

Discrete Dynamic Modeling of State Regulation of Human Capital

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Abstract—The multicriteria problem of optimizing the state regulation of the quality of human capital in an information society is considered. A discrete dynamic model of human capital is described taking into account the age dynamics of the awareness and cognitive abilities possessed by an individual as a carrier of information. On trajectories, the lifelong indices of human capital are considered on trajectories; i.e., productivity and creativity are considered taking into account the initial increase in the quality of human capital followed by its subsequent decline at an older age. The task of state regulation is described as a two-criteria maximization of the population’s mathematical expectations of indices for a generation, taking into account not only the natural distribution of individuals by the starting values but also the possibilities of socialization within a state-controlled education system. The model’s identification is based on taking into account the known population restrictions on the phase trajectory tube. The model is identified and investigated by methods using approximation by Shannon metric networks. It is shown that the optimal choice of educational priorities by the state depends on the society’s parameters such as age-specific mortality rates and the time budget available for the socialization of a nonadult. There are combinations of parameters that lead to a conflict of the regulation criteria; in this case the solution is the set of Pareto optimal strategies. For Russia, there is a dominant solution that presumes the priority development of cognitive abilities rather than awareness.

Keywords: human capital, cognitive abilities, educational governance, information society, discrete dynamic model, multicriteria optimization, metric data analysis, Pareto optimality, trajectory tube

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1. INTRODUCTION

This article considers the problem of human capital development in the context of the optimal choice of priorities for the state policy of socialization. In order to substantiate this choice, we have constructed and studied a dynamic mathematical model simulating two characteristics of human capital. Human capital modeling is one of the topical areas of modern economic research.

Human capital as a category includes several relatively independent components, including: human health, as well as a person’s qualifications and aptitudes, which determines his ability to develop. One of the conventions in its numerous conceptualizations, starting with the classical ones [1], is the idea that human capital is formed mainly at an early age (before a person starts working, within general and vocational education). The considered model makes it possible to remove this conditionality by describing its development.

Traditionally, the age dynamics of human capital are reduced to a change in health; however, in the proposed model, we take into account the change in qualifications taking a person’s ability into account. In this case, a person is treated by us as a means of processing information, which has two sides: obtaining new information and using the available information. Then the amount of information mastered by a person (work experience, life experience, mental processes of memory and representation) determines his productivity. We will call it “awareness.”

Inborn inclinations and a person’s abilities developed based on them, characterizing his ability to perceive, imagine, analyze, etc., information are traditionally summarized by the concept of cognitive ability

[2]. In some types of activity (creative), the main content of the work consists of creating new information. We are interested in the age dynamics of these two aspects (productive and creative) of human capital.

The investigated dynamic model describes age-related changes in cognitive abilities and personality awareness, which allows comparing the productive and creative human capital of people with different starting states of their mental setup. The model was proposed in [3]. Studying the model requires a sequential solution of the following problems:

1. Studying the possible trajectories of mental development. In this publication, we skip the details of solving the problem of calibrating the model to provide realistic trajectories: we will confine ourselves to demonstrating the fact that the calibrated model allows us to reproduce the core features of mental dynamics. A detailed description of the solution to this problem can be found in [4].

2. Assessing the quality of various trajectories in terms of the individual and society/state: the solution to this problem is also an independent study. In this study, we investigate the problem of assessing the quality of the tube's trajectories from society's point of view. The problem of assessing the quality of socialization from the standpoint of an individual is considered in [5].

3. Studying the problem on the rational choice of society/state in relation to the trajectories of mental development of the entire population: it is at this stage that a complete description of the model, including functionality and management, becomes possible, so the main emphasis will be placed on it.

2. MATHEMATICAL MODEL

2.1. Methodological Foundations and Tools

The model studied in this paper belongs to the class of dynamic models of optimal choice. A specific feature inherent in the problems of human choice is the presence of implicitly specified variables (and characteristics) that are not directly observed on most of the trajectory.

The method we use allows us to study the multicriteria optimal choice based on model experiments when comparing it with the particular constraints on unobservable variables known from social statistics. Modeling makes it possible to assess the consequences of choice along the entire trajectory but they can only be compared with the individual frontier values and patterns of the entire population as a whole, known from qualitative sociological studies.

