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Numerical Solution of the Inverse Stefan Problem in the Analysis of the Artificial Freezing of a Rock Mass

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Abstract—The problem of parametrization of a heat transfer model in a rock mass in the conditions of its artificial freezing is considered. To clarify the parameters of the model according to the data of measurements of the temperature of the rock mass in the control-thermal wells, it is proposed to solve Stefan's coefficient inverse problem. In this study, the formulation of the inverse problem is considered and a numerical algorithm for its solution is proposed and implemented. The numerical algorithm is based on the iterative minimization of the smoothing functional of the mismatch between the measured and calculated temperatures in the control-thermal wells. The properties of the smoothing functional in the phase space of the thermophysical properties of the rock mass and the features of the choice of the parameters of the smoothing functional are investigated.

Keywords: artificial ground freezing, frozen wall, inverse Stefan problem, mathematical model, model parameterization, Tikhonov regularization

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INTRODUCTION

Underground structures are constructed in flooded soils and rocks using special methods in order to prevent the flooding of a mine under construction. The artificial freezing of rocks is the most widespread, special way of constructing mines [1, 2]. This method consists of drilling the contour of the freezing wells around the designed mine shaft, installing the columns, and organizing the circulation of the coolant along them. As a result, the surrounding rock mass is cooled and frozen, and a frozen wall is formed, which prevents the penetration of pore water into the space of the excavation under construction (Fig. 1).

According to the current regulatory literature [3, 4], it is necessary to constantly monitor the state of the frozen rock mass. Currently, this is controlled using both experimental and theoretical methods. Each of these two classes of methods separately has its own drawbacks, as a result of which the most effective option is the synthesis of experimental and theoretical methods with their subsequent mutual refinement [5, 6].

The thermodynamic model of the rock mass is refined according to the data of the experimentally measured temperatures of the rock mass in the control-thermal wells. Refinement of the model most often consists of changing the thermophysical properties of the rock mass: the specific heat capacities [7], thermal conductivities [6, 8], thermal diffusivity [9], and humidity [5, 6]. According to [8], the thermal conductivities of the rock mass determined in the course of laboratory tests are least accurate. This refinement of the model of the frozen rock mass is mathematically a solution to the inverse Stefan problem [9].

In particular, a number of theoretical works [10–14], including studies carried out with the participation of the authors of this article [6, 7, 9], are devoted to the solution of inverse heat transfer problems and inverse Stefan problems. This study is a continuation of the works [6, 7, 9]. An iterative method for solving Stefan's coefficient inverse problem, proposed earlier in [5], is developed in relation to the problem of calibrating the thermophysical properties of a rock mass. A transition is made from the natural regularization, studied earlier, to regularization by A.N. Tikhonov's method (a regularizing functional is added). The need to introduce regularization by Tikhonov's method when solving the inverse problem is related to the fact that as a result of a number of practical problems on the adjustment of the thermophysical properties of the rock mass, it turned out that the obtained solution to the problem was not always unique. In this situation, with the wrong choice of the solution to the problem, deterioration of the predictive characteristics of the thermodynamic models of the rock mass in the future is possible.

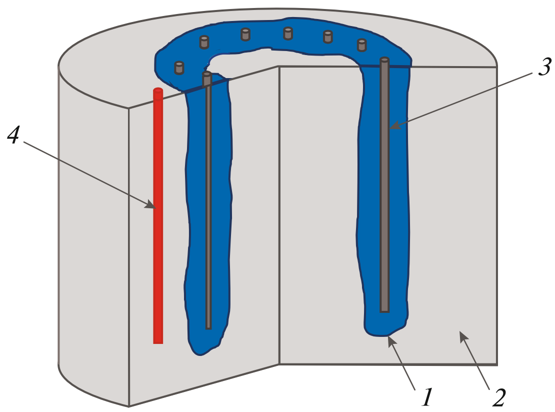

Fig. 1. Artificial freezing of the rock mass: (*1*) frozen zone, (*2*) unfrozen zone, (*3*) freezing column, and (*4*) control thermal well.

