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Absolute Permeability Upscaling for Superelement Modeling of Petroleum Reservoir

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Abstract—A technique for local upscaling of absolute permeability is proposed intended for the superelement modeling of petroleum reservoir development. The upscaling is performed for every block of an unstructured superelement grid based on solving a series of stationary one-phase flow in reservoir problems on a refined grid with the initial permeability field under various boundary conditions reflecting the characteristic structural variants of the filtrational flow and taking into account the presence or absence of wells inside the block. The resulting components of the effective permeability tensor in each superelement are sought from the solution of the problem on minimizing the deviations of the normal flows through the faces of the superelement averaged on a refined computational grid from those approximated on a coarse superelement grid. The application of the method is demonstrated by examples of the reservoir of the periodic and nonperiodic structure. The method is compared with the traditional techniques for local upscaling.

Keywords: upscaling, absolute permeability, porous medium, petroleum reservoir simulation, superelements

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1. INTRODUCTION

The superelement model of petroleum reservoir enables the fundamental acceleration of the computational process by using a coarse computational grid with the block size of the order of the distance between the wells [1–4]. The equations of this model have the same structure as the ordinary equations of two-phase flow but the functions of absolute permeability (AP) and of the relative phase permeability (RPP) differ from the original ones and result from transferring the properties of the reservoir from the detailed geological grid to the superelement grid. This process is called upscaling and is implemented in most computational and software packages such as Roxar Tempest, Schlumberger Eclipse, and TimeZYX. Earlier, we considered the RPP upscaling in a superelement model [5, 6]. The present study is devoted to the problem of absolute permeability upscaling.

There are many approaches to AP upscaling. Simplified analytical formulas of upscaling are only applicable to a small number of idealized types of heterogeneity. Porous reservoirs of a more complex structure call for different averaging methods. The mathematical averaging apparatus in the problems of flow in porous media theory is described in [7] and allows us to estimate effective flow parameters of the medium. The main results in this direction were obtained for periodically structured media [8–10]. A generalization of the averaging theory to random structures usually prevents the simplification of the initial problem with a small-scale inhomogeneity [11, 12].

The most popular techniques in the computational practice of modeling petroleum reservoir development are numerical methods for determining the AP tensor for each block of the coarse grid. The AP tensor is determined from the condition of the best approximation of the averaged fluid flow (flow-based upscaling) [13]. To this end, both global [14, 15] and more popular local upscaling methods [16, 13] are used. In contrast to the superelement modeling under consideration, in the finite superelement method [17] applied to the description of flows in the reservoir [18], the upscaling function is performed by the procedure of constructing the basis functions in each block of a coarse grid.

Obviously, upscaling should take into account not only the grid geometry but also the computational scheme for solving the problem on a coarse grid. In this paper, we propose a specialized method for the local AP upscaling of the superelement modeling of a petroleum reservoir development. The known techniques for local AP upscaling are compared based on models of a reservoir with a different heterogeneity structure.

2. DEFINING EQUATIONS

In the AP upscaling technique, we use the continuity equation with Darcy's law, neglecting the compressibility, gravitational, and capillary forces, i.e.,

$$\mathrm{div}\,\mathbf{u} = 0, \tag{1}$$

$$\mathbf{u} = -\sigma \nabla p. \tag{2}$$

Here, $p(\mathbf{x})$ is the pressure; $\mathbf{u}$ denotes the seepage velocity; $\sigma(\mathbf{x}) = k(\mathbf{x})/\mu$ is the specific hydraulic conductivity of the reservoir; $k(\mathbf{x})$ stands for the scalar field of absolute permeability; and μ is the dynamic viscosity of the fluid, which will be assumed constant within the superelement (SE). The coordinate function $k(\mathbf{x})$ is defined on a geological grid capable of accurately reproducing a complex heterogeneous reservoir structure.

Equations (1) and (2) describe the flow in the region D bounded above and below by an impermeable roof and bottom of the reservoir, and on the side, by an external cylindrical surface, on which hydrostatic pressure is maintained, and also by the internal boundaries bounded by the surfaces of the perforated sections of the wells of pipes of diameter r_{w}, on which the values of bottomhole pressure p^{w} or total production rate q of the well are set.

In accordance with the superelement approach, we select a separate element V of the grid with an outer boundary Γ consisting of faces Γ_j, and an inner boundary γ, which is a perforated part of the surface of the wells that appear inside V. We integrate Eq. (1) taking into account the divergence theorem and obtain the balance ratio

$$Q_\Gamma + Q_\gamma = 0, \tag{3}$$

$$Q_\Gamma = \int_\Gamma u_n \mathrm{d}\Gamma = \sum_j |\Gamma_j| \widetilde{u_{nj}}, \quad Q_\gamma = \int_\gamma u_n \mathrm{d}\gamma = q, \tag{4}$$

in which we introduce the mean normal flow rate on the faces Γ_j

$$\widetilde{u_{nj}} = \frac{1}{|\Gamma_j|} \int_{\Gamma_j} u_n \mathrm{d}\Gamma, \quad u_n = -\sigma \frac{\partial p}{\partial n}, \tag{5}$$

where $\mathbf{n}$ is the direction of the external normal to the SE boundary.