Let us briefly consider the general logic of the study. A dynamic model in discrete time is considered, which assumes the natural development of several (two in this study) characteristics (phase variables) of the developing object (person) under study. The value of each characteristic can be calculated by the model equations for any period of time with the known initial values. The set of trajectories for all starting states forms a tube of phase trajectories, which must satisfy the specified calibration requirements. The starting values, in turn, are considered as control elements, the parameters of the random distribution of which result from the choice made by the subject (in this case, the state system for regulating socialization). The mathematical expectations of the functionals specified on the trajectories of the characteristics are the choice criteria. Since the optimization problem is two-criteria, its solution is a set of combinations of the Pareto optimal criteria (Pareto frontier), i.e., combinations of values attainable in the model that are not improved by at least one of the criteria without deterioration in the other. The Pareto optimal combinations of criteria values correspond to the Pareto optimal (effective) solutions, i.e., combinations of the starting states of an object's characteristics. The Pareto frontier is part of the frontier of the set including all attainable combinations of the criteria values, which, in turn, is the image of the trajectory tube in the criteria space. In a particular case, the Pareto frontier may consist of one, ideal point; otherwise there is a conflict between the criteria, the presence of which requires special external decision-making mechanisms [6].

This study uses an approach to solve multicriteria optimization problems based on the explicit construction or approximation of an implicitly specified set of attainable combinations of criteria and visual analysis of its boundaries using the dialog decision maps proposed in [6]. Since the problem is nonlinear, in this study, for the approximation, the method of deep holes is used, based on C. Shannon's (ϵ, δ) -coverings [7] for the implicitly given bounded stochastic sets [8]. In order to calibrate the model, the identification sets method [9], which employs the same approximation technology, was used.

2.2. Model Relationships

Human cognitive abilities will be denoted by $x \in X \subset \mathbf{R}$ as the set of acceptable values for the cognitive abilities of x different people. Normalized (dimensionless) cognitive abilities x can vary from 1 to -1 . The subset of admissible values at a certain age n is denoted as X_n . Let us designate by y , $y \in Y \subset \mathbf{R}$, the information accumulated by a person. We introduce the notation Y_n similarly to X_n .

A dynamic system that characterizes the mental development of an individual is considered where $n = 0, 1, \dots, 100, a_1 > 0, a_2 > 0, b > 0$:

$$x_{n+1} = x_n + a_1 x_n^2 - a_2 y_n^2, \quad (1)$$

$$y_{n+1} = y_n + b x_n y_n. \quad (2)$$

We introduce the set of starting states

$$S = \{(x_0, y_0): x_0^{\min} \leq x_0 \leq x_0^{\max}, y_0^{\min} \leq y_0 \leq y_0^{\max}\}. \quad (3)$$

In order to simplify the terminology, hereinafter, the same symbol n means both the age of an individual and time (age is proportional to time). The set of values of parameters $\{a_1, a_2, b, x_0^{\min}, x_0^{\max}, y_0^{\min}, y_0^{\max}\}$ is selected by calibration. The form of Eqs (1) and (2) ensures the uniqueness of the solutions for any n , and the chosen calibration procedure ensures their existence and specified properties. As a result of the identification and calibration described below, the values

$$a_1 = 0.0210, \quad a_2 = 0.0650, \quad b = 0.0415, \\ x_0^{\min} = 0.44, \quad x_0^{\max} = 0.80, \quad y_0^{\min} = 0.23, \quad y_0^{\max} = 0.33$$

are selected.

We designate by $x(n, x_0, y_0)$, $y(n, x_0, y_0)$, $n = 0, \dots, 100$, the solution of systems (1) and (2) for $x(0) = x_0$, $y(0) = y_0$. The object of the study is a trajectory (the set of possible trajectories) of the form $\{(x_0, y_0), (x_1, y_1), \dots, (x_{100}, y_{100})\}$ characterizing the mental dynamics of the individual. The study describes ways of assessing trajectories in terms of a person and the state, the logic of choosing between available trajectories, and its dependence on external factors. The distribution of the population between the initial values x_0 and y_0 is uneven and nonrandom in the sense that they are formed in the process of socialization. In order to form this distribution, an auxiliary model is presented below. Consider on S a truncated bivariate normal distribution of componentwise independent starting states of individuals of age $n = 0$ (20 years) with a probability density of the form