MATHEMATICAL MODEL OF A FROZEN ROCK MASS

A horizontal layer of the rock mass is considered. There are many frozen wells in the rock mass, oriented vertically and located along a circular contour. Columns are installed in frozen wells, through which a coolant, which has a negative temperature, circulates at a constant speed. As a result, the rock mass and the water contained in its pores gradually cool and freeze. In the process of the freezing of a rock mass, the entire volume of the latter can be conditionally divided into a frozen zone and an unfrozen zone [1] (see Fig. 1). It is assumed that the following physical processes take place in the rock mass, which significantly affect the spread of heat in it:

(1) conductive heat transfer (thermal conductivity);

(2) phase transition of pore water;

(3) heat exchange between the rock mass and the coolant in the columns.

In order to simplify the mathematical analysis when setting the problem, a number of hypotheses are accepted:

(1) the isotropy and homogeneity of the rock mass in the frozen and unfrozen zones;

(2) the phase transition of pore water occurs completely in a certain small specified temperature range;

(3) the vertical heat transfer is negligible compared to the horizontal transfer (transition to a two-dimensional formulation);

(4) the rock mass is completely saturated with water;

(5) the water in the pore space of the rock mass is motionless;

(6) the local thermal equilibrium between different phases (dry skeleton, water, and ice).

Taking into account the considered physical processes and accepted hypotheses, we write the direct Stefan problem in enthalpy formulation as follows:

$$\frac{\partial H(T)}{\partial t} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(\lambda r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \varphi}\left(\lambda\frac{\partial T}{\partial \varphi}\right)\right], \tag{1}$$

$$\lambda = \lambda_{\text{lq}}(1-\phi) + \lambda_{\text{sd}}\phi, \tag{2}$$

$$H(T) = \begin{cases} \rho_{\text{lq}}c_{\text{lq}}(T - T_{\text{lq}}) + \rho_{\text{lq}}wL, & T_{\text{lq}} \le T, \\ \rho_{\text{lq}}wL(1-\phi), & T_{\text{sd}} \le T < T_{\text{lq}}, \\ \rho_{\text{sd}}c_{\text{sd}}(T - T_{\text{sd}}), & T < T_{\text{sd}}, \end{cases} \tag{3}$$

$$\phi(T) = \begin{cases} 1, & T < T_{\text{sd}}, \\ (T_{\text{lq}} - T)/(T_{\text{lq}} - T_{\text{sd}}), & T_{\text{sd}} \le T < T_{\text{lq}}, \\ 0, & T_{\text{lq}} \le T, \end{cases} \tag{4}$$

$$\left[\lambda \frac{\partial T}{\partial n} - h\left(T_{\text{fb}}(t) - T\right)\right]\Bigg|_{\Omega\text{fb}} = 0, \tag{5}$$

$$T\big|_{\Omega\,\text{out}} = T_0, \tag{6}$$

$$T\big|_{t=0} = T_0, \tag{7}$$

where H is the specific enthalpy of the flooded rock mass, J/m^3; r and ϕ, are polar coordinates, r in m; t is the physical time, s; λ_{lq} and λ_{sd} are the thermal conductivities of the rock mass in the unfrozen and frozen, respectively, W/(m °C); c_{lq} and c_{sd} are the specific heat capacities of the rock mass in the unfrozen and frozen zones, respectively, J/(kg °C); ρ_{lq} and ρ_{sd} are the densities of the rock mass in the unfrozen and frozen zones, respectively, kg/m^3; T_{lq} is the temperature at the start of pore water crystallization (the liquidus temperature), °C; T_{sd} is the pore ice temperature when it starts to melt (the solidus temperature), °C; ϕ is the ice volume fraction in the pores of the rock mass (ice content), m^3/m^3; L is the specific heat of the water's crystallization, J/kg; w is the initial moisture content of the rock mass, kg/kg; $T_{\text{fb}}(t)$ is the temperature of brine in the freezing columns, °C; T_0 is the temperature of the undisturbed rock mass at a distance from the freezing contour, °C; h is the heat transfer coefficient at the boundary with the freezing wells, W/(m^2°C); $\Omega_{\text{fb}} = \cup\Omega_{\text{fb}j}$ are the boundaries with all freezing wells $j = 1, \ldots, N_{\text{FB}}$; N_{FB} is the number of freezing wells; Ω_{out} is the external boundary of the modeling area; and n is the coordinate along the normal to the boundary Ω_{fb}, m.