The method for expressing normal velocities (5) is defined in terms of the average pressures in the superelements

$$\langle p \rangle = \frac{1}{|V|} \int_V p dV, \tag{6}$$

where $|V|$ is the volume of the superelement V.

The set of values $\langle p \rangle$ in the superelements determines the mean pressure field in the reservoir. The application of a particular procedure for completing the grid function $\langle p \rangle$ makes it possible to determine the gradient of the mean pressure field and use, for the averaged field of the velocity, a relation analogous to Darcy's law (2), i.e.,

$$\mathbf{U} = -\mathbf{\Sigma} \cdot \nabla \langle p \rangle. \tag{7}$$

Here, $\mathbf{\Sigma} = \mathbf{K}/\mu$; $\mathbf{K}$ is the effective tensor of absolute permeability in the SE. Taking into account the last formula, we introduce the approximation of normal velocities (5)

$$\widetilde{u_n} \approx U_n = \mathbf{U} \cdot \mathbf{n} = -(\mathbf{\Sigma} \cdot \nabla \langle p \rangle) \cdot \mathbf{n} = -\frac{\mathbf{k}}{\mu} \cdot \nabla \langle p \rangle = -\frac{|\mathbf{k}|}{\mu} \frac{\partial \langle p \rangle}{\partial \mathrm{k}}, \tag{8}$$

where $\mathbf{k} = \mathbf{n} \cdot \mathbf{K}$ can be called the total permeability vector on the face of the SE and $\partial/\partial \mathrm{k}$ is the derivative along this vector, which is constructed by the collocation method [19, 2]. The face Γ of the superelement,

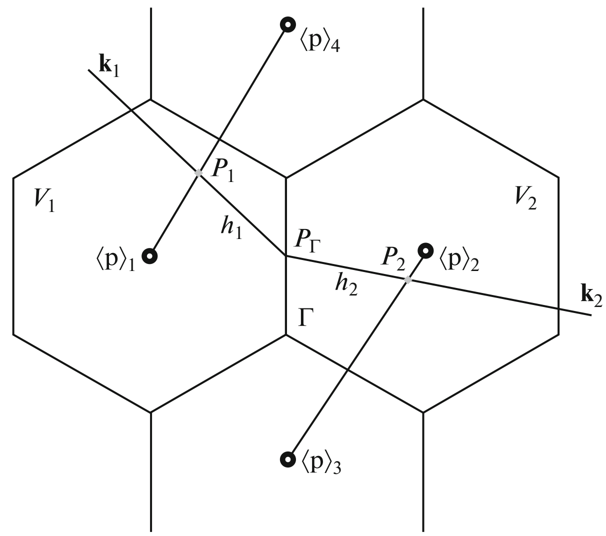

Fig. 1. Scheme of collocations for different permeability tensors in adjacent SEs.

on which the average normal flow rate (8) is calculated, divides two SEs, V_1 and V_2, with different tensors $\mathbf{K}_1$ and $\mathbf{K}_2$; therefore, in the regions of these SEs, the vectors of total permeability $\mathbf{k}_1 = \mathbf{n} \cdot \mathbf{K}_1$ and $\mathbf{k}_2 = \mathbf{n} \cdot \mathbf{K}_2$ are also different (see Fig. 1). The values P_1 and P_2 of the mean pressure on the lines of the directions k_1 and k_2 emanating from the center of the face Γ, are calculated by linear interpolation at the mean pressures in the nearest SE centers and refer to the points located at distances h_1 and h_2 from the face Γ. The finite-difference approximation of the derivative with respect to the direction in (8) yields

$$U_n^1 \approx -\frac{|\mathbf{k}_1|}{\mu}\frac{P_\Gamma - P_1}{h_1}, \quad U_n^2 \approx -\frac{|\mathbf{k}_2|}{\mu}\frac{P_2 - P_\Gamma}{h_2}. \tag{9}$$

Here, P_Γ is the unknown pressure on the face and the superscript denotes approximations of the normal flow from the side of V_1 or V_2.

From the condition stipulating the equality of the values $U_n^1 = U_n^2 = U_n$, and excluding the pressure P_Γ from (9), we finally obtain the expression of the normal velocity

$$U_n \approx -\frac{\bar{k}}{\mu}\frac{P_2 - P_1}{(h_1 + h_2)}, \quad \bar{k} = \frac{|\mathbf{k}_1||\mathbf{k}_2|(h_1 + h_2)}{|\mathbf{k}_2| h_1 + |\mathbf{k}_1| h_2} \tag{10}$$

through the values P_1 and P_2 of the mean pressure on the lines of the directions k_1 and k_2, which, in turn, are expressed in terms of the mean pressures $\langle p \rangle$ in the superelements surrounding the face Γ. Thus, the collocation algorithm is in effect a method for differentiating the mean field $\langle p \rangle$ over the node values, which do not require explicit fulfillment of this function. The values of (10), taking into account the first equality (8), are used in the balance ratio (3) and (4) for each SE. The system of such balance relationships is the system of linear algebraic equations for determining the mean pressures $\langle p \rangle$ in superelements.