$$f(x_0, y_0, \mu_x, \mu_y) = \frac{1}{A(\mu_x, \mu_y)} \exp \left\{ -\frac{(x_0 - \mu_x)^2}{2\sigma_x^2} - \frac{(y_0 - \mu_y)^2}{2\sigma_y^2} \right\}, \quad (4)$$

where μ_x is the state-established priority of cognitive abilities (peak of the normal distribution), μ_y is the priority of the awareness established by the state (peak of the normal distribution),

$$A(\mu_x, \mu_y) = \iint_S f(x_0, y_0, \mu_x, \mu_y) dx_0 dy_0 \quad (5)$$

and $\sigma_x = (x_0^{\max} - x_0^{\min})/6$, $\sigma_y = (y_0^{\max} - y_0^{\min})/6$, which ensures that rule 3 is satisfied within the acceptable range of the starting characteristics. In the case of the baseline scenario of the traditional demographic policy, we will assume that the property $\mu_x = \mu_x^*$, $\mu_y = \mu_y^*$ is satisfied, where $\mu_x^* = (x_0^{\max} - x_0^{\min})/2$, and $\mu_y^* = (y_0^{\max} - y_0^{\min})/2$.

The state is able to influence the distribution parameters (μ_x, μ_y) with the balance constraint T_μ , the limit of the time available for studies to a nonadult in the given population, where

$$d_x = (y_0^{\max} - y_0^{\min}) / (x_0^{\max} y_0^{\max} - x_0^{\min} y_0^{\min}), \\ d_y = (x_0^{\max} - x_0^{\min}) / (x_0^{\max} y_0^{\max} - x_0^{\min} y_0^{\min}): \\ d_x \mu_x + d_y \mu_y \leq T_\mu. \quad (6)$$

At these values, the area of admissible values of the maximum distributions density of the starting abilities is a rectangle truncated from the upper right corner S : $M = \{(\mu_x, \mu_y): d_x \mu_x + d_y \mu_y \leq T_\mu, \mu_x \geq x_0^{\min}, \mu_y \geq y_0^{\min}\}$.

2.3. Substantiating the Dynamic Core of the Model

Information is stored in the memory of a person but it is also replenished over time through the use of cognitive abilities. However, the assimilation of new information depends on the volume of information assimilated (through analogy, associations, etc.). Negative x_n mean that a person loses access to some of the information he has memorized. This justifies the form of Eq (2).

Cognitive abilities are used by a person for reflection and self-accelerating (modeled by quadratic dependence in (1)) the increase in cognitive abilities in the future. If a person cannot allocate sufficient resources for reflection, then there is a gradual replacement of conscious thinking with the stereotyped (habitual one).

However, the assumption of the continuous and self-accelerating nature of developing cognitive abilities would fundamentally contradict the available actual social data, which show that by the end of a person's life, his cognitive abilities are rapidly declining. In our model, we proceed from the assumption that this is due to the volume of information assimilated by a person, its storage and, in particular, the maintenance of multiple associative links between elements. This justifies the negative factor in Eq (1).

The factors external to a person's mental dynamics can be divided into two main groups. On the one hand, it is the institution of socialization that forms a human personality. The model describes the dynamics of a formed personality; therefore, it is assumed that $n = 0$ corresponds to the age at which a fully fledged independent personality is formed (conditionally equated to the age of 20 in modern society, with historical comparisons it may be a younger age). Personal development up to 20 years is the first group of external factors that determine the set of admissible values of x_0 and y_0 . The second group reflects a person's state of health, which may not allow him to live up to $n = 100$ (120 years).

2.4. Model Identification

The considered model characterizes the hidden variables of human capital, which cannot be correlated to the statistical data. The interrelation of awareness with reproductive activity and cognitive abilities with creativity makes it possible to carry out an indirect identification based on the known age-related patterns.

The works of psychologists and sociologists on the dynamics of cognitive abilities and their measurements (e.g., IQ tests) are extremely contradictory [10]. In order to calibrate this study, the concept was chosen, according to which cognitive abilities degrade with age but their peak is observed not in adolescence, as early studies have shown, but in adulthood, and memory begins to degrade even later.

We denote by $X_{\max} = \max\{x: x \in X\}$ and similarly X_{\min} , Y_{\max} , and Y_{\min} . Unlike cognitive abilities, the model admits a change in the scale of information through a change in coefficients a_2 and b . This allows us to choose any desired scale Y , and for simplifying calculations and visualization, we calibrate these so that $Y_{\max} \approx 1$.