The direct Stefan problem includes an unsteady energy balance equation (1), a formula for calculating thermal conductivity depending on the phase state of the pore water in the rock mass (2), equations of state (3) and (4), a boundary condition of the third kind on freezing columns (5), a boundary condition of the first kind on the outer boundary of the computational domain (6), and the initial condition (7).

In the enthalpy approach, all the information on the phase transition of the pore water is contained in the function of the specific enthalpy of temperature, $H(T)$. In this case, there is no need to explicitly track the front of the water–ice phase transition, which greatly simplifies the procedure for the numerical implementation of the two-dimensional model (1)–(7). From the form of the function $H(T)$ included in the model, it follows that the front of the phase transition is smeared out in the finite temperature interval [T_{sd}; T_{lq}]. In this temperature range, the phase transition heat is released as a result of the complete crystallization of the pore water.

PARAMETRIZATION OF THE MATHEMATICAL MODEL

Model (1)–(7) contains a number of thermophysical parameters the values of which must be set to obtain any numerical solution. As a rule, the thermophysical parameters of rocks are determined in laboratory conditions based on the analysis of the core samples extracted from the exploration wells. The analysis determines the thermal conductivity, density, and specific heat capacity of rocks in the frozen and thawed (cooled) states, as well as the temperatures of the onset of the freezing of the pore water and the beginning of the melting of the ice in the pores.

According to the data [5, 8], the thermophysical parameters of the studied rocks determined in this way are not always sufficiently accurate for the model. This is due both to the insufficiency of the core samples for obtaining statistically significant results of laboratory studies and the imperfection of the experimental methods for determining the thermophysical properties. This is especially true for the determination of the thermal conductivity of the rock mass. According to [5], the relative error in determining thermal conductivity can exceed 30%. Ultimately, this results in significant errors in the mathematical forecasting of the artificial freezing of the rock mass. The temperature mismatches in the control-thermal wells can exceed several degrees Celsius [15].

Thus, the correct use of thermophysical models of a rock mass for predicting its artificial freezing during the construction of underground structures is possible only if the parameters in such models are adjusted using additional data. The measured temperatures along the depth of the control-thermal wells can serve as such additional data.

STATEMENT OF THE COEFFICIENT INVERSE PROBLEM

Adjustment of the thermophysical parameters in the rock mass model (1)–(7) according to the clarifying data of the control-thermal wells is a solution to Stefan's coefficient inverse problem [10, 11]. In order to formulate the inverse problem, it is necessary to redefine the direct problem (1)–(7) by introducing the given measured temperatures $T_i^{(c)}(t)$ at the location (r_i, φ_i) of each control-thermal well no. i:

$$T(t, r, \varphi) = T_i^{(c)}(t), \quad i = 1, \ldots, N_C. \tag{8}$$

Here N_C is the number of control-thermal wells.

Thus, the solution to the inverse Stefan problem in this case consists of determining the temperature field $T(t, r, \varphi)$ and values of the thermophysical properties of the rock mass p_j $(j = 1, .., N_P)$ satisfying the system of Eqs. (1)–(8). Here N_P is the number of adjustable thermophysical properties of the rocks.

ALGORITHM FOR SOLVING THE COEFFICIENT INVERSE PROBLEM

The numerical solution of the previously posed coefficient inverse problem is carried out using Tikhonov's regularization method [10]. A regularizing functional of the form

$$\Omega = \sqrt{\frac{1}{N_P}\sum_{j=1}^{N_P}\left(\frac{p_j - p_j^*}{p_j^*}\right)^2}, \tag{9}$$

where p_j^* are the fixed values of the thermophysical properties of the rock mass, is introduced. The minimum condition of the functional Ω indicates that the adjustable thermophysical properties of the rock mass, p_j, which are a solution to the inverse problem, should differ as little as possible from the values p_j^*, which are understood in this case as the most probable or typical values of the thermophysical properties of the considered layer of rocks.

