# **Identification of Vehicle Model Parameters under External Disturbances**

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**Abstract**—The paper proposes an approach to identification of model parameters of a vehicle subjected to external disturbances. Parameter estimates are generated using iteration procedure, which consists in estimation of the vehicle state vector including the disturbances and in estimation of vehicle model parameters using the least squares method. Identified parameters are insensitive to various external disturbances. The proposed approach was tested in full-scale tests. Modeling results are presented.

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## INTRODUCTION

If a vehicle control law is synthesized using a non invariant approach with account for vehicle dynamics, the vehicle mathematical model is required to estimate its state vector. Usually, the mathematical model in state space can be constructed based on the natural forces and moments affecting the vehicle accurate to several parameters, which are refined or determined by solving the identification problem during the full scale tests. Identification methods should satisfy certain requirements, namely:

sufficient accuracy; ⎯

independence of initial conditions;

 $\equiv$  immunity to external disturbances:

consideration for the measurement errors of onboard devices.

Currently, the most widespread methods used for dentification of vehicle model parameters are based  $\frac{1}{2}$  on  $\left[1-3\right]$ 

-prediction errors;

subspaces;

multialternative filtering.

In publications [4, 5], which became classical in identification of surface ship model parameters, hydrodynamic model coefficients are determined by the error prediction method using gradient procedure. The measurement prediction is realized by extrapolat ing the value of state vector using the Kalman filter multiplied by the observation matrix. In dynamic model of Kalman filter no external disturbances are considered, which makes the identified parameters very sensitive to disturbances.

Algorithm for identification of hydrodynamic coefficients of autonomous underwater vehicle (AUV) model using Gauss-Newton gradient procedure is addressed in [6]. Navigation and dynamic motion parameters are estimated by the Kalman filter using invariant model of the vehicle motion and orientation, which allows parameter determination by IMU data with sonar receiver aiding.

An approach described in [7] uses error prediction method with model parameter identification by the least squares method (LSM). Measurements were per formed using AUV special cable-supported model under various motions reproduced by the test facility, which also measured the model motion characteris tics. The forces acting on the model were measured by dynamometers.

The method considered in [8] determines the parameters of ship linear dynamic model described in state space. Specific features of LSM application are discussed, however, external actions such as wind, heave, and stream are not taken into consideration.

Subspace identification approach has been recently studied in many publications (see for example [9]). In [9], identification is performed in two steps: (1) Hankel matrix is constructed using input and out put signals; the model order is determined by this matrix; singular value decomposition of Hankel matrix is performed; and augmented observation matrix, and vehicle dynamic and observation matrices are constructed; (2) control matrices are constructed using LSM. With this approach, the model initially has an approximating character. Construction of approxi mating dynamic models is illustrated in [10, 11]. Example of using multialternative filtering is provided in [12].

It should be noted that the described methods suf fer from significant dependency of parameter esti mates on external disturbances. Advantages and disad vantages of these methods are analyzed in detail in [13], and the list of drawbacks of methods is given in Table 1.



External disturbances critically complicate identi fication of model parameters. Then as described in above-mentioned publications, the identified model parameters strongly depend on the disturbance level, which degrades the parameter estimates if the contri bution of disturbances is not considered in identifica tion. The paper considers the techniques used to reduce the effect of external disturbances on parame ter determination accuracy if identification problem is solved using prediction error method. Modeling results are presented which demonstrate the efficiency of the proposed methods. This approach was used to identify the model parameters for a hydrographic ship.

#### *Main Aspects of Identification Problem*

Effective application of identification methods is mostly determined by correct understanding of associ ated limitations, so here we provide the statement of identification problem.

Let the vehicle dynamic model be described by an equation

$$
X_{K+1} = A(\alpha) X_K + B(\beta) u_K + W + W_K, \qquad (1)
$$

where  $X_K$  is model state vector at time  $K$ ,  $u_K$  is the control vector (input signal); *A* is the dynamic matrix lin early depending on parameter vector  $\alpha$ ,  $\beta$  is the control matrix linearly depending on parameter vector β, *W* and  $W_K$  are the vectors characterizing low-frequency (constant and slowly varying) and high-fre quency components of disturbances.

Measurement vector (output signal) is given by<br> $Y_K = H X_K + V_K$ , (

$$
Y_K = H X_K + V_K, \tag{2}
$$

where  $V_K$  is the vector of measurement errors.