Based on these principles, in [5] the set of requirements (not taking into account physiological anomalies such as congenital mental retardation) on the dynamics of the model is formulated on the set $N = \{0, 1, \dots, 100\}$, $x_n \leq 1$, $n \in N$: $X_{\max} \approx 1$; $X_{\min} \approx -1$, $x_{60} < 0$; $x_{20} > 0$; $\max\{x_{20}: x_{20} \in X_{20}\} \approx 1$; $x_0 \in (0.62 \pm 0.18)$.

As shown by [4], the set of possible values for the calibration coefficients ensuring the realistic behavior of the model is rather narrow in the range of each criterion. The selected combination of coefficients within this range (0.0210, 0.0650, 0.0415) meets all the specified limitations. An example of a trajectory and its phase portrait ($x_0 = x_0^{\max}$, $y_0 = y_0^{\max}$) are shown in Fig. 1. The trajectory tube for the entire set of initial states S and its significant time (age) layers in the projection on the x and y axes are shown in Fig. 2.

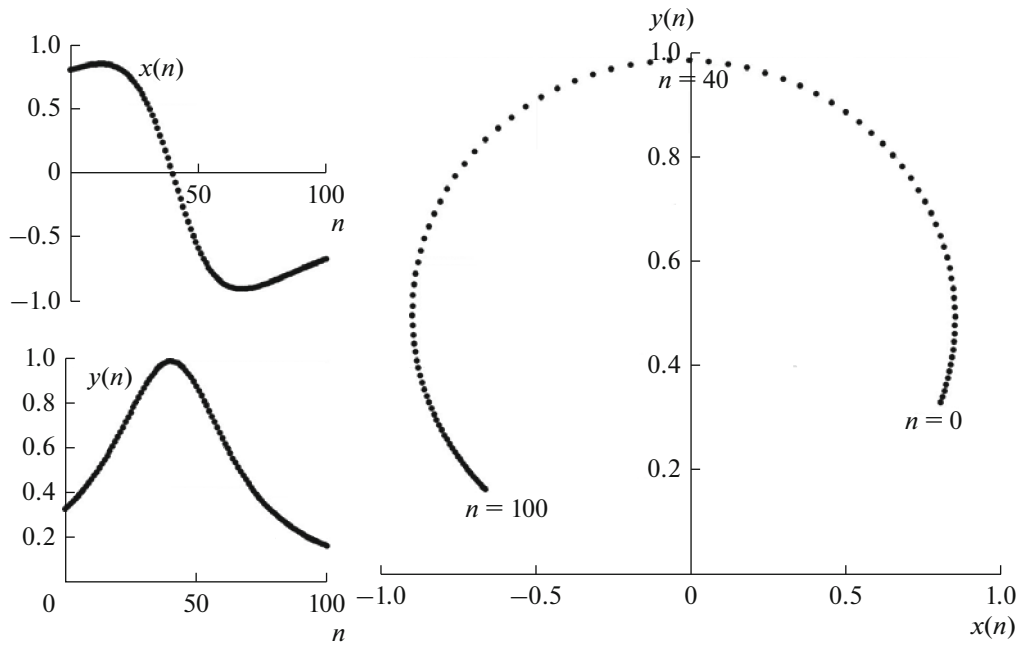


Fig. 1. Trajectory for $x_0 = x_0^{\max}, y_0 = y_0^{\max}$ and its phase portrait.