The solution to the inverse problem is reduced to minimizing the smoothing functional of the form

$$I = E\left(T(t), T_i^{(c)}(t)\right) + \alpha\Omega = \sqrt{\frac{1}{N_C}\frac{1}{\Delta T^2}\sum_{i=1}^{N_C}\left(T_i(t) - T_i^{(c)}(t)\right)^2} + \alpha\Omega, \tag{10}$$

where $E = E(T(t), T_i^{(c)}(t))$ is the functional of the temperature mismatch, α is a scalar dimensionless parameter selected empirically, and ΔT is the characteristic temperature difference in the problem under consideration, °C. The difference between the initial temperature of the array and the temperature of the coolant in the freezing columns can be taken as ΔT.

Both terms in the functional I are dimensionless: it is for the purpose of nondimensionalization that the characteristic temperature ΔT is introduced in (10). The first term in the functional I in the physical sense is the relative root-mean-square error of the model prediction of temperatures at the locations of control-thermal wells.

When minimizing the functional I, the principle of iterative minimization is used, according to which a sequential refinement of the solution is performed in accordance with the formula [10]:

$$p_j^{(k+1)} = p_j^{(k)} + \Delta p_j^{(k)}, \tag{11}$$

where k is the iteration number and $\Delta p_j^{(k)}$ is the increment for the jth adjustable parameter of the problem, which ensures a decrease in the target functional I. Correction $\Delta p_j^{(k)}$ is calculated based on the modified gradient descent method. The use of the gradient descent method rather than the descriptive regularization method developed in [12] for solving the inverse Stefan problems is due to the fact that the latter turns out to be less effective in relation to the problem of forming a frozen wall, as shown in [7].

The functional minimization algorithm using the modified gradient descent method is presented below:

(1) Determination of the minimization parameters p_j, their initial values $p_j^{(0)}$, the maximum number of iterations of the algorithm N_{Iter}, fixed values of the thermophysical properties of the rock mass p_j^*, and the initial value of the parameter ω which determines the speed of the gradient descent.

(2) Start of the kth iteration. Solution of the direct Stefan problem at the current values of the thermophysical properties of the rock mass $p_j^{(k)}$. Calculation of the current value of the smoothing functional $I^{(k)}$

and errors $E^{(k)}(T,T_i^{(c)})$. If the current error value $E^{(k)}(T,T_i^{(c)})$ is below some given acceptable error $1 \geqslant \varepsilon > 0$, completion of the algorithm; otherwise, go to the next iteration.

(3) Calculation of the partial derivatives of the smoothing functional $I^{(k)}$ by the adjustable parameters $p_j^{(k)}$ using finite difference formulas:

$$\frac{\Delta I}{\Delta p_j}(p_j^{(k)}) = \frac{I(p_j^{(k)} + \beta\delta p_j p_j^{(k)}) - I(p_j^{(k)})}{\beta\delta p_j p_j^{(k)}}. \tag{12}$$

Here, β is a parameter taking values $+1$ or -1 and δp_j is the relative variation of the parameter $p_j^{(k)}$ of order not higher than 10^{-2}.

If the partial derivative (12) with respect to parameter $p_j^{(k)}$ is greater than zero (i.e., changing parameter $p_j^{(k)}$ by the amount $\beta\delta p^{(k)} p_j^{(k)}$ entails an increase in the minimized functional), it is necessary to change the sign of β and recalculate derivative (12). If the partial derivative (12) with respect to parameter $p_j^{(k)}$ is again greater than zero, the partial derivative (12) is taken equal to zero (a local extremum is set with respect to the minimization variable $p_j^{(k)}$) and parameters ω and δp_j are halved.

(4) After determining all the partial derivatives, corrections are calculated for each jth adjustable parameter of the problem:

$$G^{(k)} = -\omega\sqrt{I(p_j^{(k)})}\frac{\Delta I}{\Delta p_j}(p_j^{(k)}), \tag{13}$$

$$\Delta p_j^{(k)} = (1-\mu)G^{(k)} + \mu G^{(k-1)}, \tag{14}$$

$$p_j^{(k+1)} = p_j^{(k)} + \Delta p_j^{(k)}. \tag{15}$$

Here $\mu \in [0;1)$ is the factor of prehistory and $G^{(k)}$ is the calculated value of the correction at the kth iteration.