To account for disturbances we include them in the state vector and describe by the following models: con- For  $V_K = H X_K + V_K$ , (2)<br>where  $V_K$  is the vector of measurement errors.<br>To account for disturbances we include them in the<br>state vector and describe by the following models: con-<br>stant disturbances  $W_i$ , by  $W_i = 0$ , and slow disturbance components, by first order Markov pro cess.

Based on the model (1) and additionally intro duced state vector components the output can be pre stant disturbances  $W_i$ , by  $W_i = 0$ , and slowly varying<br>disturbance components, by first order Markov pro-<br>cess.<br>**Based on the model (1) and additionally intro-**<br>duced state vector components the output can be pre-<br>dicted described by the relations:

models have the same identification procedure  
\n
$$
\tilde{X}_{K+1} = A\hat{X}_K + Bu_K, \quad \tilde{Y}_{K+1} = H\tilde{X}_{K+1},
$$
\n(3)  
\n
$$
\tilde{X}_{K+1}
$$
 is the predicted state vector;  $\hat{X}_K$  is the state

where  $\tilde{X}_{K+1}$  is the predicted state vector;  $\hat{X}_K$  is the sate vector estimate for the current step determined by the equation:  $K_{K+1} = A\ddot{X}_K + Bu_K, \quad \ddot{Y}_{K+1} = H\ddot{X}_K.$ <br>
<sup>1</sup> is the predicted state vector;  $\dot{X}_{K+1}$  is the predicted state vector;  $\dot{X}_K$ <br>  $\dot{X}_K = \tilde{X}_K + K_K(Y_{K-1} - H\hat{X}_{K-1}).$ 

$$
\hat{X}_K = \tilde{X}_K + K_K (Y_{K-1} - H\hat{X}_{K-1}).
$$
\n(4)

In (3), vector  $\hat{X}_K$  is the augmented vector including constant and slowly varying disturbances; matrices of extended system are denoted by A and B and are given by

$$
A = \begin{bmatrix} A(\alpha) & 0 \\ 0 & A_W \end{bmatrix}, B = \begin{bmatrix} B(\beta) \\ 0 \end{bmatrix},
$$

where  $A_W$  is the matrix of dynamic models of disturbances *W*, 0 denotes the corresponding zero matrixes.

Kalman gain *K* and covariance matrix *P* are given by

$$
K_{K} = AP_{K}H^{T} (HP_{K}H^{T} + R)^{-1}; \quad P_{K+1} = AP_{K}A^{T}
$$
  
+ GG<sup>T</sup> - AP<sub>K</sub>H<sup>T</sup> (HP<sub>K</sub>H<sup>T</sup> + R)<sup>-1</sup> HP<sub>K</sub>A<sup>T</sup>. (5)

Here  $P_0$  is the initial covariance matrix of state vector components, *R* is *V* covariance matrix*, G* is the matrix of intensities of generating noises included in disturbance model.

Let us make two comments on generation of filter  $(4)$ ,  $(5)$ .

First, the Kalman filter should be calculated with account for matrix *A*, whose components should be determined. Further suppose that components of A are known but to a low accuracy and thus the filter should be robust to uncertainty of model parameters. This filter can be constructed by studying the sensitiv ity to dynamic matrix components. Constructing robust filters is addressed for example in [14]. If the model parameters are unknown, it is difficult to con struct a filter.

Second, matrices *A* and *H* should be such that observability condition (all components of state vector can be estimated using the measurements) is met.



### *The Problem of Identification of Vehicle Model Parameters*

We have a set of data

$$
Z^N = \{Y_1, u_1, Y_2, u_2, \dots Y_N, u_N\}.
$$
 (6)

The structure of dynamic and control matrices is set accurate to unknown parameters. Information in  $Z<sup>N</sup>$  should be used to determine the parameters of model (1), i.e., identify unknown parameters of matri ces  $A(\alpha)$  and  $B(\beta)$ .

In this context, the criterion of model parameter determination will be the prediction error [2, 3]. Thus a good model is a model which predicts well, i.e., gen erates small prediction errors for the available observa tion data. Prediction error for one step is determined by the equation

$$
\varepsilon_K = Y_K - H\hat{X}_K. \tag{7}
$$

Model parameters are determined by error mini mization criterion

$$
J(\alpha, \beta, Z^N) = \frac{1}{N} \sum_{K=1}^N \varepsilon_K^2.
$$
 (8)

Then the estimate of model parameters is given by

$$
(\hat{\alpha}, \hat{\beta}) = \underset{\alpha, \beta}{\arg \min} J(\alpha, \beta, Z^N). \tag{9}
$$

After introduction of criterion the identification problem is reduced to searching for its extremum. Computational methods used to determine the param eters are described in sufficient detail in [2, 3].