3. UPSCALING METHOD

The problem of local AP upscaling consists of determining the matrices of the effective tensors $\mathbf{K}$ in each SE. The coefficients of these tensors are sought from the condition of the best approximation of the mean normal velocities (8), while quantity (5) is used as the exact solution for the velocities $\widetilde{u_{nj}}$ on the faces Γ_j, in which the pressure p is obtained as the solution of problem (1) and (2) with the initial coefficient of hydraulic conductivity $\sigma(\mathbf{x})$ on a sufficiently fine grid in the region V of this superelement.

In order to reproduce different variants of the flow through an SE, linear boundary conditions are set, i.e.,

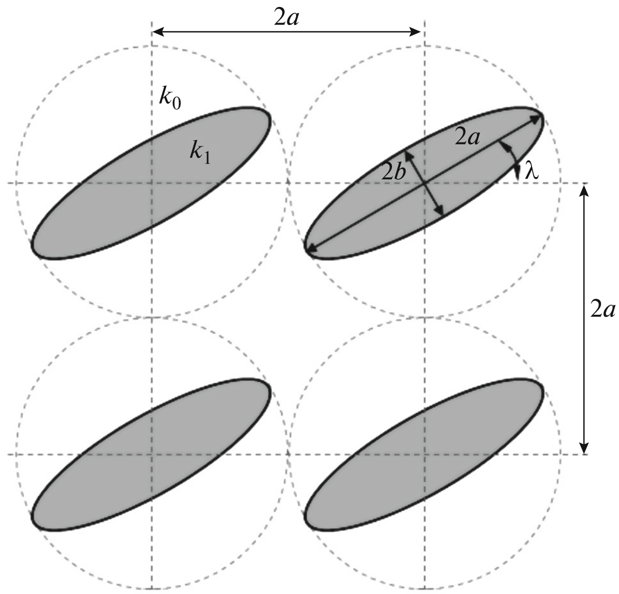

Fig. 2. Scheme of periodic structure of permeability field.

$$p^b\Big|_{\mathbf{x}\in\Gamma} = \sum_i \delta_i^b x^i, \quad b = 1,\ldots,B, \tag{11}$$

where x^i stands for Cartesian coordinates of the points of the SE external boundary, δ_i^b is the Kronecker symbol, and B is the number of variants of boundary conditions that coincides with the dimension of the problem. On upscaling the AP and SE containing the well γ, in addition to conditions (11), we set the characteristic value of the well production rate, i.e.,

$$\int_\gamma u_n d\gamma = q. \tag{12}$$

The components K^{ij} of the effective matrix tensor $\mathbf{K}$ for SE V are obtained from the condition for minimizing the residual functional

$$R^2\left(K^{ij}\right) = \frac{1}{B}\sum_{b=1}^{B} \rho^b\left(K^{ij}\right) \to \min_{K^{ij}}, \quad \rho^b\left(K^{ij}\right) = \frac{1}{M}\sum_{m=1}^{M}\left(\frac{|\Gamma_m|\left(\tilde{u}_{nm}^b - U_{nm}^b\left(K^{ij}\right)\right)}{\max\limits_{l=1..M}\left(|\Gamma_l|\left|\tilde{u}_{nm}^b\right|\right)}\right)^2, \tag{13}$$

where the values $U_{nm}^b\left(K^{ij}\right)$ on each face Γ_m for every variant b of the boundary conditions are calculated from the collocation scheme (9), in which the value P_Γ is calculated in the center of the face Γ_m from the boundary conditions (11), and the value P_1 is determined by interpolation from the values of the pressure on the faces and the mean $\langle p \rangle$ in an SE, which is calculated from (6) by the numerical integration of the pressure field obtained on a fine grid.

It should be noted that the formulated scheme for calculating the coefficients K^{ij} does not impose a symmetry condition on the matrix of the effective permeability tensor.

4. TESTING THE UPSCALING METHOD

The presented method of superelement upscaling (MSU) is tested on a two-dimensional problem in the horizontal plane (x, y). Such a model satisfactorily describes the petroleum reservoir, bounded above and below by weakly permeable bridges (overrides) with a permeability field that depends weakly on the vertical coordinate z.