(5) Completion of the kth iteration. Switch to a new iteration and return to step 2.

In fact, the algorithm proposed above is in effect the gradient descent method, in which the rate of the gradient descent is specifically selected:

$$\hat{\lambda} = -\omega\sqrt{I(p_j^{(k)})} \tag{16}$$

and a factor of prehistory is additionally introduced.

NUMERICAL SIMULATION

The algorithm for solving the direct Stefan problem (1)–(7) is implemented numerically using the finite difference method on a polar grid. The center of the circular contour of the freezing columns was taken as the origin of coordinates. Due to the need for the mesh refinement near the freezing columns, it was decided to use a composite mesh of several annular blocks (see Fig. 2). Inside each block, a polar grid that is not uniform along the radial coordinate and uniform along the angular coordinate was set. The grid density, the size of the cells near the freezing columns, and the distance between the outer boundary of the computational domain Ω_{out} and the contour of the freezing columns were selected by preliminary modeling based on the condition that the obtained numerical solution is independent of the method of discretization of the computational domain.

In order to discretize Eq. (1), we used the central spatial discretization scheme of the second order of accuracy and the explicit Euler time integration of the first order of accuracy. The time step was determined based on the Courant condition for the thermal diffusion equation

$$\Delta t < \frac{1}{2\max\limits_{i,j}\left(\frac{1}{\Delta r_i^2} + \frac{1}{r_i^2\Delta\varphi_j^2}\right)\max\left(\frac{\lambda_{\text{lq}}}{\rho_{\text{lq}}c_{\text{lq}}};\frac{\lambda_{\text{sd}}}{\rho_{\text{sd}}c_{\text{sd}}}\right)}, \tag{17}$$

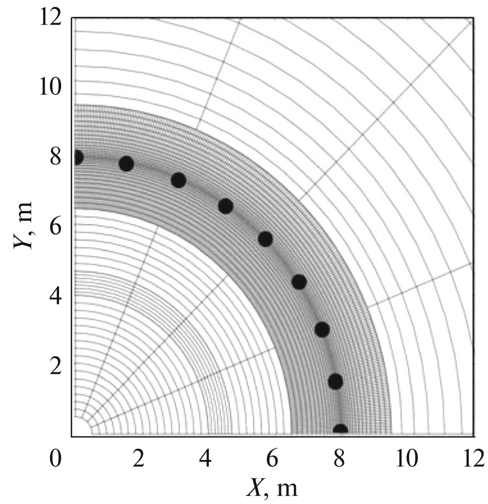

Fig. 2. Finite-difference polar grid for the sector $\varphi \in (0°; 90°)$ of the rock mass.

where Δr_i is the step along the radial coordinate, m; $\Delta \varphi_j$ is the step along the angular coordinate, rad; r_i is the radial coordinate of the ith node, m; and index i numbers the nodes in the radial coordinate while index j nodes in the angular coordinate.

There are two reasons for selecting an explicit time integration scheme, which requires much smaller time steps than an implicit time integration scheme. First, in view of the specifics of the problem about the artificial freezing of the rock mass to be solved, for the correct modeling of unsteady heat transfer under conditions of high temperature gradients near the freezing columns, it is necessary to reduce not only the step of the spatial mesh near the columns but also the time step. Second, within the enthalpy formulation of the Stefan problem, it is required to solve a nonlinear system of equations for the temperature [16], and the use of an implicit time integration scheme in this situation would lead to additional labor costs for solving the system of nonlinear equations at each time iteration.

The geometrical parameters of the computational domain and the thermophysical properties of the investigated model for a sand layer are presented in Tables 1 and 2, respectively. The thermophysical properties correspond to the layer of sand lying in the depth interval of 0 to 14.7 m, in the conditions of the industrial site of the potash mine of the Petrikovsky Mining and Processing Plant in Belarus.

