ISSN 2070-0482, Mathematical Models and Computer Simulations, 2018, Vol. 10, No. 4, pp. 519–528. © Pleiades Publishing, Ltd., 2018.
Original Russian Text © A.G. Tonkoshkur, 2018, published in Matematicheskoe Modelirovanie, 2018, Vol. 30, No. 1, pp. 103–116.

Modeling Conductive Heat Transfer in Ground Air Coolers

A. G. Tonkoshkur*

Yuri Gagarin State Technical University of Saratov, Saratov, Russia

**e-mail: alex-g-t@bk.ru*

Received March 6, 2017

Abstract—We propose a mathematical model for the heat transfer in ground tubular air coolers to construct the temperature field in it. The aim is to find the least admissible distance between the tubes. We visually present the results obtained in a program developed according to this model on *MATLAB* and in the *PDE Toolbox MATLAB* environment. Comparing the obtained results, we see that they are close to each other.

Keywords: ground heat exchanger, mathematical modeling, conductive heat transfer, unsteady thermal conductivity

DOI: 10.1134/S2070048218040142

1. INTRODUCTION

In modern heating and cooling systems for buildings, energy-efficient technologies with ground heat exchangers are widespread (see [1–5]).

The ground of the Earth's surface layer is actually a heat accumulator of infinite capacity such that its temperature varies slightly during the year, is positive, but not high, and substantially differs from the air temperature. In summer, it is lower than the air temperature. In winter, it is higher than the air temperature. Therefore, the ground represents considerable interest from the following viewpoints:

(1) it is a low-grade heat source for the operation of thermal pumps in heating systems or heating recuperation in ventilation units;

(2) it is a heat absorber in air-conditioning systems.

In such a case, ground heat exchangers are used to exchange heat with the ground.

There are ground heat exchangers of various designs. For example, in airconditioning systems, heat exchangers frequently consist of a row of horizontal underground tubes at a depth of at least 1.5 m so that the air from the atmosphere is pumped through these tubes. Passing through the tubes, the air is cooled by the heat exchange with the ground and enters the building. The tubes are slightly inclined (to sink the condensate water) and are connected between each other; e.g., the connection is parallel and uses U-shaped collector systems (see [1, 6, 7]).

An important parameter of ground heat exchangers is the least admissible distance between the tubes, ensuring guaranteed and sufficiently fast heat scattering (in the air-cooling case) in the ground.

Currently, only general recommendations are used to design and assemble ground heat exchangers; they do not take into account the specific properties of the designed object, which can lead to a decrease of the efficiency and an unreasonable increase in investments. The recommended distances between the tubes do not depend on the operating regime and heat load of the unit (e.g., at least 1 m in [8] or from 0.5 to 0.7 m in [9]). However, if the heat dissipation is intense, then the recommended distance between the tubes might be insufficient. This causes an overheating (in the air-cooling case) of the ground block around the heat exchanger tubes, their temperature fields intersect each other, and the heat scattering in the ground is impeded. This leads to the instability of the operation of the airconditioning system and an increase of the ground's accumulating ability in the neighborhood of the tubes. Considerable time is required to restore the ground block's accumulating ability in a natural way, while it is not always possible to apply special measures.

Thus, the distance between the tubes has to be such that their temperature fields are disjoint and, respectively, the surrounding ground block is not overheated and does not lose its accumulation ability during its exploitation. By solving this problem, we can determine the necessary boundaries of the area for a field and arrange the territory optimally.

The existing methods to solve this problem, based on constructing a heat balance equation (see [5]) is applicable for the engineering design of a ground heat exchanger, i.e., for the case where the heat load of the ground heat exchanger and the temperature of the heat carrier at the tube output are given. However, in the case of the calibrating computation of the heat exchanger, these parameters are unknown. The mentioned methods are not applicable in such a case. This problem can be solved based on the numerical modeling and programming of the conductive heat transfer; in such a case, it is treated as a nonstationary boundary-value heat problem to construct and analyze the temperature field of the heat exchanger. The present paper is devoted to resolving this problem.

2. MATHEMATICAL MODEL

The heat transfer in a horizontal tubular ground air cooler is a complex heat exchange consisting of the heat dissipation under the forced air flow in tubes of the ground's heat exchanger and the heat conductivity between the wall of the heat exchanger's tube and the ground block. Hence, the problem can be conditionally divided into two parts:

- the inner part consists of the processes inside the heat exchanger tube;
- the outer part consists of the processes outside the inner wall of the heat exchanger tube.

Modeling problems related to finding the temperature field is based on resolving the following Fourier-Kirchhoff differential equation for the nonstationary heat transfer by the heat conductivity and connecting the temporal and spatial changes of the temperature at each point of the body (see [10]):

$$\rho c \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial t}{\partial z}\right) + Q_w(x, y, z, \tau, t), \tag{1}$$

where $t = t(x, y, z, \tau)$ is the temperature measured in °C, ρ is the density measured in kg/m^3, c is the specific heat measured in J/(kg K), λ is the heat conduction coefficient measured in W/(m K), τ is the time measured in s, and $Q_w(x, y, z, \tau, t)$ is the power of the internal heat generation sources measured in W/m^3.