#### *Approach to Model Parameter Identification*

Here, we consider the case where the parameters of dynamic and control matrices are unknown. There fore, data required to construct the filter and to esti mate the state vector are lacking.

Make the following assumptions: (1) observation matrix is such that all components of state vector of dynamic model (1) are observed, some part of which are observed jointly with the disturbances; distur bances are not observed directly; (2) conditions of state vector observability are met, i.e., disturbances included in the state vector can also be estimated.

With these assumptions, observation matrix *H* can be represented as

$$
H = [E \ H_1], \tag{10}
$$

where  $E$  is the unity matrix of the relevant dimensions, matrix  $H_1$  has at least one nonzero component.

To estimate the model parameters in the first approximation, we use only available data: data set *ZN* according to (6) and the structure of extended dynamic and control matrices.

As a state vector estimate, we use the measurement at the previous step  $\hat{X}_{K-1} \approx Y_{K-1}$  according to the first formula in (3) and the corresponding control  $u_{K-1}$ , assuming that disturbances are zero. Summing the 1 matrices.<br> *x x*<sub>*K*-1</sub>  $\approx$  *Y<sub>K-1</sub>* 

prediction errors (7), we come to criterion (8). Mini mizing this criterion by unknown model parameters, we obtain the equation to determine the first approxi mations of these parameters. Minimization is per formed using LSM method.

We generate the matrix

$$
Y^N = [Y_1 \ Y_2 \ ... \ Y_N]
$$
 (11)

and matrix

$$
Z_0^N = \begin{bmatrix} Y_0^0 & Y_1^0 & \dots & Y_{N-1}^0 \\ u_0 & u_1 & \dots & u_{N-1} \end{bmatrix} . \tag{12}
$$

Then matrix equation to determine the first approximations of model parameters using the obser vations of input-output pairs will be given by

$$
[A \; B]Z_0^N = Y^N. \tag{13}
$$

Equation (13) has the only solution with respect to the block matrix [*A B*]  $[A \ B]Z_0 = I$ .<br>
tion (13) has the only solution wit<br>  $[AB] = Y^N (Z_0^N)^T [Z_0^N (Z_0^N)^T]^{-1}$ ,

$$
\left[A \hspace{0.2cm} B\right] = Y^N \left(Z_0^N\right)^T \left[Z_0^N \left(Z_0^N\right)^T\right]^{-1},\tag{14}
$$

if there are no linearly depending columns among the matrix columns  $Z_0^N (Z_0^N)^T$  (otherwise, Eq. (13) has infinitely many solutions, among which we choose the solution with the minimal norm).

The first approximations of parameters are used to construct the Kalman filter according to (4) and (5). The estimates of state vector components including the disturbances obtained using this Kalman filter and data set (11) and (12) are used to determine the approximations of model parameters as follows. Gen erate the updated measurements

$$
Y_i^1 = Y_i - H_1 W_i^1,
$$
 (15)

by subtracting the estimates of disturbances obtained at each filtering step from the available measurements. Generate matrix  $Z_1^N$  (12) with the updated measurements  $Y_i^1$ . Using (14) and replacing  $Z_0^N$  by  $Z_1^N$ , we obtain the next approximation of the model parame ters. We repeat the calculation procedure in the same manner as for the first approximation with updating of measurements  $Y_i^2$  to determine the next approximation, etc.  $Y_i^1$ . Using (14) and replacing  $Z_0^N$  by  $Z_1^N$ ,

The described iteration procedure refines the disturbances and the parameter estimates at each step.