In the model, it was assumed that two vertical control-thermal wells were drilled in the layer of the rock mass. The first well has the Cartesian coordinates (8 m; 4 m), while the second one has the Cartesian coordinates (8 m; –4 m). Thus, they are diametrically opposite to each other. The choice of the number and locations of the model control-thermal wells differs from the real conditions that took place during the construction of shafts at the industrial site of the mine in question. This is done primarily for the convenience of analyzing the solution of the inverse problem.

For the numerical solution of Stefan's inverse problem, it was necessary to set the temporal diagrams of temperature on the control-thermal wells. For the first well, the temperature temporal diagram was obtained as a result of solving the direct Stefan problem for the problem parameters presented in Table 2. For the second well, the parameters of the problem were also taken from Table 2, with the exception of thermal conductivities λ_{lq} and λ_{sd}, which were taken 7% higher than the values presented in Table 2. This was done in order to take into account the possible inconsistency of the readings of various control-ther-

Table 1. Geometric parameters of the computational domain

Parameter	Value
Freezing column's outer radius, m	0.073
Radius of the contour of freezing columns, m	8
Number of freezing columns	40
Radius of the outer boundary of the computational domain, m	40

Table 2. Thermophysical properties of the flooded sand layer

Parameter	Value
Thermal conductivity in the frozen zone, W/(m °C)	3.79
Thermal conductivity in the unfrozen zone, W/(m °C)	2.46
Heat capacity in the frozen zone, J/(kg°C)	910
Heat capacity in the unfrozen zone, J/(kg °C)	1266
Initial temperature of the mass, °C	7.3
The temperature of the beginning of crystallization of pore water (liquidus temperature), °C	–0.05
The temperature of the beginning of the ice melting in the pores (solidus temperature), °C	–3.05
Density, kg/m^3	2640
Initial moisture, kg/kg	0.127
Specific heat of crystallization of water, J/kg	330000

mal wells, which most often occurs in practice. The inconsistency can be related to both the error of the measuring equipment and the deviations of the wells' positions from the design values that were not taken into account in the model, as well as the inhomogeneity and anisotropy of the thermodynamic properties of the real rock mass.

Figure 3 shows the temperature temporal diagrams calculated for both control-thermal wells. The temperature difference is due to the difference in the values of the thermal conductivity. These diagrams were further used to numerically solve Stefan's coefficient inverse problem.

The algorithm for solving the inverse Stefan problem is also implemented numerically. Programming was carried out in the C# language in the Microsoft Visual Studio environment. It was assumed that the unknown parameters to be determined in the course of solving the inverse problem were the thermal conductivity of the mass in the frozen and unfrozen zones. We varied the deviations of the initial values of the thermal conductivity from the true values given in Table 2.

Figure 4 shows the calculated isolines of the smoothing functional (10) in the phase space of the thermal conductivities for simulation times of 15 and 50 days; value $\alpha = 0.05$; and parameters $\delta p_1 = \delta p_2 = 0.05$, $\mu = 0.3$, and $\omega = 3$. Figure 4 shows that the functional has a single minimum. The form of the functional changes over time. Although in the early stages of freezing (when the frozen zone had not yet reached the control-thermal wells), the functional had an elongated shape along the sufficiently flat G_1 axis, for a longer freezing time, the functional is elongated along the G_2 axis, tilted much more strongly and almost parallel to the y axis. Physically, this means that in the initial time interval (less than 30 days), the solution largely depends on the thermal conductivity in the unfrozen zone, while for long freezing times (more than 30 days), it depends on the thermal conductivity in the frozen zone. In both cases, functional (10) does not vanish at the minimum point: its minimum value is about 0.002. This is due to the inconsistency of the readings of the control-thermal wells.

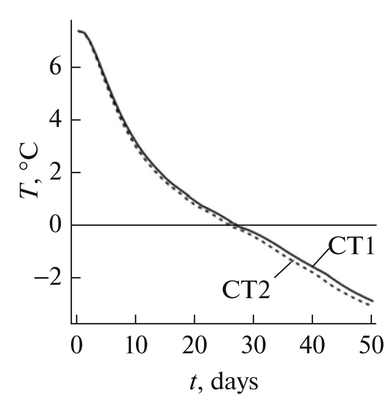

Fig. 3. Temporal diagrams of temperature at control-thermal wells.

