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New Concept of the Discrete Sources Method in Electromagnetic Scattering Problems

N. V. Grishina[a], **Yu. A. Eremin**[b], **and A. G. Sveshnikov**[a]

[a]*Faculty of Physics, Moscow State University, Moscow, 119991 Russia*
[b]*Faculty of Computational Mathematics and Cybernetics, Moscow State University, Moscow, 119991 Russia*
e-mail: ngrishina@inbox.ru, eremin@cs.msu.ru, sveshnikov@phys.msu.ru

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Abstract—We propose and implement a new concept of the discrete sources method; by applying this concept, one can investigate dielectric scatterers with large wave numbers. It is shown that the total scattering cross section can be determined analytically by using the amplitudes of discrete sources. The numerical results are presented; they demonstrate a considerable gain obtained by using the new concept in comparison with the conventional concept.

Keywords: electromagnetic scattering, dielectric particles of large sizes, discrete sources method, numerical algorithm

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INTRODUCTION

Problems of electromagnetic scattering by local objects are found in many practical applications. One such application is the sounding of atmospheric particles both by electromagnetic waves and by laser emission. There are very many methods for the simulation of the scattering properties of penetrable particles [1, 2], including rigorous and approximate methods [3, 4]. Since particle sizes can significantly exceed the emitting wave length, the use of approximate methods (such as the method of physical or geometrical optics) is very attractive [4, 5]. At the same time, the application domain of the considered approximations is unknown. It is necessary to have standard results obtained by rigorous methods in a domain where the accuracy of rigorous methods can be ensured.

The discrete sources method (DSM) [6] is a universal tool for the analysis of the scattering properties of particles. However, until recently the use of the DSM was limited by the resonance range of the wave lengths (when the wave length is comparable with the size of the scatterer). Here, for particles with large wave sizes, only strongly extended scatterers of the type of cylindrical bodies or extended spheroids could be considered [7]. In the present paper, a new concept of the DSM is considered; by applying this concept, one can analyze nonspherical atmospheric particles that have the wave parameter $ka \geq 50$ and possess a high refraction index with a complex component. We assume that it is difficult to obtain such results by applying other numerical analytical methods (NAMs) even with the use of advanced computational systems [5, 8].

The significant advantages of NAMs (such as the T-matrix method) include the possibility of determining the integral scattering characteristics (such as the total scattering cross section) in the analytical form [1]. This is the consequence of the orthogonality of spherical functions on a unit sphere. The DSM uses the distributed lower-order multipoles that are not orthogonal [7]. It is shown in the present paper that the total scattering cross section can be determined in the analytical form by using only the amplitudes of discrete sources.

FORMULATION OF THE SCATTERING PROBLEM

Consider the scattering of an electromagnetic plane-wave $\{\mathbf{E}_0, \mathbf{H}_0\}$ by an axially symmetric penetrable particle D_i located in $D_e = R^3 / \overline{D_i}$, with the axis of symmetry Oz. Assume that the plane wave is propagated at an angle of $\pi - \theta_0$ about the Oz axis. Then the mathematical formulation of the problem can be presented as follows:

$$\begin{gathered}\operatorname{curl}\mathbf{H}_{e,i} = jk\varepsilon_{e,i}\mathbf{E}_{e,i};\quad \operatorname{curl}\mathbf{E}_{e,i} = -jk\mu_{e,i}\mathbf{H}_{e,i}\ \text{in}\ D_{e,i};\\ \mathbf{n}_p \times (\mathbf{E}_i(P) - \mathbf{E}_e(P)) = \mathbf{n}_p \times \mathbf{E}_0(P),\\ \mathbf{n}_p \times (\mathbf{H}_i(P) - \mathbf{H}_e(P)) = \mathbf{n}_p \times \mathbf{H}_0(P),\quad P \in \partial D_i;\\ \lim_{r\to\infty} r\left(\sqrt{\varepsilon_e}\mathbf{E}_e \times \frac{\mathbf{r}}{r} - \sqrt{\mu_e}\mathbf{H}_e\right) = 0,\quad r = |M| \to \infty.\end{gathered} \tag{1}$$

Here, $\{\mathbf{E}_e, \mathbf{H}_e\}$ is the scattered field, $\{\mathbf{E}_i, \mathbf{H}_i\}$ is a complete field inside the particle, and $\mathbf{n}_p$ is a unit normal to the surface $\partial D_i \in C^{(2,\alpha)}$; in this case, $\operatorname{Im}\varepsilon_e, \mu_e = 0$ and $\operatorname{Im}\varepsilon_i, \mu_i \le 0$. We suppose that the temporal relation was taken in the form of $\exp\{j\omega t\}$. Then the diffraction problem (1) is uniquely solvable.