The given equation, together with the uniqueness conditions, forms the complete mathematical formulation of the heat boundary-value conditions.

A simplification is needed to model the outer problem because there are too many elements affecting the heat conduction in that problem and those elements are not completely determined from the informational viewpoint.

The ground is treated as a solid isotropic medium, where the heat is propagated only by means of the heat conductivity. Obviously, the ground is a complex multiphase system such that the following three forms of the heat transfer take place in it simultaneously (see [11]):

(1) the heat conductivity inside the particle elements of the solid phase and at the points of their direct contact;

(2) the radiation from a particle to another one;

(3) the convection and heat conductivity in the pore space under a damp transfer.

It is a hard problem to find the temperature field in the field, where all these processes take place; however, taking into account the low values of the ground block temperature in the natural state observed below the freezing depth (for Saratov city, they are between +1 and +17°C, see [12]), we can ignore the impact of the convection, radiation, and damp transfer (according to [13]).

In the considered case, we have only one direction of the heat propagation; it is orthogonal to the tube's axis. Then this is a one-dimensional nonstationary conductive heat transfer with cylindrical symmetry. Taking into account this and the fact that there are no internal sources in the tube wall and the ground ($Q_w = 0$), we can represent Eq. (1) in the following one-dimensional form:

$$\rho c \frac{\partial t}{\partial \tau} = \frac{\lambda}{r} \frac{\partial}{\partial r}\left(r \frac{\partial t}{\partial r}\right), \tag{2}$$

where r is the radial coordinate.

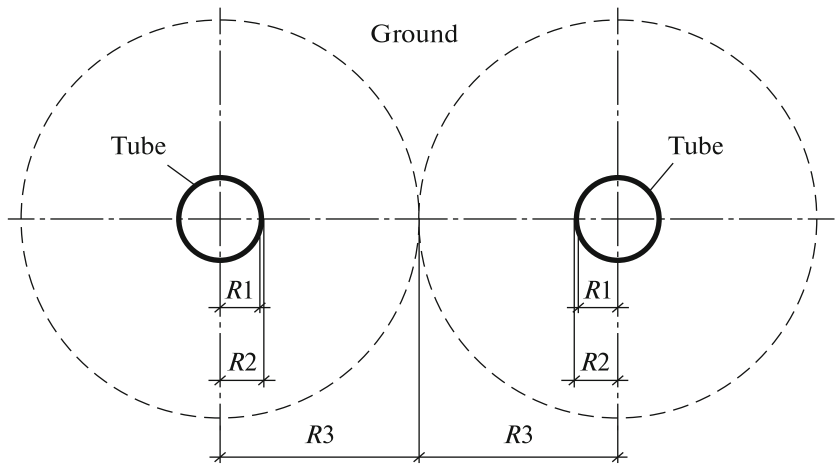

Fig. 1. Scheme to compute distance between neighboring tubes of horizontal ground heat exchanger, where $R1$ is inner radius of tube, $R2$ is its outer radius, and $R3$ is smallest radius of ground cylinder such temperature fields of neighboring tubes are disjoint.

A single heat exchanger tube and the surrounding ground block can be geometrically represented as a two-layer cylinder with different thermophysical properties (Fig. 1). For this case, the mathematical formulation of problem (2) is as follows:

$$\begin{cases} \rho_{tb} c_{tb} \dfrac{\partial t_{tb}}{\partial \tau} = \dfrac{\lambda_{tb}}{r} \dfrac{\partial}{\partial r}\left(r \dfrac{\partial t_{tb}}{\partial r} \right), & R1 \le r < R2 \\ \rho_{gr} c_{gr} \dfrac{\partial t_{gr}}{\partial \tau} = \dfrac{\lambda_{gr}}{r} \dfrac{\partial}{\partial r}\left(r \dfrac{\partial t_{gr}}{\partial r} \right), & R2 < r \le R3, \end{cases} \tag{3}$$

where ρ_{tb} and ρ_{gr} are the density of the material of the tube wall and of the ground block, respectively, measured in kg/m^3; c_{tb} and c_{gr} are the heat capacity of the material of the tube wall and of the ground block, respectively, measured in J/(kg K); t_{tb} and t_{gr} are the temperature of the tube wall and of the ground, respectively, measured in °C; λ_{tb} and λ_{gr} are the heat conductivity of the material of the tube wall and of the ground, respectively, measured in W/(m K); $R1$ and $R2$ are the inner and outer radii of the tube, respectively, measured in m; and $R3$ is the smallest radius of the ground cylinder such that the temperature fields of the neighboring tubes are disjoint (measured in m).

System (3) describes many variants of the development of the heat-conductivity process. To select one of them (together with its complete mathematical description), add uniqueness conditions to relations (3). Those conditions contain geometric, physical, initial-value and boundary-value conditions. For example, for the tube on the left (Fig. 1), they take the following form.

