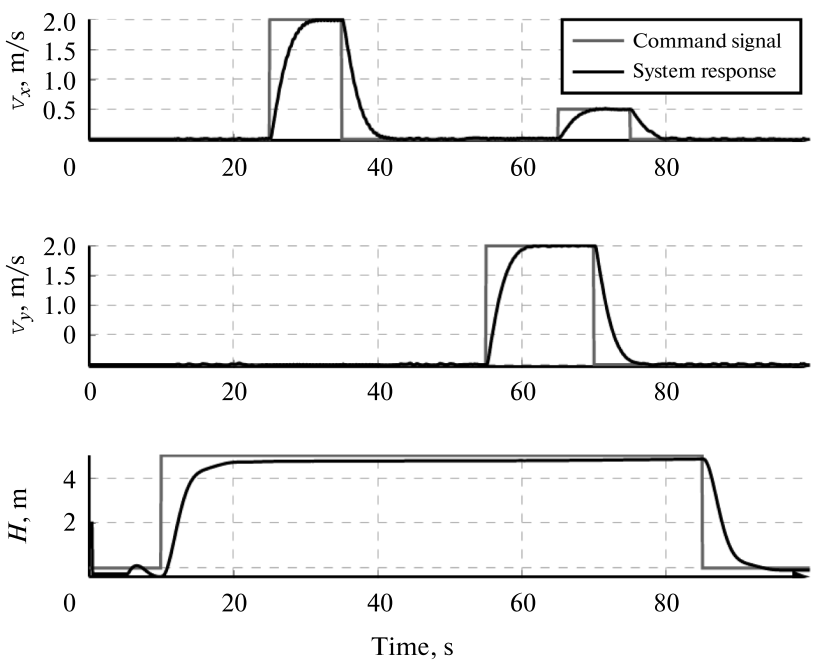

Fig. 6. Simulation results of the basic control algorithm concerning the velocity and height control. The results and the conclusion are basically the same as for the control of attitude: the controller works slowly but stable. The command signal is the input $w = [v_x, v_y, H]$ into the reference model.

ing on the ground and initializing its systems. This is represented in the simulation environment in order to deliver results as close as possible to the real behavior. During initialization for example the global navigation satellite systems (GNSS) are powered up and getting a position fix. Next to the initialization of the GNSS the inertial measurement unit (IMU) and therefore the inertial navigation system (INS) and the Kalman filter are powered up. During this stage the offset values of the gyroscopes and accelerometers are estimated in

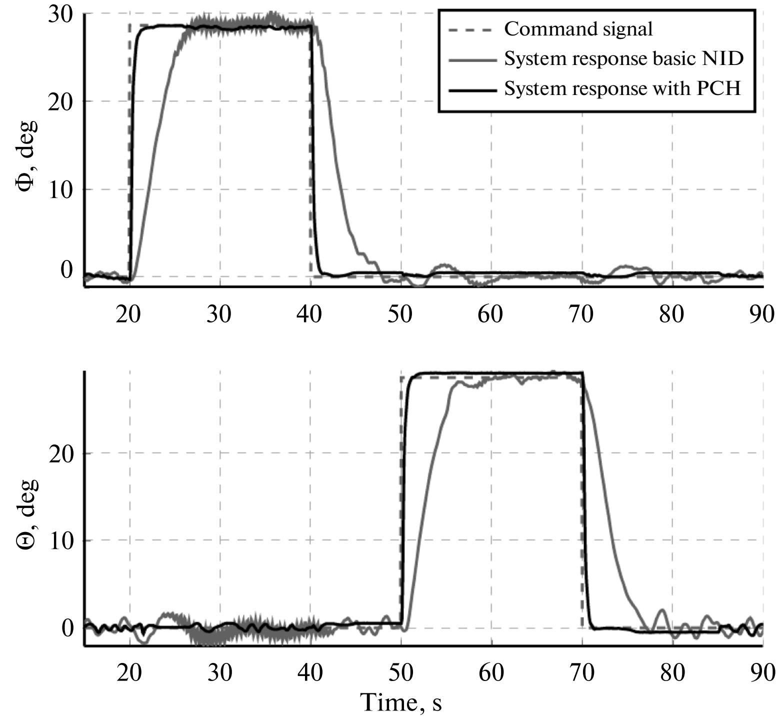

Fig. 7. Simulation results of a change in roll and pitch angle with pseudo control hedging activated in the control loop. The increased performance is indicated by the faster but stable system response to a step in the command signal. The command signal $w = [\Phi, \Theta]$ is fed from the guidance system into the reference model.

order to provide a navigation solution with the slowest drift possible.

The oscillations in the plots and the little disturbances even if the attitude should be zero mainly result from the mechanical setup of the system. The tilting angle σ has a backlash at which the angle is free to move around at about one degree. In the simulation environment this backlash is modeled and the angle σ is put to one end or the other depending on a calculation of the resulting torque.

Considering the Figs. 5 and 6 the decoupled control of attitude and velocity is demonstrated. As this highly nonlinear system behaves basically as a system of first order in all cases it can be stated stable.

4.2. Pseudo Control Hedging

The influence of pseudo control hedging is demonstrated by the simulation results given in Fig. 7. The curves denote the commanded signal, the system response of the basic control algorithm and the system reaction using pseudo control hedging. The system response with pseudo control hedging is much faster compared to the one without pseudo control hedging

5. Conclusions

A quadrotor with tiltable rotors was presented as a new design approach to micro aerial vehicles. The used algorithm was derived from an earlier airplane control algorithm and adapted to control this quadrotor. The basic challenge to control such an over actuated system was successful demonstrated by simulation results. An extension to further improve the basic control algorithm called pseudo control hedging was introduced. This led to an increased performance of the controller which was demonstrated by simulation results as well.

The stated task of control was successfully fulfilled and the used control algorithm as well as the improvements was demonstrated. The whole system consisting of the mechanical setup and the control algorithm is capable to maneuver the MAV in situations which require a decoupled control of attitude and velocity. In order to prepare the vehicle for flight tests these results proofed the control algorithm to be able to guarantee a safe flight. This work presented a new opportunity concerning the use of MAVs in cases of difficult demands regarding the support of rescue forces.

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