In order to solve the formulated diffraction problem (1), we use the DSM [6]. Let us find the approximate solution; here, we take into account the axial symmetry and the polarization of the external excitation. In the case of the linear polarization, the fields of the plane wave for the P/S-polarization are presented as follows:

$$\begin{gathered}\mathbf{E}_0^P = (\mathbf{e}_x\cos\theta_0 + \mathbf{e}_z\sin\theta_0)\psi,\quad \mathbf{H}_0^P = -\mathbf{e}_y\cos\theta_0 n_0\psi,\\ \mathbf{H}_0^S = \{\mathbf{e}_x\cos\theta_0 + \mathbf{e}_z\sin\theta_0\}\psi,\quad \mathbf{E}_0^S = \mathbf{e}_y\psi,\\ \psi = \exp\{-jk_e(x\sin\theta_0 - z\cos\theta_0)\},\end{gathered} \tag{2}$$

where $k_e = k\sqrt{\varepsilon_e\mu_e}$ and $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ is the Cartesian basis. We expand the plane wave into a Fourier series in terms of the azimuth angle φ and obtain

$$\exp\{\pm jk_e\sin\theta_0\cos\varphi\} = \sum_{m=0}^{\infty}(2 - \delta_{0m})(\pm j)^m J_m(k_e\sin\theta_0)\cos m\varphi,$$

where J_m is a cylinder Bessel function and δ_{0m} is a Kronecker symbol. Then the Fourier harmonics of the tangential components of field P of the polarized plane wave (2) in the cylindrical system of coordinates are presented as follows:

$$\begin{gathered}\mathbf{E}_0^P \cdot \boldsymbol{\tau} \Rightarrow e_{m+1,\tau}^P(\eta)\cos(m+1)\varphi,\quad \mathbf{E}_0^P \cdot \mathbf{e}_\varphi \Rightarrow e_{m+1,\varphi}^P(\eta)\sin(m+1)\varphi,\\ \mathbf{H}_0^P \cdot \boldsymbol{\tau} \Rightarrow h_{m+1,\tau}^P(\eta)\sin(m+1)\varphi,\quad \mathbf{H}_0^P \cdot \mathbf{e}_\varphi \Rightarrow h_{m+1,\varphi}^P(\eta)\cos(m+1)\varphi,\end{gathered} \tag{3}$$

where $\boldsymbol{\tau}$ is a vector that is tangent to the generating line of the surface of revolution. In order to take into account the axial symmetry and polarization (3), we use the following vector potentials for finding an approximate solution:

$$\begin{gathered}\mathbf{A}_{mn}^{1,e,i} = \{Y_m^{e,i}(\eta, w_n^{e,i})\cos(m+1)\varphi;\ -Y_m^{e,i}(\eta, w_n^{e,i})\sin(m+1)\varphi; 0\},\\ \mathbf{A}_{mn}^{2,e,i} = \{Y_m^{e,i}(\eta, w_n^{e,i})\sin(m+1)\varphi;\ Y_m^{e,i}(\eta, w_n^{e,i})\cos(m+1)\varphi; 0\},\\ \mathbf{A}_n^{3,e,i} = \{0; 0; Y_0^{e,i}(\eta, w_n^{e,i})\}.\end{gathered} \tag{4}$$

Here, $Y_m^e(\eta, w_n^e) = h_m^{(2)}(k_e R_{\eta w_n^e})(\rho/R_{\eta w_n^e})^m$, $Y_m^i(\eta, w_n^i) = h_m^{(2)}(k_i R_{\eta w_n^i})\,(\rho/R_{\eta w_n^i})^m$, $R_{\eta w_n^{e,i}}^2 = \rho^2 + (z - w_n^{e,i})^2$, $\eta = (\rho, z)$, and $h_m^{(2)}$ are spherical Hankel functions; $w_n^{e,i}$ are discrete sources' complex coordinates located in such a way that the sources' images for the external and internal fields belong to the interior domain D_i and the exterior domain D_e, respectively: $\operatorname{Re}(w_n^e) \in D_i$ and $\operatorname{Re}(w_n^i) \in D_e$ [9].