• The initial-value conditions are as follows:

$$\tau = 0:\ t = t_0,\ \ R1 \le r \le R3, \tag{4}$$

where t_0 is the initial temperature of the tube and ground (at time $\tau = 0$), measured in °C.

• The boundary-value conditions are as follows.

At the left-hand boundary, the third-type boundary-value conditions are posed because the heat transfer from the air to the inner wall of the tube takes place:

$$r = R1:\ -\lambda_{tb} \frac{\partial t_{tb}}{\partial r} = 2\pi r \alpha \left(t_{air} - t_1 \right), \quad \tau > 0, \quad \alpha > 0, \tag{5}$$

where α is the coefficient of the heat transfer from the air to the inner wall of the tube, measured in W/(m^2 K), while t_{air} and t_1 are the air temperature and the temperature of the inner wall of the tube, respectively, measured in °C.

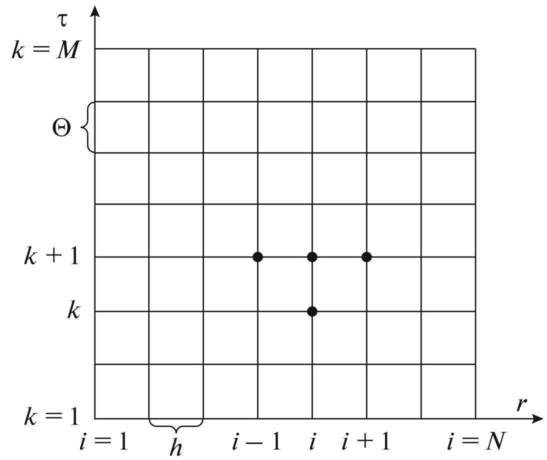

Fig. 2. Template of implicit four-point difference scheme.

At the boundary between the tube and the ground, the fourth-type boundary-value conditions are posed; they characterize the ideal contact between the tube and the ground and guarantee that the temperatures and heat flows from both sides of the boundary are equal to the each other:

$$\begin{cases} t_{\rm tb}(\tau, r^*) = t_{\rm gr}(\tau, r^*), \\ -\lambda_{\rm tb} \left.\dfrac{\partial t_{\rm tb}}{\partial r}\right|_{r=r^*} = -\lambda_{\rm gr} \left.\dfrac{\partial t_{\rm gr}}{\partial r}\right|_{r=r^*}, \end{cases} \quad (r^* = R2). \tag{6}$$

At the right-hand boundary, the first-type boundary-value conditions are posed such that the ground temperature is constant and equal to its initial value t_0:

$$r = R3:\ t_{\rm gr} = t_0, \quad \tau > 0. \tag{7}$$

The physical uniqueness conditions set the values $\lambda_{\rm tb}$, $\lambda_{\rm gr}$, $\rho_{\rm tb}$, $\rho_{\rm gr}$, $c_{\rm tb}$, $c_{\rm gr}$, and α.

The geometric uniqueness conditions set the values $R1$, $R2$, and $R3$. Since the value $R3$ refers to the sought parameter, which is not known before the computation, its value is set by the iteration procedure.

The value of α is found from the criteria equations describing the heat transfer from gases to a wall under the air movement in a circular horizontal tube. Depending on the motion mode of the air in the tube determined by the Reynolds criterion

$$\mathrm{Re} = \frac{w_{\rm air} d_{\rm in}}{\upsilon_{\rm air}}, \tag{8}$$

the coefficient α is found from one of the following expressions (see [14]):

• if $\mathrm{Re} < 2300$ (the laminar motion mode of the air in the tube), then

$$\alpha = 0.17 \frac{\lambda_{\rm air}}{d_{\rm in}} \mathrm{Re}^{0.33}\, \mathrm{Pr}_{\rm air}^{0.43}, \tag{9}$$

• if $2300 \le \mathrm{Re} \le 10000$ (the transitional motion mode of the air), then

$$\alpha = 0.00365 \frac{\lambda_{\rm air}}{\upsilon_{\rm air}} w_{\rm air}\, \mathrm{Pr}_{\rm air}, \tag{10}$$

• if $\mathrm{Re} > 10000$ (the turbulent motion mode of the air), then

$$\alpha = 0.023 \frac{\lambda_{\rm air}}{d_{\rm in}} \mathrm{Re}^{0.8}\, \mathrm{Pr}_{\rm air}^{0.4}, \tag{11}$$

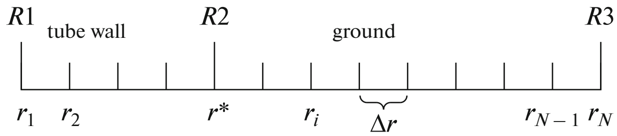

Fig. 3. Finite-difference grid, where $r_2,\ldots r_i,\ldots r_{N-1}$ are coordinates of inner nodes, while $r_1,\ldots r^*,\ldots r_N$ are coordinates of outer nodes.

where w_{air} is the air velocity in the tube measured in m/s, $d_{\text{in}} = 2R1$ is the inner diameter of the tube measured in m, υ_{air} is the kinematic viscosity coefficient of the air measured in m^2/s, λ_{air} is the heat-conductivity coefficient of the air measured in W/(m K), and Pr_{air} is the Prandtl number for the air.