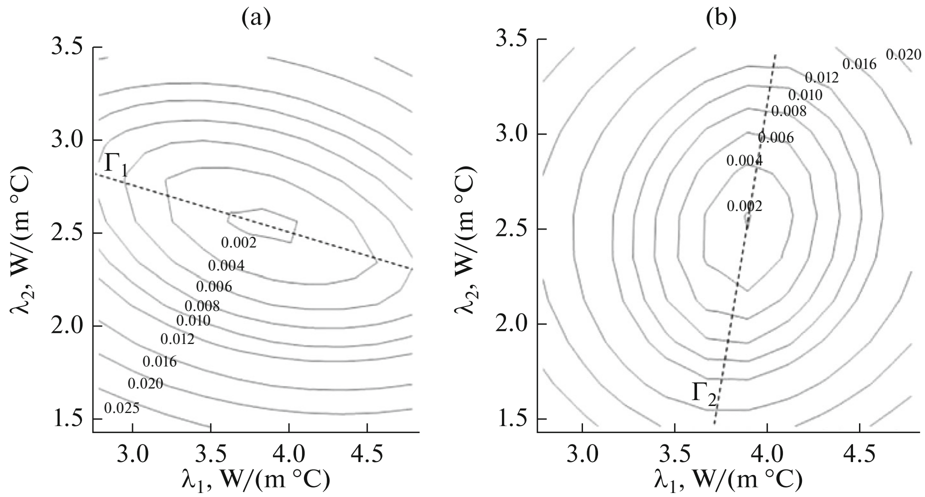

Fig. 4. Smoothing functional in the phase space of the thermal conductivity of the rock mass.

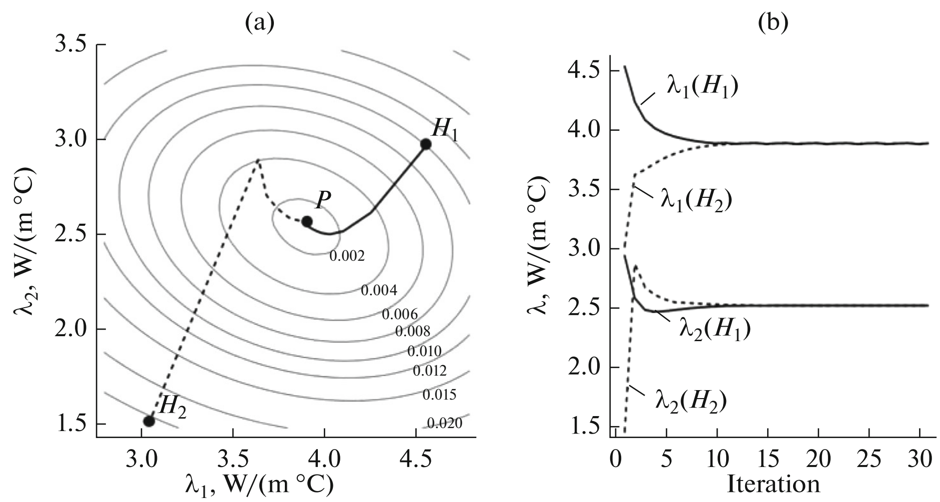

Fig. 5. Iterative procedure for solving the inverse Stefan problem: (a) curves for finding the minimum of the functional in the phase space of thermal conductivities and (b) dynamics of thermal conductivities of the rock mass in the course of the iterative procedure.

Figure 5 shows the results of solving the inverse problem: the curves of the changes in the thermal conductivity of the rock mass in the process of performing the iterative procedure described above. In the case considered $\alpha = 0.05$ and there are two initial deviations of the thermal conductivity of the rock mass from the true values in Table 2. In the first initial deviation (H_1), both thermal conductivities increased by 20%; in the second initial deviation (H_2), the thermal conductivity in the frozen zone was reduced by 20% and the thermal conductivity in the unfrozen zone was reduced by 60%. For each of the initial deviations, the contour lines of the smoothing functional (10) are presented. The number of iterations in both cases is 30, the simulation time of the direct Stefan problem is 30 days, and the initial values of the thermal conductivities (points H_1 and H_2, respectively) are taken as p_j^*. Figure 5 shows that in both cases, as a result of the iterative procedure, the resulting solution (point P) coincides with the minimums of the functionals.