Then the approximate solution for the scattered field in D_e and for the complete field in D_i is

$$\begin{gathered}\mathbf{E}_{e,i}^N = \sum_{m=0}^{M}\sum_{n=1}^{N_{e,i}^m}\left\{p_{mn}^{e,i}\frac{j}{k\varepsilon_{e,i}\mu_{e,i}}\operatorname{curlcurl}\mathbf{A}_{mn}^{1,e,i} + q_{mn}^{e,i}\frac{1}{\varepsilon_{e,i}}\operatorname{curl}\mathbf{A}_{mn}^{2,e,i}\right\} + \sum_{n=1}^{N_{e,i}^0} r_n^{e,i}\frac{j}{k\varepsilon_{e,i}\mu_{e,i}}\operatorname{curlcurl}\mathbf{A}_n^{3,e,i};\\ \mathbf{H}_{e,i}^N = \frac{j}{k\mu_e}\operatorname{curl}\mathbf{E}_{e,i}^N.\end{gathered} \tag{5}$$

In the case of S-polarization, the Fourier harmonics of the plane wave are presented from (2) as follows:

$$\mathbf{E}_0^S \cdot \boldsymbol{\tau} \Rightarrow e_{m+1,\tau}^S(\eta)\sin(m+1)\varphi, \quad \mathbf{E}_0^S \cdot \mathbf{e}_\varphi \Rightarrow e_{m+1,\varphi}^S(\eta)\cos(m+1)\varphi,$$
$$\mathbf{H}_0^S \cdot \boldsymbol{\tau} \Rightarrow h_{m+1,\tau}^S(\eta)\cos(m+1)\varphi, \quad \mathbf{H}_0^S \cdot \mathbf{e}_\varphi \Rightarrow h_{m+1,\varphi}^S(\eta)\sin(m+1)\varphi. \tag{6}$$

Finding the approximate solution depends on the following potentials:

$$\mathbf{A}_{mn}^{1,e,i} = \{Y_m^{e,i}(\eta, w_n^{e,i})\sin(m+1)\varphi;\ \ Y_m^{e,i}(\eta, w_n^{e,i})\cos(m+1)\varphi; 0\},$$
$$\mathbf{A}_{mn}^{2,e,i} = \{Y_m^{e,i}(\eta, w_n^{e,i})\cos(m+1)\varphi;\ \ -Y_m^{e,i}(\eta, w_n^{e,i})\sin(m+1)\varphi; 0\}, \tag{7}$$
$$\mathbf{A}_n^{3,e,i} = \{0;\ 0; Y_0^{e,i}(\eta, w_n^{e,i})\}.$$

Correspondingly, the representation for the approximate solution takes the form

$$\mathbf{E}_{e,i}^N = \sum_{m=0}^{M}\sum_{n=1}^{N_{e,i}^m}\left\{p_{mn}^{e,i}\frac{j}{k\varepsilon_{e,i}\mu_{e,i}}\operatorname{curlcurl}\mathbf{A}_{mn}^{1,e,i} + q_{mn}^{e,i}\frac{1}{\varepsilon_{e,i}}\operatorname{curl}\mathbf{A}_{mn}^{2,e,i}\right\} + \sum_{n=1}^{N_{e,i}^0} r_n^{e,i}\frac{1}{\varepsilon_{e,i}}\operatorname{curl}\mathbf{A}_n^{3,e,i};$$
$$\mathbf{H}_{e,i}^N = \frac{j}{k\mu_e}\operatorname{curl}\mathbf{E}_{e,i}^N. \tag{8}$$

Since the particle is axially symmetric and the external excitation (4) and (6), as well as the approximate solution (5) and (8), are represented by the expansion in terms of the azimuth variable φ, the unknown amplitudes of the discrete sources $\{p_{mn}^{e,i}, q_{mn}^{e,i}, r_n^{e,i}\}$ are determined from boundary condition (1) sequentially for each Fourier harmonic m. The amplitudes of the discrete sources are determined from the requirement of minimizing the Fourier harmonics of the residual of boundary condition (1) in the network norm $L_2(\partial D_i)$. The computational algorithm represents the generalized collocation method with the subsequent pseudo-solution of the corresponding overdetermined systems of linear algebraic equations. Due to this, one can find the approximate solution immediately for all angles of incidence of the wave θ_0 and for both P/S-polarizations. In addition, by finding the value of the surface residual of the fulfillment of boundary conditions (1) at intermediate points with a relation to the collocation points, we obtain a posteriori estimate for the error of solution (5) and (8).