To solve the posed boundary-value problem, we use the method of finite differences (see [15, 16]) based on an implicit four-point scheme and uniform grids (both with respect to time and the spatial variable), representing a solid body by a set of nodes (see [10]) and assuming that the thermophysical properties of the tube material and ground block do not depend on the temperature.

Introduce the uniform spatial grid (see Fig. 2)

$$r_i = (i-1)h, \quad i = 1,\ldots,N, \quad r_1 = R1,\ldots,r_N = R3, \quad h = \frac{R3 - R1}{N-1},$$

where i is the number of the step with respect to the spatial coordinate, while h and N are the integration step and the number of nodes with respect to the radial coordinate, respectively.

In the same way, introduce the temporal grid (see Fig. 2)

$$\tau_k = k\theta, \quad k = 0,1,\ldots M, \quad \tau_0 = 0,\ldots,\tau_M = \tau_{\text{finite}}, \quad \tau > 0,$$

where k is the number of the step with respect to time, while θ and M are the integration step and the number of nodes with respect to the temporal coordinate, respectively.

Decompose the width of the tube wall and the ground block into $N-1$ equal intervals; this yields a finite-difference grid with step Δr with respect to the radial coordinate (see Fig. 3).

Define the value of the temperature at the ith node at time $\tau = \tau_k = k\theta$ as $t(r_i, \tau_k) = t_i^k$. Further, change the differential operators in (2) for their finite-difference analogs; the scheme displayed in Fig. 2:

$$\frac{\partial t}{\partial \tau} = \frac{t_i^{k+1} - t_i^k}{\theta}$$

and

$$\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right) = \frac{1}{h^2}\left\{r_{i+(1/2)}t_{i+1}^{k+1} - \left[r_{i-(1/2)} + r_{i+(1/2)}\right]t_i^{k+1} + r_{i-(1/2)}t_{i-1}^{k+1}\right\},$$

where $r_{i-(1/2)} = \dfrac{r_{i-1} + r_i}{2}$ and $r_{i+(1/2)} = \dfrac{r_i + r_{i+1}}{2}$.

As a result of the approximation of the partial derivatives by the corresponding finite differences, we get the following system of linear algebraic equations:

$$\rho c\frac{t_i^{k+1} - t_i^k}{\theta} = \frac{\lambda}{r_i h^2}\left\{r_{i+(1/2)}t_{i+1}^{k+1} - \left[r_{i-(1/2)} + r_{i+(1/2)}\right]t_i^{k+1} + r_{i-(1/2)}t_{i-1}^{k+1}\right\}, \tag{12}$$
$$i = 2,\ldots,N-1, \quad k = 0,1,\ldots,M.$$

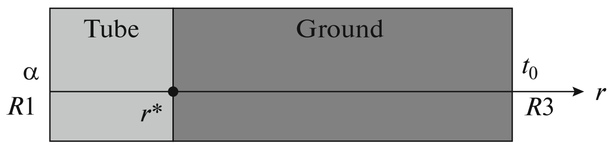

Fig. 4. The geometry of the problem.

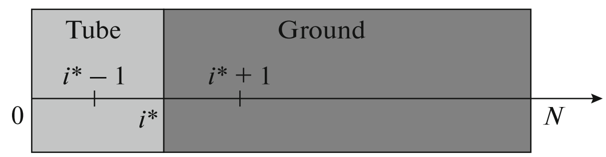

Fig. 5. Template of difference grid.

For the two-layer cylinder, system (12) is expressed as follows:

$$\begin{cases} \text{for } r_1 = R1 < r_i < r^* = R2 \\ \rho_{\text{tb}} c_{\text{tb}} \dfrac{t_i^{k+1} - t_i^k}{\theta} = \dfrac{\lambda_{\text{tb}}}{r_i h^2} \left\{ r_{i+(1/2)} t_{i+1}^{k+1} - \left[r_{i-(1/2)} + r_{i+(1/2)} \right] t_i^{k+1} + r_{i-(1/2)} t_{i-1}^{k+1} \right\}, \\ \text{for } r^* = R2 < r_i < r_N = R3 \\ \rho_{\text{gr}} c_{\text{gr}} \dfrac{t_i^{k+1} - t_i^k}{\theta} = \dfrac{\lambda_{\text{gr}}}{r_i h^2} \left\{ r_{i+(1/2)} t_{i+1}^{k+1} - \left[r_{i-(1/2)} + r_{i+(1/2)} \right] t_i^{k+1} + r_{i-(1/2)} t_{i-1}^{k+1} \right\}. \end{cases} \tag{13}$$