**Example.** Here we provide an example of using the proposed identification approach for the model of lateral motion of hydrographic ship: The described iteration procedure<br>bances and the parameter estimate<br>**Example**. Here we provide an exproposed identification approach<br>praise in proposed in the probability of the probability:<br> $\lambda^2 = a_0 V V + a_0 V \omega + b_0 V^2 \delta + F \$ 

$$
\dot{V}_y = a_{11} V V_y + a_{12} V \omega + b_1 V^2 \delta + F, \quad \dot{\omega} = a_{21} V V_y \n+ a_{22} V \omega + b_2 V^2 \delta + I F, \quad \dot{F} = -\mu_F F + \sigma_F \sqrt{2\mu_F} w_1; \tag{16}
$$
\n
$$
\dot{V}_{Ty} = -\mu_V V_{Ty} + \sigma_V \sqrt{2\mu_V} w_2,
$$

where  $V_y$  is the ship lateral speed through the water;  $\omega$ is the yawing angular velocity;  $F$  is the normalized slowly varying lateral component of the wind force (force divided by the sum of the ship weight and the additional mass);  $V_{T_y}$  is the unaccounted slowly varying current component being the systematic error of measurement;  $\mu_F$  and  $\mu_V$  are the values inverse to correlation interval;  $\sigma_F$  and  $\sigma_{V_T}$  are RMS disturbances;  $w_i$ is the independent generating white noise of unit intensity; *V* is the ship speed through the water (mea sured by water speed log and assumed known to suffi cient accuracy);  $\delta$  is the control action (rudder displacement);  $a_{ij}$  and  $b_i$  are the model parameters; *l* is the known normalized arm of lateral component of wind force (the arm multiplied by the sum of the ship weight and the additional mass and divided by the sum of ship inertia moment and the added mass moment of inertia of water with respect to the vertical axis; the formulas to calculate them are given in all reference books on ship control theory).

Generate the discrete model using the continuous model (16):

$$
V_{yK+1} = a_{11}V_{yK} + a_{12}\omega_K + b_1\delta_K + F_K, \quad \omega_{K+1} = a_{21}V_{yK}
$$
  
+  $a_{22}\omega_K + b_2\delta_K + lF_K, \quad F_{K+1} = \mu_F F_K + g_F w_{1K}, \quad (17)$   

$$
V_{TyK+1} = \mu_V V_{TyK} + g_V w_{2K},
$$

with the parameters  $a_{ij}$  and  $b_i$  to be identified denoted as in earlier formulas.

Measurement equations are given by

$$
y_{VK} = V_{\text{GPSK}} \sin\left(\text{COG}_{\text{GPSK}} - K_K\right) + \upsilon_{VK};
$$
  

$$
y_{\omega K} = \omega_K + \upsilon_{\omega K},
$$
 (18)

where  $V_{\text{GPS}}$  is the speed over ground,  $\text{COG}_{\text{GPS}}$  is the course over ground measured by GNSS receivers;  $v_{\omega}$ , ν*<sup>V</sup>* are white noise processes.

For identification purposes, the vehicle performs a zigzag maneuver set by the following rudder control. At the initial moment, the ship heading  $K_0$  is fixedd. Yawing angle  $\psi$  is calculated by the formula  $\psi = K K_0$ , where K is the current heading. At initial moment the rudder is displaced to the starboard through the angle  $\delta = \delta_0$  with maximum speed. The rudder is kept in this position until yawing angle  $\psi$  reaches the preset value  $\psi_0$ . Then the rudder is displaced to the port through the angle  $\delta = -\delta_0$  and is kept in this position until the yawing angle reaches  $(-\psi_0)$ , etc. Denote this maneuver by  $(\psi_0/\delta_0)$ .

Due to methodical reasons, Eq. (17) does not account for the dependency of discrete model param eters  $a_{ij}$  and  $b_i$  on the speed of motion (this dependency is known, and parameters of discrete model can always be recalculated for continuous model with account for the speed through the water measured by the water speed log or speed through the water can be estimated by the proposed method using the equations of the ship longitudinal motion). As confirmed by the practice, during zigzag maneuver speed through the water is nearly constant after the second rudder dis placement.

First measurement (18) is related to the state vector components as follows

$$
y_{VK} = V_{\text{GPSK}} \sin\left(\text{COG}_{\text{GPSK}} - K_K\right) + \upsilon_{V_T K}
$$
  
= 
$$
V_{0K} \sin \alpha_{DK} + V_T \sin\left(K_T - K_0 - \psi_K\right) + \upsilon_{V_T K}, \quad (19)
$$

where  $V_0$  is the speed through the water,  $\alpha_D$  is the drift angle;  $V_T$  is the current speed,  $K_T$  is the current heading.