Once the amplitudes of the discrete sources are found, one can determine the field pattern of scattered $\mathbf{F}(\theta,\varphi)$:

$$\mathbf{E}(\mathbf{r})\Big/\left|\mathbf{E}^0(\mathbf{r})\right| = \frac{\exp\{-jk_0 r\}}{r}\mathbf{F}(\theta,\varphi) + O(1/r^2), \quad r \to \infty. \tag{9}$$

The components θ and φ of the scattered-field pattern for the P-polarization take the form

$$F_\theta^P(\theta,\varphi) = j\sum_{m=0}^{M}(j\sin\theta)^m\cos(m+1)\varphi\sum_{n=1}^{N_e^m}\left\{p_{mn}^e\cos\theta + q_{nm}^e\right\}\exp\left\{-jk_e w_n^e\cos\theta\right\}$$
$$- j\sin\theta\sum_{n=1}^{N_e^0} r_n^e\exp\left\{-jk_e w_n^e\cos\theta\right\}, \tag{10}$$
$$F_\varphi^P(\theta,\varphi) = -j\sum_{m=0}^{M}(j\sin\theta)^m\sin(m+1)\varphi\sum_{n=1}^{N_e^m}\left\{p_{mn}^e + q_{nm}^e\cos\theta\right\}\exp\left\{-jk_e w_n^e\cos\theta\right\},$$

while for S-polarization, we have

$$F_\theta^S(\theta,\varphi) = j\sum_{m=0}^{M}(j\sin\theta)^m\sin(m+1)\varphi\sum_{n=1}^{N_e^m}\left\{p_{mn}^e - q_{nm}^e\cos\theta\right\}\exp\left\{-jk_e w_n^e\cos\theta\right\},$$
$$F_\varphi^S(\theta,\varphi) = j\sum_{m=0}^{M}(j\sin\theta)^m\cos(m+1)\varphi\sum_{n=1}^{N_e^m}\left\{p_{mn}^e\cos\theta - q_{nm}^e\right\}\exp\left\{-jk_e w_n^e\cos\theta\right\} \tag{11}$$
$$+ j\sin\theta\sum_{n=1}^{N_e^0} r_n^e\exp\left\{-jk_e w_n^e\cos\theta\right\},$$

Hence, once the amplitudes of the discrete sources are determined, one can find the components of the scattering pattern (10) and (11). However, in the case of large wave parameters ka, the DSM's classical

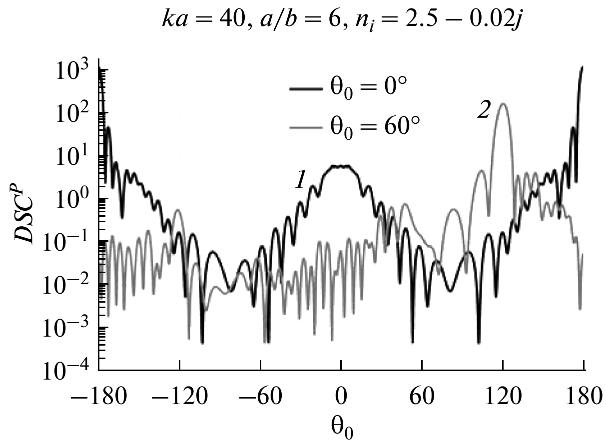

Fig. 1. The results of DSC^P in the incidence plane of the wave in relation to the observation angle θ for $a/b = 6$, $ka = 40$, and $n_i = 2.5 - 0.02j$. For curve *1*, $\theta_0 = 0°$ and for curve *2*, $\theta_0 = 60°$.

scheme (where one determines the a posteriori estimate for the error of the results from satisfying the boundary conditions) is very laborious. Its practical implementation requires great computational resources and often takes time, which is longer by a factor of 2 or 3 than the time taken to solve the scattering problem itself.

NEW CONCEPT OF THE DISCRETE SOURCES METHOD

We consider the intensity of the scattered DSC field; this intensity on a unit sphere is determined as follows:

$$DSC^{P,S}(\theta_0,\theta,\varphi) = \left|F_\theta^{P,S}(\theta_0,\theta,\varphi)\right|^2 + \left|F_\varphi^{P,S}(\theta_0,\theta,\varphi)\right|^2. \quad (12)$$

We also consider the total scattering cross section (the summary intensity of a scattered field)

$$\sigma^{P,S}(\theta_0) = \int_\Omega DSC^{P,S}(\theta_0,\theta,\varphi)d\omega. \quad (13)$$

Here, Ω is a unit sphere: $\{180° \geq \theta \geq 0°, 360° \geq \varphi \geq 0°\}$. In view of (9), a unit of measure for the intensity and the total scattering cross section (SC) is nm^2.