Since a nonstationary problem is considered, it follows that system (12) has to be solved at each time step. To do this, we reduce the specified system to a two-point first-order equation (see [10]), assuming that there exist number sets n_i and m_i $(i = \overline{1, N-1})$ such that

$$t_i^{k+1} = n_i t_{i+1}^{k+1} + m_i, \tag{14}$$

where

$$n_i = \frac{A_i}{B_i - C_i n_{i-1}}, \quad m_i = \frac{C_i m_{i-1} - F_i}{B_i - C_i n_{i-1}}, \quad A_i = \frac{\lambda}{h^2} \frac{r_{i+(1/2)}}{r_i},$$

$$B_i = \frac{\lambda}{h^2} \frac{r_{i-(1/2)} + r_{i+(1/2)}}{r_i} + \frac{\rho c}{\theta}, \quad C_i = \frac{\lambda}{h^2} \frac{r_{i-(1/2)}}{r_i}, \text{ and } F_i = -\frac{\rho c}{\theta} t_i^k.$$

To construct the temperature field according to (14), we use the sweep method; at the first stage, the sweep coefficients n_i and m_i are found.

For the left-hand boundary-value condition, digitize the third-type boundary-value conditions with error $O(h)$. The initial sweep coefficients m_1 and n_1 are found from the relation

$$t_1 = n_1 t_2 + m_1. \tag{15}$$

Thus, $-\lambda_{\text{tb}} \left.\dfrac{\partial t_{\text{tb}}}{\partial r}\right|_{r=r_1} = 2\pi r \alpha \left(t_{\text{air}} - t|_{r=r_1} \right) \Rightarrow -\lambda_{\text{tb}} \dfrac{t_2 - t_1}{h} = 2\pi r \alpha \left(t_{\text{air}} - t_1 \right).$

Introduce the notation $\dfrac{\alpha h}{\lambda_{\text{tb}}} \equiv Bi$. Then

$$t_1 - t_2 = 2\pi r Bi \left(t_{\text{air}} - t_1 \right) = 2\pi r Bi t_{\text{air}} - 2\pi r Bi t_1. \tag{16}$$