A more complex situation arises when moisture w is added to the optimization parameters. In this case, the minimum of functional (10) can be achieved not at one point but at a set of points along some line G_{m} in the phase space of the optimized parameters. This can lead to the fact that the obtained solution of the inverse problem will strongly depend on the initial conditions and on the chosen method for minimizing the functional (see Fig. 6a). With the proper selection of parameter α in (10), it is possible to achieve the

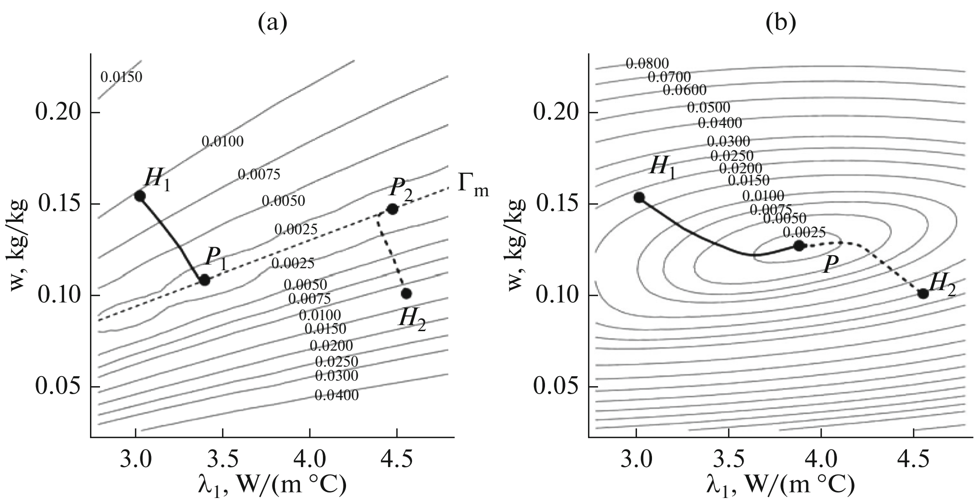

Fig. 6. Curves for searching for the minimum of the functional in the phase space of the adjustable parameters: thermal conductivity in the ice zone and the moisture content of the rock mass: (a) $\alpha = 0$ and (b) $\alpha = 0.1$.

appearance of a clearly pronounced unique minimum of the functional and a unique solution to the inverse problem (see Fig. 6b). Within a certain range of parameters α—when the second term in (10) does not have a dominant effect on the value of the smoothing functional—this minimum will be stable with respect to small variations in α.

However, the position of the minimum of the smoothing functional will in any case strongly depend on the selected values p_j^*, since in this case, we additionally require the resulting solution to be as close as possible to the values p_j^*. For this reason, it is necessary to be especially careful when choosing the values p_j^*. The most reasonable solution here is to set p_j^* to the typical values of the thermophysical properties of the considered layer of the rock mass as p_j^*; in this case, the solution obtained will not depend on the initial parameters of the problem $p_j^{(0)}$.

CONCLUSIONS

Let us formulate the main conclusions obtained as a result of the research carried out.

(1) A numerical algorithm for solving the inverse Stefan problem in relation to the problem of adjusting the thermophysical parameters of a frozen rock mass is proposed and implemented. The numerical algorithm is based on minimizing the smoothing functional of the mismatch between the measured and calculated temperatures in the control-thermal wells.

(2) The properties of the smoothing functional in the phase space of the adjustable parameters of the problem are investigated. It is shown that for a certain choice of the adjustable parameters of the problem, the minimum of the smoothing functional can be achieved not at one point but at a set of points along a certain line in the phase space of the parameters.

(3) It is shown that it is necessary to introduce a regularizing functional to obtain a unique solution to the inverse problem, independent of the selected initial values of the adjustable parameters. When introducing a regularizing functional, it is necessary to be especially careful in choosing its parameters, since the resulting solution can be sensitive to them. The most reasonable solution here is to accept the typical values of the thermophysical properties of the considered layer of the rock mass as the parameters of the regularizing functional.

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