In fact, if it is assumed that representation (5) and (8) is based on spherical functions, then, due to their orthogonality on Ω, the CS is represented as the sum of the squares of the amplitudes of these harmonics. In our case, it is easy to check that the functions $\sin^m\theta\exp\{-jk_ew_n^e\cos\theta\}$ within the limits of one Fourier harmonic (m), which correspond to various positions of the discrete sources w_n^e, are not orthogonal. Let us show that, nevertheless, the SC can be determined analytically. The SC is represented as follows:

$$\sigma^{P,S}(\theta_0) = \int_0^\pi\int_0^{2\pi}\left(F_\theta^{P,S}(\theta_0,\theta,\varphi)F_\theta^{P,S*}(\theta_0,\theta,\varphi) + F_\varphi^{P,S}(\theta_0,\theta,\varphi)F_\varphi^{P,S*}(\theta_0,\theta,\varphi)\right)\sin\theta d\theta d\varphi.$$

By substituting expressions (10) and (11) and integrating with respect to φ, one can readily check that for determining the CS it is sufficient to compute integrals

$$\int_0^\pi \sin^{2m}\theta\exp\{\gamma_{nl}\cos\theta\}\sin\theta d\theta = I_m^{(0)} := \int_{-1}^1\left(1-x^2\right)^m\exp\left(\gamma_{nl}x\right)dx,$$
$$I_m^{(1)} := \int_{-1}^1\left(1-x^2\right)^m x\exp\left(\gamma_{nl}x\right)dx; \quad I_m^{(2)} := \int_{-1}^1\left(1-x^2\right)^m x^2\exp\left(\gamma_{nl}x\right)dx. \quad (14)$$

Here, $\gamma_{nl} = -jk_e(w_n^e - w_l^e)$. Integrating by parts, we can readily obtain the following basic recurrence relations:

$$I_m^{(0)} = a_m I_{m-2}^{(0)} - b_m I_{m-1}^{(0)}, \quad m \geq 2, \tag{15}$$

where

$$\begin{gathered} a_m = 4m(m-1)/\gamma_{nl}^2, \quad b_m = 2m(2m-1)/\gamma_{nl}^2, \quad I_0^{(0)} = (e^{\gamma_{nl}} - e^{-\gamma_{nl}})/\gamma_{nl}, \\ I_1^{(0)} = 2(e^{\gamma_{nl}} + e^{-\gamma_{nl}})/\gamma_{nl}^2 - 2(e^{\gamma_{nl}} - e^{-\gamma_{nl}})/\gamma_{nl}^3 \text{ for } \gamma_{nl} \neq 0; \\ I_m^{(0)} = 2mI_m^{(0)}/(1+2m), \quad m \geq 1. \end{gathered} \tag{16}$$

Here, $I_0^{(0)} = 2$ for $\gamma_{nl} = 0$.

It is also easily seen that $I_m^{(1)}(\gamma) = \dfrac{\partial}{\partial \gamma} I_m^{(0)}(\gamma), I_m^{(2)}(\gamma) = \dfrac{\partial}{\partial \gamma} I_m^{(1)}(\gamma)$.

Hence, the total scattering cross section is determined analytically as soon as the amplitudes of the discrete sources are found. We note this fact, because due to it one can determine the absorption cross section by using the optical theorem [1].

The importance of eliminating the numerical integration for a unit sphere can be easily seen from Fig. 1. We present in it the results of DSC^P in the incidence plane of the wave with the angles of incidence $\theta_0 = 0°$, $60°$ for the flattened spheroid having the ratio of axes $a/b = 6$, wave parameter $k_e a = 40$, and refraction index $n_i = 2.5 - 0.02j$; this requires 37 Fourier harmonics. The figure shows that numerically integrating each of 37 harmonics will take plenty of time.

Note that the value of the wave parameter $k_e a$ determined by the ratio of the scatterer's radius to the emitting-wave length a/λ is very significant. In the Western literature, the wave parameter is known as the size parameter. When finding the amplitudes of the discrete sources, it is involved in the matrix elements determined by the representation for fields (5) and (8), as well as in the right side, which depends on the external excitation. The main objective of the present work consists in the advance of the DSM to a high-frequency region, i.e., the region of large values of $k_e a$.