Grouping the terms of (16), we obtain the relations

$$t_1 + 2\pi r Bi t_1 = t_2 + 2\pi r Bi t_{\text{air}}$$

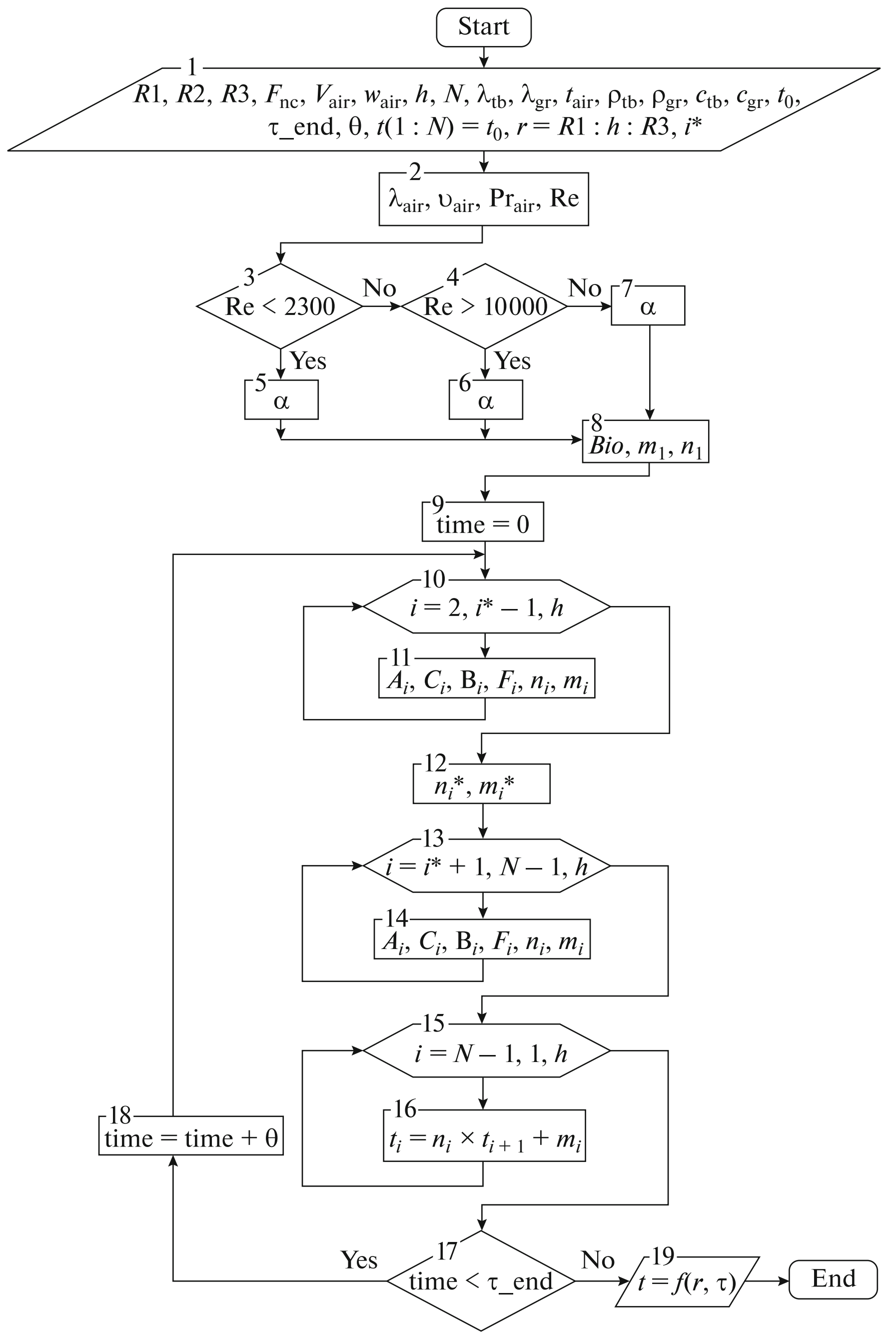

Fig. 6. Block diagram for computation of temperature field in ground air cooler.

and

$$t_1\left(1 + 2\pi rBi\right) = t_2 + 2\pi rBit_{\text{air}},$$

which means that

$$t_1 = \frac{1}{1 + 2\pi rBi}t_2 + \frac{2\pi rBi}{1 + 2\pi rBi}t_{\text{air}}.$$

Thus, according to (15), the sweep coefficients for the first node are expressed as follows:

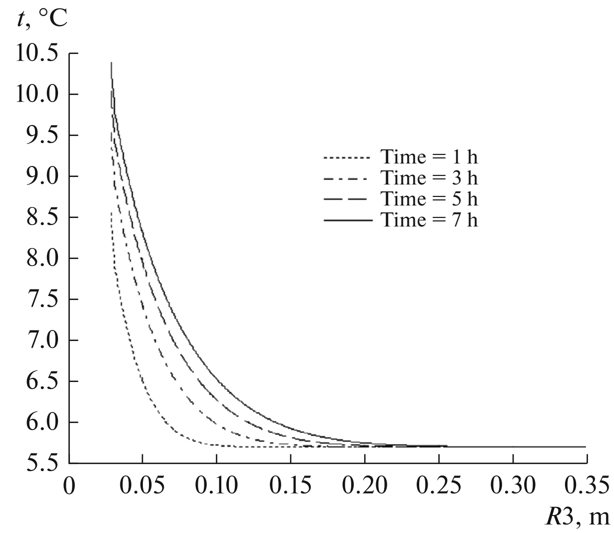

Fig. 7. Predicted temperature field in ground air cooler after various time intervals.

$$\begin{cases} n_1 = \dfrac{1}{1 + 2\pi r Bi} = \dfrac{\lambda_{tb}}{\lambda + 2\pi r\alpha h}, \\ m_1 = \dfrac{2\pi r Bi}{1 + 2\pi r Bi} t_{air} = \dfrac{2\pi r\alpha h}{\lambda_{tb} + 2\pi r\alpha h} t_{air}. \end{cases} \tag{17}$$

The sweep coefficients for the boundary between the tube and ground are found by the fourth-type boundary-value conditions (6) at the contact point of two media with different thermophysical properties, located at the i^*th node with coordinate r^* (see Fig. 4).

Deduce the sweep coefficients at the contact point for two media, considering the first-order approximation with respect to the step of the radial coordinate:

$$\begin{cases} t_{tb,i^*} = t_{gr,i^*}, \\ -\lambda_{tb} \left.\dfrac{\partial t_{tb}}{\partial r}\right|_{r=r^*} = -\lambda_{gr} \left.\dfrac{\partial t_{gr}}{\partial r}\right|_{r=r^*}. \end{cases}$$

Taking into account that $t = t_{tb}$ for $i < i^*$ and at $t = t_{gr}$ for $i > i^*$, eliminate the indices characterizing the medium. According to Fig. 5, we obtain the relations

$$\begin{cases} t_{tb,i^*} = t_{gr,i^*}, \\ -\lambda_{tb} \dfrac{t_{tb,i^*} - t_{i^*-1}}{h} = -\lambda_{gr} \dfrac{t_{i^*+1} - t_{gr,i^*}}{h}. \end{cases} \tag{18}$$

Introduce the notation $t_{tb,i^*} = t_{gr,i^*} \equiv t_{i^*}$. Using the sweep relation $t_{i^*-1} = n_{i^*-1} t_{i^*} + m_{i^*-1}$ and the second relation of conditions (18), we conclude that

$$t_{i^*} - n_{i^*-1} t_{i^*} - m_{i^*-1} = \frac{\lambda_{gr}}{\lambda_{tb}} t_{i^*+1} - \frac{\lambda_{gr}}{\lambda_{tb}} t_{i^*},$$

i.e., $t_{i^*} = \dfrac{\lambda_{gr}}{\lambda_{gr} + \lambda_{tb}(1 - m_{i^*-1})} t_{i^*+1} + \dfrac{\lambda_{gr} n_{i^*-1}}{\lambda_{gr} + \lambda_{tb}(1 - m_{i^*-1})}$, which implies the following expressions for the sought coefficients:

$$\begin{cases} n_{i^*} = \dfrac{\lambda_{gr}}{\lambda_{gr} + \lambda_{tb}(1 - m_{i^*-1})} \\ m_{i^*} = \dfrac{\lambda_{tb} n_{i^*-1}}{\lambda_{gr} + \lambda_{tb}(1 - m_{i^*-1})}. \end{cases} \tag{19}$$