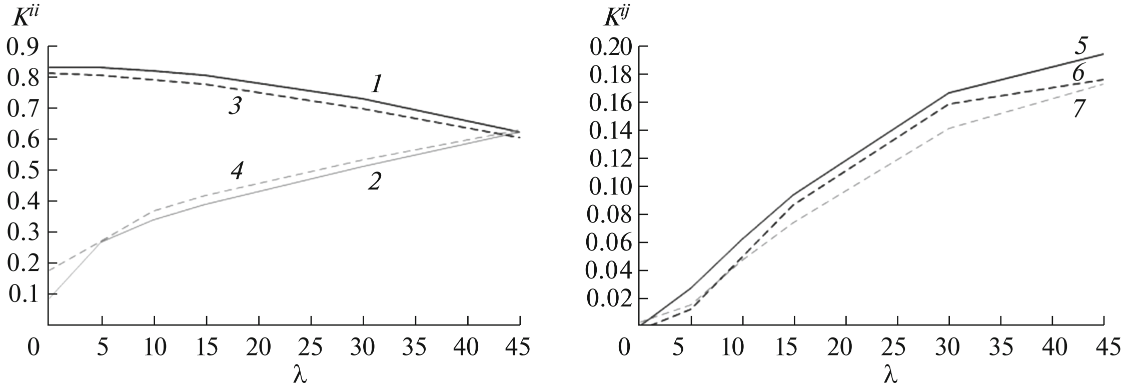

Fig. 3. Components of effective permeability tensor at $\varepsilon = 0.15$: (*1*) K_e^{xx}, (*2*) K_e^{yy}, (*3*) K^{xx}, (*4*) K^{yy}, (*5*) $K_e^{xy} = K_e^{yx}$, (*6*) K^{xy}, (*7*) K^{yx}.

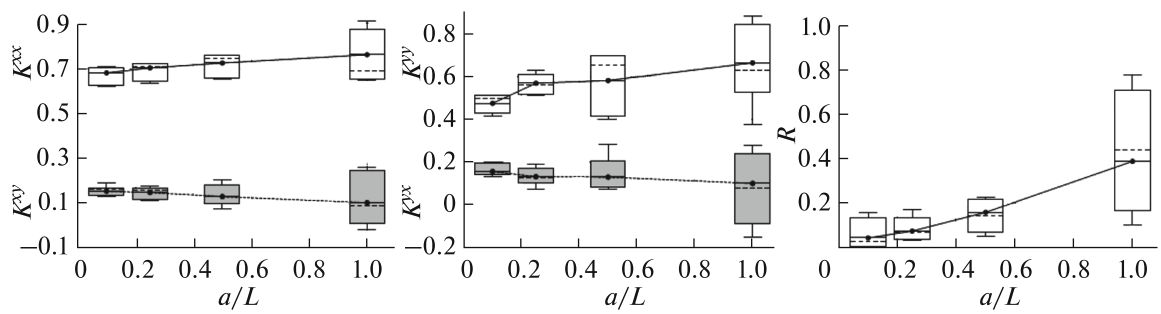

Fig. 4. Distribution of components K^{ij} and residual R over superelements with different sizes of ε: minimum and maximum values, percentiles of 5% and 95%, dotted line denotes percentile of 50%, line connects mean values.

4.1. Reservoir with a Periodic Structure

First, we compare the presented MSU with the classical averaging theory. To do this, we consider a reservoir with a periodic AP field. The periodicity cell contains an elliptic inclusion (Fig. 2), inside which $k = k_1 = 0.01$, and outside, $k = k_0 = 1$. The ratio of the minor and major semiaxes is $b/a = 0.2$. The orientation of the inclusions is set by the angle λ between the major axis of the ellipse and the axis *Ox*. The size of the periodicity cell is specified by the parameter $\varepsilon = a/L$, where L is the average radius of the superelements.

The area of such a reservoir was covered with a grid of hexagonal superelements. For each SE, a AP upscaling problem was solved. Figure 3 shows the dependence on the orientation angle λ observed for the effective permeability tensor, which is obtained from the solution of the upscaling problem for SE, and for the components K_e^{ij} obtained from the problem solution on a periodicity cell, which is a square with sides $2a$.

Simple calculations confirm that the components K^{ij} satisfy the most stringent constraints imposed in the theory of averaging on the effective permeability tensor in an inhomogeneous periodic structure consisting of two different isotropic components with different permeabilities obtained by Lurie and Cherkaev [20] and Murat and Tartar [21].

It should be noted that the values of the components K^{ij} for different SEs differ but their scatter, as well as the average value of the minimized functional of residual (13), decrease as the relative size ε of the periodicity cell shrinks (Fig. 4).

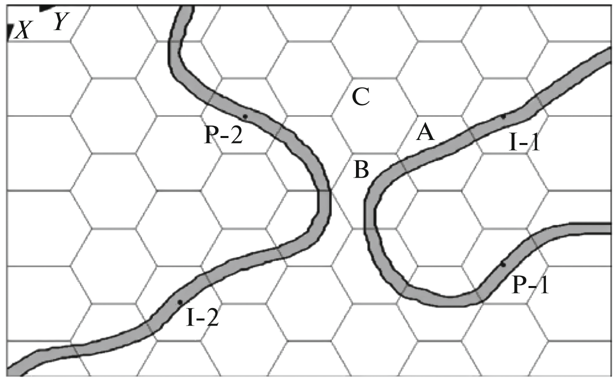

Fig. 5. Scheme of area of reservoir with nonperiodic structure of permeability field.