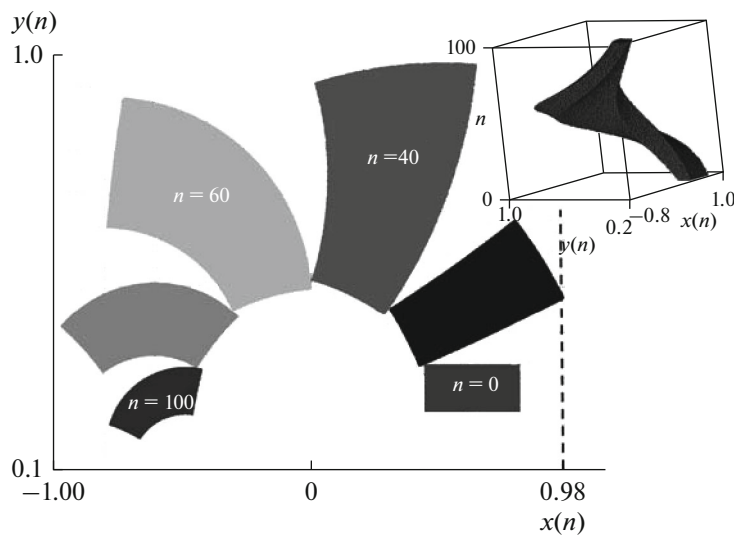


Fig. 2. Three-dimensional trajectory tube (top) and its significant time layers.

2.5. Rationale for the Socialization Model

With respect to the objectives of the study, the developmental outcome of the nonadult is described by his cognitive abilities and awareness. They depend on a number of factors, such as inborn potential, microenvironment (family, friends), macroenvironment (mass media, social environment), and educational system (secondary and higher schools). Depending on the studied subject, only one of these categories will act as the control, while the others will be factors of deviation from the desired result. Since only the state explicitly sets the task of choosing between cognitive abilities and awareness (defining these priorities in educational standards), it is advisable to choose the state as the subject of management.

As the working hypothesis, we have chosen the assumption that the distribution of cognitive abilities and awareness is componentwise independent normal around the priority value established by the state.

This statement cannot be proven strictly from statistical data since most of the required variables are not directly observed but is justified by the lack of data that would unequivocally indicate a strong correlation (direct or inverse) between creativity and the participants' productivity.

For the sake of simplicity, we will not consider cases of progressive state policy when state priorities are very different from the strict background cultural restrictions imposed on the development of the individual (for example, when it is prohibited for girls to attend secondary school or when boys have to receive only traditional military education) hindering the implementation of the priority chosen by the state.

The choice that the state can make is limited by the standard study time: it cannot set the goal of achieving arbitrarily large values of cognitive abilities and awareness. The state may not fully utilize this standard instruction time but cannot arbitrarily increase it. Taking this into account requires introducing balance constraint (6), the coefficients of which are calculated from the definition of the set \mathcal{S} .

For purposes of historical comparison, it would be necessary to take into account the gradual growth T_μ due to the increase in the number of years of nonadulthood, the burden on the student, and the development of learning technologies. For cross-country comparisons and analysis of the state educational policy in a particular historical period, T_μ can be taken as a constant. Further, in order to compare societies with different systems of socialization of the nonadult, T_μ will be considered as a dimensionless parameter, $T_\mu = 1$ corresponds to modern Russia, $T_\mu < 1$ is used for societies/countries with a shorter expected duration of study (including ancient ones), and $T_\mu > 1$ signifies societies/countries with a longer expected duration of study (e.g., Germany).

Since extreme mental deviations (psychopathology, geniuses, etc.) are not considered in the model, a truncated two-dimensional normal distribution is used, containing in its domain the area 3σ . If the state by its educational policy shifts the distribution center, then it remains normal with the same dispersion characteristic.

2.6. Assessment of Trajectory Quality

Assessing the preferability of trajectories is an independent auxiliary problem since it is necessary to take into account the contribution made by another component of human capital: human health. The traditional approach proposed by the classical theorists of human capital [1] consists in constructing life-long indices reflecting the return on capital throughout a person's life with allowance for its expected duration. In their work, qualification is assumed to be a constant value determined by the education system; however, we will apply this term to the dynamic model of awareness and cognitive abilities.

We denote by p_n , $n = 0, \dots, 100$, the relative size of the population at age n assuming $p_0 = 1$. Consider the dynamics of this size in the form

$$p_\gamma(n+1) = \begin{cases} (1 - \gamma k(n))p_\gamma(n), & \gamma k(n) \leq 1 - \delta \\ \delta p_\gamma(n), & \gamma k(n) > 1 - \delta. \end{cases} \quad (7)$$

Here, $k(n)$ is the proportion of deaths at the age n to those who survived up to this age in Russia (column 4, Mortality tables, see [11]), γ is the comparative mortality rate reflecting the demographic situation in comparison with the situation in Russia, and δ is a small number—in our study it is assumed that $\delta = 0.01$. When detailed mortality statistics are available for a particular country, the mortality rates can be used directly instead of the value γk . As the indicators of an individual's human capital, we take the cumulative indices of cognitive abilities and accumulated information of a person, constructed by analogy with [1], i.e.,

$$x'_\gamma(x_0, y_0) = \sum_{n=0}^{100} p_\gamma(n)x(n, x_0, y_0), \quad (8)$$

$$y'_\gamma(x_0, y_0) = \sum_{n=0}^{100} p_\gamma(n)y(n, x_0, y_0). \quad (9)$$

The conceptual meaning of these indices is the accumulated volume of productive and creative activity during a person's life with the given starting x_0 and y_0 .