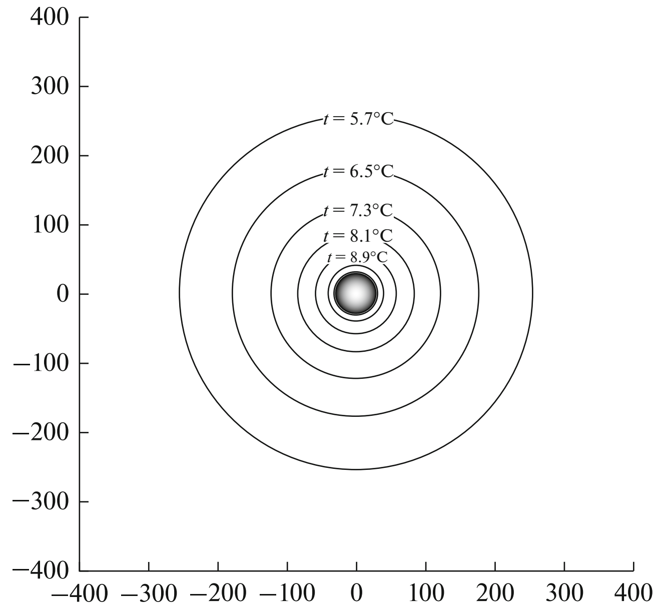

Fig. 8. Predicted temperature field in ground air cooler, constructed in *PDE Toolbox MATLAB* environment.

Figure 6 presents the block diagram for the computation of the temperature field in the considered ground heat exchanger.

In block 1, the input data are given: the preliminary values of parameters $R1$, $R2$, and $R3$, $t_{\rm air}$, t_0, h, θ, N, $\lambda_{\rm tb}$, $\lambda_{\rm gr}$, $\rho_{\rm tb}$, $\rho_{\rm gr}$, $c_{\rm tb}$, and $c_{\rm gr}$; the open flow area $F_{nc} = \pi R1^2$ of the tube; the airflow $V_{\rm air}$ through the tube; the air velocity $w_{\rm air} = (V_{\rm air}/F_{nc})(t_{\rm air} + 273)/273$; and the final time τ_end for the computation. Also, we set the initial temperature distribution $t(1:N) = t_0$ over all the grid nodes, the radius $r = R1:h:R3$ with respect to the grid, and the number i^* of the node corresponding to the boundary between the tube and the ground.

In block 2, the following thermophysical parameters of the air are computed as functions of the air temperature: the heat-conductivity coefficient, the kinematic viscosity, and the Prandtl number. Also, the Reynolds coefficient Re is computed according to (8).

Depending on the value of Re (see blocks 3 and 4), we use relations (9), (10), or (11) to compute the heat-transfer coefficient α (see blocks 5, 6, or 7).

In block 8, the value of Bi and the sweep coefficients n_1 and m_1 are computed for the first node.

In block 9, the initial time is given: $time = 0$.

For the spatial grid, for the nodes from the second one to the one preceding the boundary between the tube and the ground (see block 10), we use the direct sweep method to compute the parameters A_i, C_i, B_i, and F_i and the sweep coefficients n_i and m_i (see block 11).

In block 12, the sweep coefficients n_{i*} and m_{i*} are computed for the boundary between the tube and the ground.

For the spatial grid, for the nodes from the first node after the boundary between the tube and the ground to the next-to-last node of the grid (see block 13), we use the direct sweep method to compute the parameters A_i, C_i, B_i, and F_i and the sweep coefficients n_i and m_i (see block 14).

For the spatial grid, for the nodes from the next-to-last one to the first one (see block 15), we use the inverse sweep method to compute the temperature field (see block 16).

In block 17, the time is counted. Once the given time is passed ($time = \tau\_end$), the obtained results are printed out (see block 19) and the computation is complete. Otherwise, the current time (time) is increased by one integration step θ with respect to the temporal grid (see block 18) and the computation is repeated, beginning from the 10th block.