The first summand in (19) is  $V_y$ , and the second summand after expanding the sinus of angle difference with account for first order smallnesses for angle  $\psi$  can be presented as  $V_T \sin(K_T - K_0)$  + The first summand in (19) is  $V_y$ , and the second<br>summand after expanding the sinus of angle difference<br>with account for first order smallnesses for angle  $\psi$  can<br>be presented as  $V_T \sin(K_T - K_0) +$ <br> $V_T \cos(K_T - K_0) \psi_K$ . Then  $V_T$ and the second summand can be neglected if heading  $K_0$  is crossing the main current stream. *V<sub>y</sub>*, and the sinus of angle difference of angle difference  $V_T$  sin  $(K_T - K_0)$ 

The measurement matrix becomes  $H = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ .

State vector components are observable in accor dance with observability criterion.

Error in measurement prediction  $y_{VK+1}$  using the first model equation (17) according to the proposed approach is generated using the previous measure ments

$$
\varepsilon_{K+1} = y_{VK+1} - \hat{y}_{VK+1} = y_{VK+1} - (a_{11}y_{VK} + a_{12}\omega_K + b_1\delta_K).
$$

Using all the measurements from the first to the *N*-th measurement, summing the prediction errors and minimizing criterion (8), determine the first approximation of model parameters for the first equa tion. Similarly, we determine the model parameters in the system second equation (17). After estimating the system state vector (17) using the first approximation of model parameters, we obtain the estimates of state vector components given in Fig. 1.

The plots demonstrate that the yawing angular velocity is estimated rather accurately (true values and estimates agree); for the component  $V<sub>v</sub>$  the estimate is between the measurement and true value curves; dis turbing force is practically not estimated (true value of normalized force is  $5 \times 10^{-3}$  m/s<sup>2</sup>); the current component is estimated as a half value (true value is 0.3 m/s).

Generate the prediction error using the parameters of the first approximation for the second approxima tion with account for the estimated wind force and current Generate the prediction error using the p<br>the first approximation for the second ap<br>in with account for the estimated wind<br>rrent<br> $\epsilon_{K+1} = y_{VK+1} - \hat{y}_{VK+1} = y_{VK+1}$ <br>- { $\left[\tilde{a}_{11} \left(y_{VK} - \hat{V}_{TK}\right) + \tilde{a}_{12} \omega_K + \tilde{b}_1 \delta_K +$  $\frac{c}{t}$ <br> $\frac{1}{t}$ 

$$
\varepsilon_{K+1} = y_{VK+1} - \hat{y}_{VK+1} = y_{VK+1} - \left\{ \left[ \tilde{a}_{11} \left( y_{VK} - \hat{V}_{TK} \right) + \tilde{a}_{12} \omega_K + \tilde{b}_1 \delta_K + \hat{F}_K \right] - \hat{V}_{TK+1} \right\}.
$$

For the next iterations, the procedure is repeated.

Estimate the parameter second approximation using the set of prediction errors. Use the similar pro cedure for the second equation of the system (17). After several iterations we obtain the refined model parameters (Fig. 2) and parameter estimates (Fig. 3). Horizontal straight lines in Fig. 2 correspond to the true values.



**Fig. 1.** Estimate of system (16) state vector based on the first approximation of model parameters.

Figure 3 shows that the disturbances are estimated sufficiently accurately, the estimates of lateral velocity component practically coincide with the true values.

The 25th iteration provided the following accura cies of model parameter estimation: for  $a_{11}$ —10%, for *a*<sub>12</sub>—10%, for *a*<sub>21</sub>—25%, for *a*<sub>22</sub>—20%, for *a*<sub>1</sub>—20%, for  $a_2$ —5% of the true values.

Modeling results demonstrate that iterative process of refining the model parameters converges, then parameter estimation error is 10–15% of the true value.

This procedure was checked in the tests of control channel of navigation-control system onboard a hydrographic ship. The identified model parameters were used to synthesize a control law, whose quality is confirmed by highly stable stabilization during tack ing [15].

## **CONCLUSIONS**

Analysis of traditional methods used to identify the vehicle model parameters in state space has shown that their disadvantages are associated with parameter high susceptibility to the acting disturbances.

We propose an approach to identify model param eters of sea vehicles exposed to slowly varying distur bances (with neglected high-frequency components) with initially unknown model parameters, which makes the identified parameters immune to external disturbances.

The proposed approach was implemented in navi gation control system software to stabilize the hydro graphic ship. The results of sea trials reveal fairly good ship stabilization using the control law synthesized



**Fig. 2.** Modeling the refinement of model parameters in 25 iterations.

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**Fig. 3.** Estimate of state vector based on the 25th approximation of model parameters.

with the help of identified model parameters in condi tions of wind and wave disturbances.

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