4.2. Reservoir of NonPeriodic Structure

The periodic structure of an inhomogeneity is convenient for applying the averaging theory but it remains far-removed from the real structure of the reservoir. With large-block flow modeling in a reservoir, macroinhomogeneities such as paleochannels or tectonic faults that form the extensive prevailing paths for filtration flows are of the greatest interest.

Consider the structure of the field $k(\mathbf{x})$ formed by the presence of a high permeability inclusion (channel, fault) with permeability $k_1 = 1$ in a reservoir with absolute permeability $k_0 = 0.001$. The computational domain is specified as a rectangular region of a reservoir, which will be covered by a superelement grid consisting of 50 blocks (Fig. 5). The ratio of the channel width to the average radius of the superelements is set to 0.25. We assume that there are two production wells (P-1, P-2) and two injection wells (I-1, I-2) in the area with rates equal in absolute values in such a way that wells I-1, P-1 and I-2, P-2 are connected in pairs by high-permeability channels. At the outer boundaries of the section, we set the constant hydrostatic pressure.

In order to estimate the presented upscaling method, the matrices of the effective AP tensors in each SE will be sought by four techniques, i.e., the method of volume averaging (MVA), the rate averaging method (RAM), the minimum dissipation method (MDM), and the MSU.

The method of volume averaging where the medium in each block of the coarse grid is considered to be isotropic with the permeability weighted by the volume of this block is the simplest way of rescaling AP. This method is the least reliable and can yield valid results only when the inhomogeneity scale of the initial permeability field is much higher than the average step of the computational grid.

More adequate results can be obtained by the numerical methods of local upscaling, among which the most common are the RAM and the MDM [13]. These methods are based on the solution of the auxiliary stationary problem of single-phase flow (1) and (2) on a refined computational grid in the rectangular area $\Omega = L_x \times L_y \times L_z$ bounding the block of a coarse computational grid, with periodic boundary conditions for the RAM and linear boundary conditions for the MDM.

It should be noted that the equation, the solution of which reduces to the MDM is analogous to the variational formulation of the problem on a cell (periodicity), which is written in the theory of averaging of a periodically oscillating permeability field from the condition that the solutions of the initial and averaged equation coincide with the period of oscillations tending to zero [7].

Table 1. Examples of matrices of effective tensors

Method	Superelement A				Superelement B			
	K^{xx}	K^{xy}	K^{yx}	K^{yy}	K^{xx}	K^{xy}	K^{yx}	K^{yy}
MVA	0.152	0	0	0.152	0.179	0	0	0.179
RAM	0.002	0.0015	0.0015	0.004	0.003	0.001	0.001	0.002
MDM	0.057	0.070	0.070	0.155	0.103	0.062	0.062	0.060
MSU	0.041	0.082	0.069	0.144	0.106	0.056	0.062	0.033

Table 2. Residuals of mean normal flows through faces and mean pressures

Residual	Method			
	MVA	RAM	MDM	MSU
r_u	0.130	0.222	0.079	0.063
r_p	0.158	5.523	0.186	0.139
R_p	0.503	18.76	0.653	0.434

Table 3. Limit tracer concentrations in the output of producing wells

Variant of calculation	Tracer injection into well I-1		Tracer injection into well I-2	
	C in well P-1	C in well P-2	C in well P-1	C in well P-2
Exact solution	0.375	0.000	0.000	0.498
MSU	0.291	0.025	0.039	0.440
MDM	0.233	0.144	0.084	0.372
MVA	0.164	0.275	0.148	0.294

The criterion for assessing the quality of upscaling procedures will be the consistency of the results of the calculations performed on a coarse grid using the obtained permeability tensor with the calculation results on a fine grid using the original scalar AP field.

The average step of the fine computational grid, which was used to cover the entire computation domain and the area of each SE in solving problems of AP upscaling, was approximately 1% of the average step of the coarse grid.

Figure 5 shows three characteristic types of SEs, differing by the location of a relatively high-permeability channel: A is a channel that passes through the SE predominantly in one direction, B is a channel that changes its direction within the SE, C is a channel that does not pass through the SE. Upscaling AP in the SE C by all the above-listed methods yields similar results; i.e., the effective tensor $\mathbf{K}$ is characterized by isotropy and its diagonal components coincide with the permeability k_0. The components of the AP tensor obtained by different methods for SE A and B are shown in Table 1.

In the presented coefficients, especially prominent are the results of the RAM, the coefficients of the effective tensor, which are underestimated by an average of 1–2 orders of magnitude and are a consequence of using the periodic boundary conditions in the model problems that are unsuitable for a nonperiodic structure of the field $k(\mathbf{x})$. Simulation of the reservoir development in the mode of prespecified production rates using such coefficients leads to a multiple overestimation of the reservoir's pressure gradient.