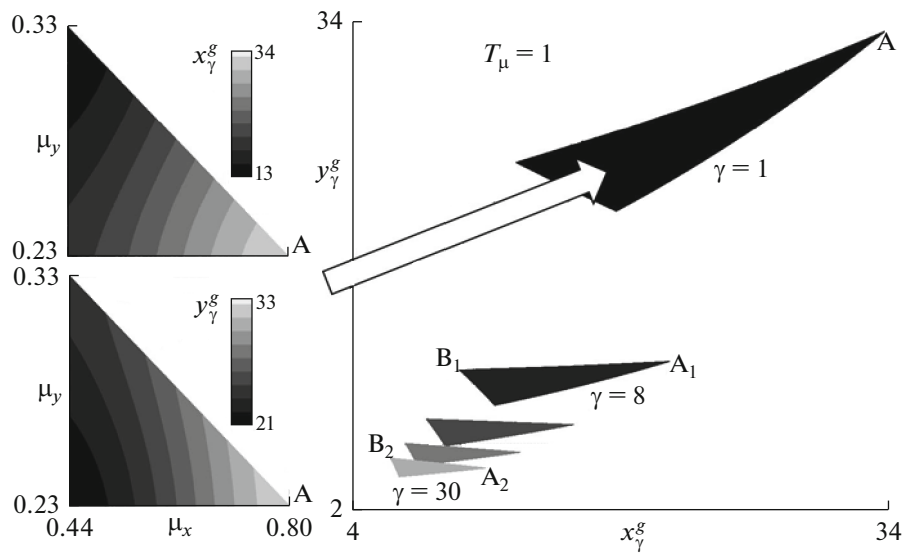


Fig. 3. Base budget constraint $T_\mu = 1$. On the left, the behavior of indicators on the set of admissible controls for $\gamma = 1$; and on the right, the dependence of the set of attainable combinations of indicator values on parameter γ .

2.7. Assessing the Quality of State Regulation

Based on these indices of an individual, the quality of the educational policy pursued by the state that seeks to maximize the human capital of the population can be assessed. To do this, we introduce indicators reflecting the mathematical expectation for the cumulative indices of cognitive abilities and the accumulated information on the starting distribution of individuals:

$$x_\gamma^g(\mu_x, \mu_y) = \frac{1}{A(\mu_x, \mu_y)} \iint_S \theta(x_\gamma^l(x_0, y_0)) x_\gamma^l(x_0, y_0) f(x_0, y_0, \mu_x, \mu_y) dx_0 dy_0, \quad (10)$$

$$y_\gamma^g(\mu_x, \mu_y) = \frac{1}{A(\mu_x, \mu_y)} \iint_S \theta(y_\gamma^l(x_0, y_0)) y_\gamma^l(x_0, y_0) f(x_0, y_0, \mu_x, \mu_y) dx_0 dy_0. \quad (11)$$

Here, θ denotes the Heaviside function equal to zero for negative values of the argument; and otherwise, to unity.

The problem of state regulation is reduced to a two-criteria optimization depending on parameter γ : $(x_\gamma^g, y_\gamma^g) \rightarrow \max, (\mu_x, \mu_y) \in M$. Indicators x_γ^g and y_γ^g with different μ_x and μ_y are calculated in this study on a uniform grid covering the set of admissible values of S . Integration in this case is replaced by summing the contribution of the trajectory from the corresponding node in the set of initial states.

The complexity of the calculation on a uniform grid is estimated as $O(100L^2)$, where L is the number of grid nodes for the stencil of the population's starting states, a parameter of the model algorithm. All the calculations below are carried out for $L = 10$. Comparison with $L = 100$ showed that a smaller number of grid points coarsens the set of attainable values of the criteria but does not introduce fundamental changes in the relationship of the criteria.