3. TEST COMPUTATIONS

Figure 7 presents the temperature field in the ground heat exchanger after 1, 3, 5, and 7 working hours, respectively, computed with respect to the above algorithm in the *MATLAB* environment. The following parameters are treated as the input data: $t_{air} = 32.5°C$ (the air temperature), $t_0 = 5.7°C$ (the initial ground temperature); the tube's material is ethylene homopolymer, $\lambda_{tb} = 0.29$ W/(m °C) (the tube's heat-conductivity coefficient), $\rho_{tb} = 1315$ kg/m^3 (the tube's density), $c_{tb} = 990$ J/(kg °C) (the tube's heat capacity); the ground's heat-conductivity coefficient $\lambda_{gr} = 0.658$ W/(m °C), $\rho_{gr} = 1700$ kg/m^3 (the ground's density), $c_{gr} = 2010$ J/(kg °C) (the ground's heat capacity), $h = 0.0005$ m (the integration step with respect to the radial coordinate), $\theta = 0.6$ s (the integration step with respect to time), $R1 = 0.029$ m (the inner radius of the tube), $R2 = 0.0315$ m (the outer radius of the tube), and $R3 = 0.3$ m (the radius of the ground cylinder). It is assumed that the temperature field from the tube is propagated for seven hours: from 12:00 a.m. to 7:00 p.m. (in daylight), i.e., during the period of the highest diurnal temperature and the strongest heat flow in the heat exchanger.

From Fig. 7 we see that the depth of the ground heat penetration is about 0.26 m ($R3_{min} = 0.26$ m) during this seven-hour period and under the assumed input data. No further propagation of the temperature field is observed because the outer air temperature falls in the evening and the heat flow in the heat exchanger decreases.

Thus, according to Fig. 1, the shortest distance between the axes of the neighboring tubes of the exchanger such that their temperature fields are disjoint is $2R3_{min} = 2 \times 0.26 = 0.52$ m.

To compare, we compute the temperature field in a ground air cooler, using the finite-element technology in the *PDE Toolbox MATLAB* environment; the result is presented in Fig. 8. The input data is the same as for the computation given above according to the proposed model.

Comparing the graphs in Figs. 7 and 8, we see that the proposed finite-difference model and the standard finite-element model yield identical results to each other. In both cases, the propagation of the temperature field from the tube ends at a distance of about 0.25 m from the tube's center, which confirms that the proposed model is correct and justified.

REFERENCES

1. S. O. Filatov and V. I. Volodin, "The work of heat exchangers for utilization of ground heat," Tr. BGTU, Khim. Tekhnol. Neorg. V-v, No. **3** (141), 179–184 (2011).
2. Ground Heat Exchangers for Geothermal Geat Pump, 2013. http://solarsoul.net/gruntovyeteploobmenniki-dlya-geotermalnogo-teplovogo-nasosa
3. E. V. Korepanov, "Determining method for the thermal potential of a vertical U-shaped ground collector of a heat pump plant," Vestn. IzhGTU im. and M. T. Kalashnikova, No. **1** (65), 108–112 (2015).
4. Fundamentals of Design Methodology for Heat Pump Systems with Horizontal Ground Heat Exchangers. http://www.rosteplo.ru/Tech_stat/stat_shablon.php?id=405
5. I. A. Zhurmilova, "Improvement of heat and cold supply systems for buildings using ground heat exchangers," Extended Abstract of Cand. Sci. (Tech. Sci.) Dissertation (Far Eastern Fed. Univ., Vladivostok, 2016).
6. Free Heat. Ground Heat Exchanger. http://dachseason.ru/besplatnoe-teplo-gruntovyj-teploobmennik.html
7. Ground Heat Exchanger Ventilation in a Private House. http://domekonom.su/gruntovyi-teploobmennik.html
8. Installation Manual NIBE F1145. Geothermal Heat Pump. IHB RU 1343-2231699. NIBE AB Sweden. http://nibe.com.ru/sites/default/files /catalogs/Technical-NIBE-F1145.pdf.
9. Heat Pump Systems. Design Instructions, 5829 122-2-GUS (Viessmann Werke, 2000). http://forum.abok.ru/index.php?act=ttach&type=post&id=19331.
10. G. V. Kuznetsov, *Differential Methods for Heat Conductivity Problems Solving, the School-book* (Tomsk. Politekh. Univ., Tomsk, 2007) [in Russian].
11. E. P. Shubin, "A new method for calculating the heat losses of several pipes laid in the ground," Izv. VTI, No. 8, 42–50 (1934).
12. *Reference Book on the Climate of the USSR. Air and Soil Temperature* (Gidrometeoizdat, Leningrad, 1965) [in Russian].
13. A. F. Chudnovskii, *Thermophysical Characteristics of Disperse Materials* (FM, Moscow, 1962) [in Russian].
14. K. F. Pavlov, P. G. Romankov, and A. A. Noskov, *Examples and Problems on Chemical Technology Processes and Devices, The School-Book for Higher Schools*, Ed. by P. G. Romankov (Khimiia, Leningrad, 1987) [in Russian].
15. L. I. Turchak, *Principles of Numerical Methods* (Nauka, Moscow, 1987) [in Russian].
16. J. C. Tannehill, D. A. Anderson, and R. H. Pletcher, *Computational Fluid Mechanics and Heat Transfer,* 2nd ed. (Taylor and Francis, London, 1997).

Translated by A. Muravnik