Recall that the matrix in the MSU for an SE containing wells was calculated according to the second scenario with boundary conditions (12). An example can be the comparison of the coefficients for superelement A presented in Table 1 with the coefficients obtained for the SE containing well I-1: $K^{xx} = 0.235$, $K^{xy} = 0.404$, $K^{yx} = 0.404$, and $K^{yy} = 0.702$.

The quality of the upscaling procedures is estimated by the magnitude of the residuals

$$r_u = \sqrt{\frac{1}{M}\sum_{i=1}^{M}\left(\frac{\widetilde{u_{ni}} - U_{ni}}{u_*}\right)^2}, \quad u_* = \max_{k=1..M}\left|\widetilde{u_{nk}}\right|,$$

$$r_p = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\langle p\rangle_i - P_i}{\Delta P}\right)^2}, \quad \Delta P = \max_{i=1..N}\langle p\rangle_i - \min_{i=1..N}\langle p\rangle_i,$$

$$R_p = \max_{i=1,\dots,N}\left|\frac{\langle p\rangle_i - P_i}{\Delta P}\right|,$$

where M and N are the total number of faces and blocks of a superelement grid; the $\widetilde{u_{ni}}$ values are calculated by formula (5) for each SE face by numerically integrating the solution obtained on a fine grid and

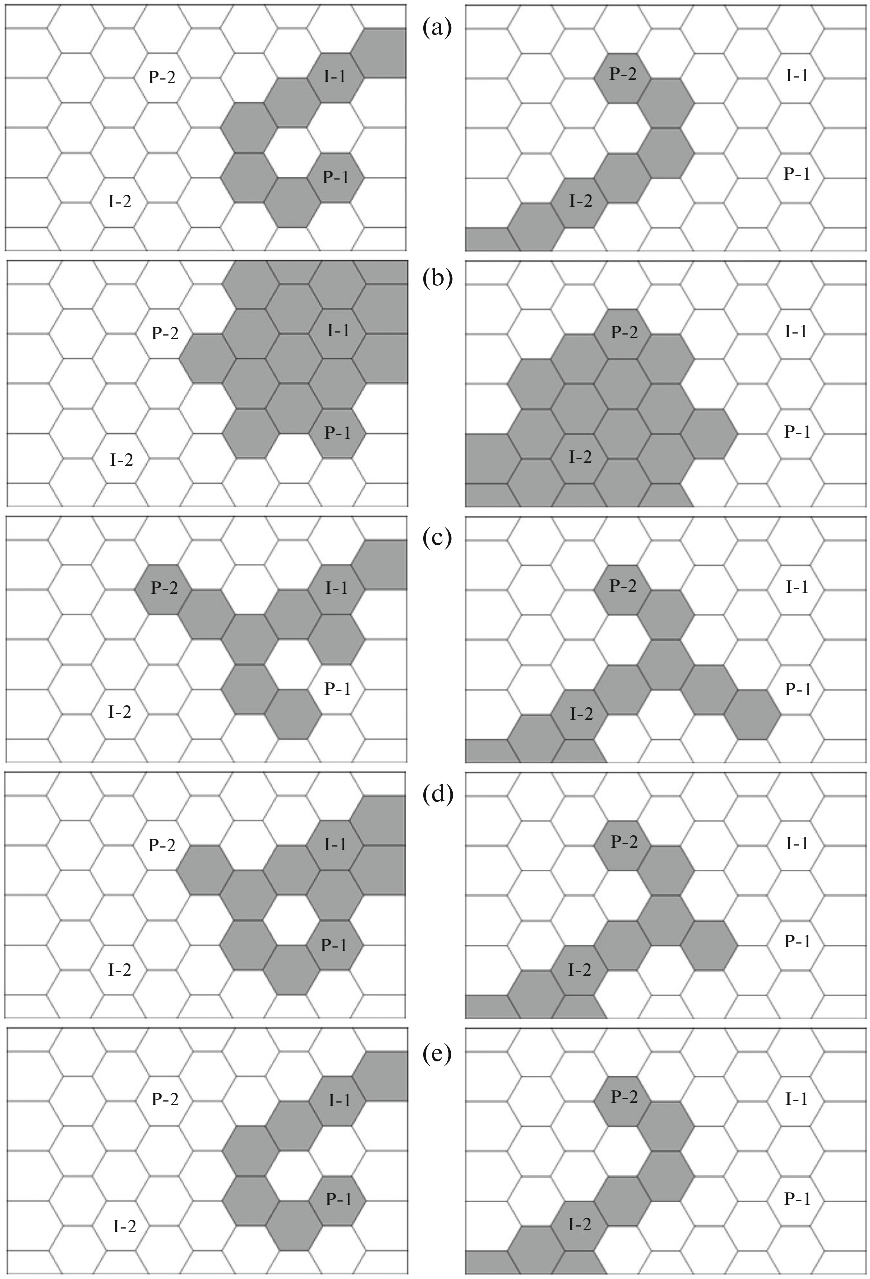

Fig. 6. Distribution of tracer from injection well I-1 (left) and well I-2 (right): (a) exact solution, (b) RAM, (c) MVA, (d) MDM, (e) MSU.

the values $\langle p \rangle_i$ are computed by formula (6) for each SE; U_{ni} and P_j are the values of the normal flux through the faces of the superelements and the mean pressure in the superelements obtained from the solution on a coarse grid.