3. MODEL STUDY

Consider the problem of the state for the basic scenario corresponding to the conditions of modern Russia: $\gamma = 1$ and $T_\mu = 1$. Figure 3 on the left shows the sets of admissible combinations of the control parameters μ_x (abscissa axis) and μ_y (ordinate axis). The upper part of the figure shows the lines of the criterion x_γ^g level; and the lower one, the y_γ^g level. It can be seen from the figures that in the case under consideration the maximum of the criteria is achieved at the same point A corresponding to the combination (μ_x^{**}, μ_y^{**}) , where $\mu_x^{**} = x_0^{\max}, \mu_y^{**} = y_0^{\min}$. Each admissible combination of values of the control parameters

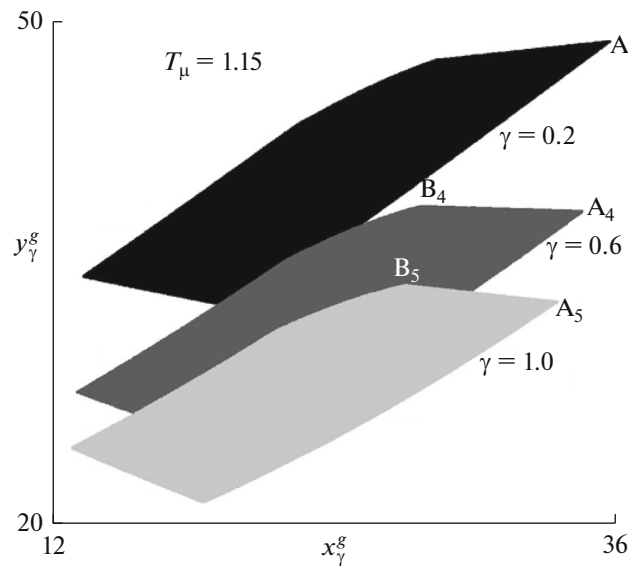


Fig. 4. Dependence of the set of achievable combinations of indicator values on parameter γ at an increased budget constraint of $T_\mu = 1.15$.

(μ_x, μ_y) corresponds to an achievable combination of values of the indicators $(x_\gamma^g(\mu_x, \mu_y), y_\gamma^g(\mu_x, \mu_y))$, i.e., a point in the two-dimensional space of the indicators. The set of all indices (abscissa) and y_γ^g (ordinate) attainable at $\gamma = 1$ and $T_\mu = 1$ is shown on the top right in Fig. 3. It is clear that in this case there is only one dominant point (an achievable combination of indicator values in which no indicator can be increased without harming another one): point A. Thus, in the considered case, the Pareto frontier (the set of non-dominated combinations of indicators) consists of only one, the so-called ideal, point. This means that in the problem of the state regulation of the socialization process for Russia with a basic budget constraint (expected duration of training), no conflict is expected between the indicators of the cumulative indices of cognitive abilities and the accumulated information.

Let us now consider the influence of parameter γ on the state's decision (right side of Fig. 3). The character of the Pareto frontier changes gradually: at $\gamma \leq 8$ there is the same dominant solution (point A_1) but at a high γ (ultrahigh mortality characteristic of preindustrial societies), the situation approaches the appearance of a weak (almost horizontal) Pareto frontier or, in other words, the Slater frontier (e.g., $[B_2, A_2]$).

Similarly, a change in T_μ (level of funding of socialization) in the range from 0.8 to 1.25 expands the set of admissible controls from empty to full; see (4). At $\gamma = 1$ this has little effect on the character of the relationship between the criteria. More significant changes are observed under the simultaneous variation of T_μ and γ .

Consider typical cases for $T_\mu = 1.15$ (see Fig. 4). At $\gamma > 0.8$ we observe a contradiction and the Pareto frontier (e.g., $[B_5, A_5]$ at $\gamma = 1$). In this case, for the indicator x_γ^g , the optimal solution is still $(\mu_x^{**}, \mu_y^{**}) = (x_0^{\max}, y_0^{\min})$; however, for y_γ^g the optimal solution is $(\mu_x^{**}, \mu_y^{***})$, where $\mu_y^{***} = y_0^{\min} + 0.7(y_0^{\max} - y_0^{\min})$. At $0.4 < \gamma < 0.8$, we observe a slight contradiction and the Slater boundary, e.g., $[B_4, A_4]$ under $\gamma = 0.6$. At $\gamma < 0.4$ there is no contradiction and an ideal point exists (e.g., A_3 at $\gamma = 0.2$ corresponding to the combination (μ_x^{**}, μ_y^{**})). Thus, under an increase in the budgetary constraint, we can expect a conflict between the values of the indices of cognitive abilities and the accumulated information.