The magnitudes of these residuals for the considered methods are shown in Table 2. The best results are demonstrated by the solutions constructed using the MDM and the MSU; moreover, the latter method results in smaller residuals.

Minimizing the listed mean residuals for the area is not the only criterion of the quality of the AP upscaling procedures. It is equally important to adequately reproduce the structure of the flow which determines the interaction of the wells on the coarse grid. In order to analyze this characteristic, we solve the problem of simulating a tracer injection alternately into each injection well, assuming that the presence of a dissolved tracer does not change the properties of the liquid, so that the velocity field is still deter-

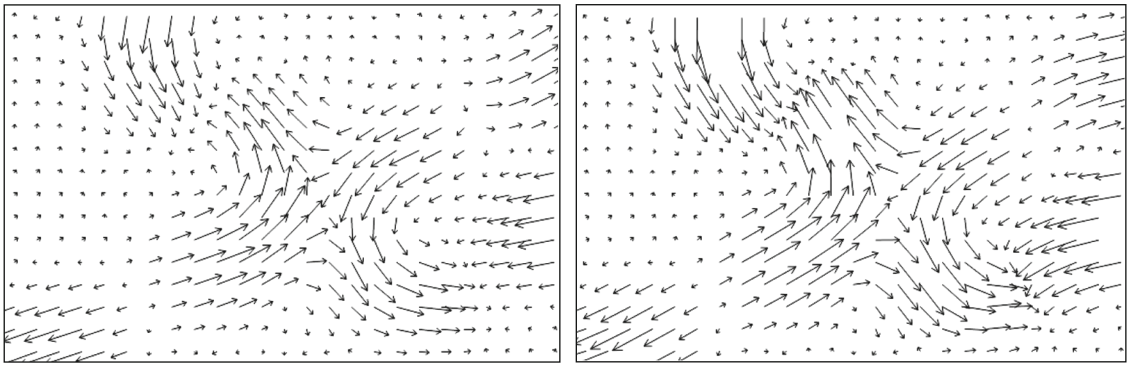

Fig. 7. Vector field of filtration rates, average with respect to SE, constructed on refined (left) and superelement (right) grid.

mined from the solution of the stationary problem (1) and (2). The obtained tracer distribution in the reservoir will demonstrate the trajectory of the injected liquid into the injection wells, and the limiting tracer concentrations in the production wells will correspond to the degree of their interaction with the respective injection wells. The exact solution of the problem will be understood as a solution constructed on a fine grid. We construct the solution on the superelement grid using various upscaling methods

Figure 6 shows the tracer distribution over the reservoir at a fixed time obtained by various methods. SEs, in which the concentration of the tracer exceeds 20%, are designated by color. According to the results of the exact solution, such SEs were determined by the numerical integration of the concentration field obtained on a fine grid over the areas of an SE of a coarse grid.

It can be seen that upscaling the AP by the RAM with periodic conditions does not adequately describe the flow structure on the nonperiodic permeability field. Among the remaining methods, the MSU yielded the closest distribution to the true tracer distribution in the reservoir. Upscaling by the MVA and the MDM does not provide sufficient isolation of the faces separating the SE from the sections of two independent highly permeable channels, as a result of which the tracer from the injection well appears to enter the production well, with which in fact there is no interaction. The limiting concentrations C of the tracer in the output producing wells are given in Table 3.

In conclusion, we demonstrate the possibility of constructing a vector field of the filtration rate in the area of the reservoir based on the results of superelement modeling. This requires the following actions:

(i) to construct the triangulation of the section using the SE centers and their vertices on the boundary of the section as reference points; at all these points, the pressure is known—the mean pressure at the SE centers and the specified hydrostatic pressure at the boundary;

(ii) to calculate the gradient of the mean pressure at all the vertices of the triangles;

(iii) to calculate the vector of the velocity $\mathbf{U} = -\mathbf{K} \cdot \nabla \langle p \rangle / \mu$ knowing the matrix of the effective AP tensor from the pressure gradient found at each inner vertex of the triangle (at the SE center).