The most interesting solutions to the optimization problem arise in conditions of the extremely limited resources of the education system and socialization: $T_\mu = 0.85$ in a society with a high mortality rate of $\gamma \sim 10$. In this case, the boundary of the set of achievable criteria values becomes concave (see Fig. 5), and two

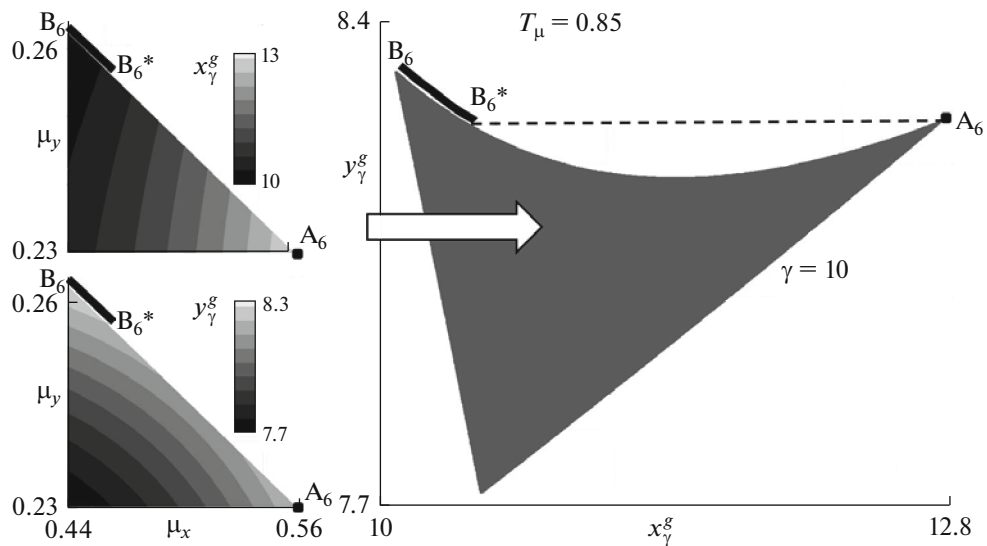


Fig. 5. Case of low budget constraint and high mortality ($T_\mu = 1$, $\gamma = 10$). On the left, behavior of indicators on set of admissible controls and set of optimal solutions from two clusters. On the right, the set of attainable combinations of values of indicators and its Pareto frontier $\{[B_6, B_6^*], A_6\}$.

isolated conflicting clusters of Pareto optimal points ($[B_6, B_6^*]$ and A_6 on the right side in Fig. 5) and their corresponding effective solutions (the preimage of the set $\{[B_6, B_6^*], A_6\}$ in on the left side of Fig. 5). The presence of two clusters of Pareto optimal solutions means the possibility of two fundamentally different strategies for managing the socialization process.

4. CONCLUSIONS

The multicriteria problem of optimizing the state regulation of the quality of human capital in an information society was considered based on the a discrete dynamic nonlinear model that takes into account the age dynamics of the awareness and cognitive abilities of an individual as a carrier of information. It was shown that the limited time budget in the state education system largely eliminates the contradictions between the goals of developing the creative and productive human capital. For countries with a rather high life expectancy (including Russia), the optimal decision of the state corresponds to the maximum development of students' cognitive abilities (which is manifested in a gradual transition to a competence-based approach in education).

The model also reveals a contradiction between the criteria for low life expectancy, both in the case of the long time spent on socialization (the case of the ruling elite of a preindustrial society) and in the case of the short time spent on socialization (the majority of the population). However, the character of this contradiction is different. In the first case, there is a weak Pareto boundary and the choice can be made according to the Pareto Rule (20% of efforts give 80% of the result). In the second case, the set of attainable values of the criteria is nonconvex, the Pareto frontier is disconnected, and the choice is nontrivial.

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