The obtained discrete set of vectors at the internal vertices of the triangulation can be filled up to a continuous vector field using the interpolation algorithm. With the help of this procedure, the velocity fields are constructed based on the results of the superelement solution of problem (1) and (2) using the MSU, as well as its solution on a fine grid with the initial scalar field of permeability. For an adequate comparison of the results, the vector field from the fine grid was rescaled to a coarse grid by calculating the average integral values for each component of the velocity vector in the area of each SE. As can be seen from Fig. 7, despite the rather coarse size of the superelement grid blocks, the results of the SEM, that is, the mean block pressures and effective AP tensors, are sufficient for the correct structural reproduction of the vector field of the flow velocity in the reservoir.

CONCLUSIONS

A special method for the local upscaling of the absolute permeability that makes it possible to minimize the error in the calculation of the flow rates during superelement modeling of petroleum reservoir and to accurately reflect the structure of the flow is formulated.

The comparative analysis has shown a significant advantage of the presented method of upscaling over the most popular methods of averaging the flow rate and minimum energy dissipation used for upscaling the permeability field from a fine geological grid to the coarse hydrodynamic grid.

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REFERENCES

1. A. B. Mazo and D. V. Bulygin, “Super elements. New approach towards oid field development modeling,” Neft’. Gaz. Novatsii, No. **11**, 6–8 (2011).
2. A. B. Mazo, K. A. Potashev, E. I. Kalinin, and D. V. Bulygin, “Oil reservoir simulation with the superelement method,” Mat. Model. **25**, 51–64 (2013).
3. D. V. Bulygin, A. B. Mazo, K. A. Potashev, and E. I. Kalinin, “Geological and technical aspects of super element method of petroleum reservoir simulation,” Georesursy **53** (3), 31–35 (2013).
4. A. Mazo, K. Potashev, and E. Kalinin, “Petroleum reservoir simulation using super element method,” Proc. Earth Planet. Sci. **15**, 482–487 (2015).
5. A. B. Mazo and K. A. Potashev, “Relative phase permeability upscaling for super element model of petroleum reservoirs,” Math. Models Comput. Simul. **9** (5) (2017, in press).
6. K. A. Potashev, “Upscaling of relative phase permeabilities in an isolated stratified reservoir,” Uch. Zap. Kazan. Univ., Fiz.-Mat. Nauki **156**, 120–134 (2014).
7. A. Yu. Belyaev, *Homogenization in Problems of Filtering Theory* (Nauka, Moscow, 2004) [in Russian].
8. N. S. Bakhvalov, “Averaged characteristics of bodies with periodic structure,” Sov. Phys. Dokl. **19**, 650–651 (1974).
9. A. Bensoussan, J. L. Lions, and G. Papanicolaou, *Asymptotic Analysis for Periodic Structures,* Vol. 5 of *Studies in Mathematics and Its Applications* (North-Holland, Amsterdam, 1978).
10. A. L. Berdichevskii, “On effective heat conductivity of media with periodically disposed inclusions,” Sov. Phys. Dokl. **24**, 652 (1979).
11. S. M. Kozlov, “Averaging differential operators with almost periodic, rapidly oscillating coefficients,” Mat. Sb. **107**, 199–217 (1978).
12. M. B. Panfilov and I. V. Panfilova, *Average Models of the Processes of Flow through a Porous Medium with Inhomogeneous Internal Structure* (Nauka, Moscow, 1996) [in Russian].
13. X. H. Wu, Y. Efendiev, and T. Y. Hou, “Analysis of upscaling absolute permeability,” Discrete Contin. Dyn. Syst., Ser. B **2**, 185–204 (2002).
14. C. D. White and R. N. Horne, “Computing absolute transmissivity in the presence of fine scale heterogeneity,” Soc. Petrol. Eng. J. **16011**, 209–221 (1987).
15. L. Holden and B. F. Nielsen, “Global upscaling of permeability in heterogeneous reservoirs; the output least squares (OLS) method,” Transp. Porous Media **40**, 115–143 (2000).
16. L. J. Durlofsky, “Numerical calculation of equivalent grid block permeability tensors for heterogeneous porous media,” Water Resour. Res. **27**, 699–708 (1991).
17. L. G. Strakhovskaya and R. P. Fedorenko, “A version of the finite element method,” USSR Comput. Math. Math. Phys. **19**, 162–173 (1979).
18. A. Kh. Pergament, V. A. Semiletov, and P. Yu. Tomin, “On some multiscale algorithms for sector modeling in multiphase flow in porous media,” Math. Models Comput. Simul. **3**, 365–374 (2011).
19. N. G. Nikitin, “Nonlinear finite volume method for two-phase flow in porous media,” Mat. Model. **22** (11), 131–147 (2010).
20. K. A. Lur’e and A. V. Cherkaev, “G-closure of a set of anisotropic conducting media in two dimensions,” Sov. Phys. Dokl. **26**, 657 (1981).
21. F. Murat and L. Tartar, “Les methodes de l’homogeneisation: theorie et applications en physique,” in *Collection de la Direction des Etudes et Recherches d’Electricité de France* (Paris, 1985), pp. 319–370.

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