We mention an important point of the previous concept [6]: the closeness between the approximate and exact solutions is determined by the fields' residual on the surface of a scatterer. However, according to the recent investigations, a surface residual is not the necessary condition of the closeness between the approximate and exact solutions in a wave zone. For example, it is established that the 20% residual on the surface of a scatterer can ensure an of less than 1% of the results in intensity. This was found by using the computer module of the DSM in various computing environments: with 32, 64, and 128 digits. It turned out that the same DSC^P can correspond to different values of the residual. In implementation 32 this value comprises 14%, whereas in implementation 128 this value is 0.12%.

This is not a surprise, because the usability conditions of the NAMs near a scatterer in the far zone can differ fundamentally [10]. This is explained physically by the availability of nonemitting waves near a body, which appreciably appear and show up for scatterers whose sizes are considerably larger than the wave length. Hence, it is suggested to determine the error in $DSC^{P,S}$ by the internal convergence by checking the residual for the case of the axial excitation $\theta_0 = 0°$ where only one Fourier harmonic is available.

The next change in the DSM concept lies in the fact that "corporate" discrete sources are used. In other words, for each external source with the coordinate w_n^e, two additional sources are found; these sources are located at points with the coordinates $w_n^e \pm \delta$, where δ is determined in relation to the position of the basic source w_n^e. Such a technique made it possible to ensure the monotonic convergence of the results by increasing the number of collocation points, especially by increasing the number of discrete sources by 6–10 units at once.

Thus, the new DSM concept is implemented in several steps:

(i) For the maximum angle of incidence, the number of Fourier harmonics M is determined with allowance for the error of the plane wave's approximation by a Fourier series on the surface of the scatterer of $\leq 0.05\%$; here, the value of M does not depend on the refraction index of a particle.

(ii) We choose the number of collocation points and discrete sources at the axial incidence of the wave ($m = 0$) by using heuristic formulas and determining the residual on the surface of the particle.

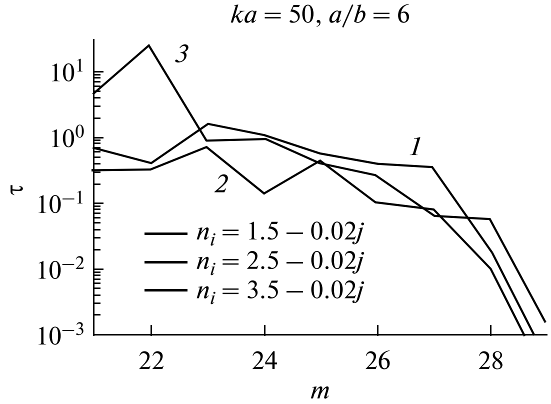

Fig. 2. The values of τ for $a/b = 6$ and $ka = 50$ in relation of the number of a harmonic m. For curve *1*, $n_i = 1.5 - 0.02j$; for curve *2*, $n_i = 2.5 - 0.02j$; and for curve *3*, $n_i = 3.5 - 0.02j$.

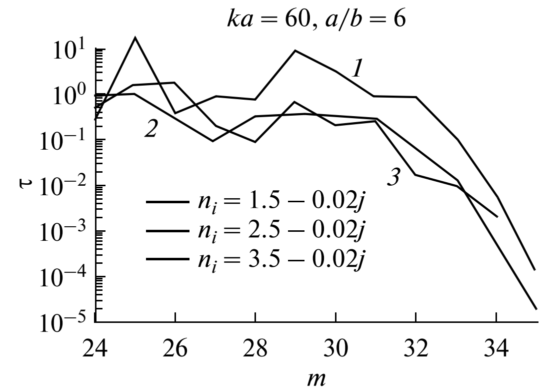

Fig. 3. The values of τ for $a/b = 6$ and $ka = 60$ in relation to m. For curve *1*, $n_i = 1.5 - 0.02j$; for curve *2*, $n_i = 2.5 - 0.02j$; and for curve *3*, $n_i = 3.5 - 0.02j$.

(iii) The amplitudes of the discrete sources are sequentially determined by the harmonics from the 0th to Mth. In this case, for the maximum angle of incidence, 12 observation angles $\{\theta_l\}_{l=1}^{12}$ (in direction $\theta_0, \pi - \theta_0$, together with 10 intermediate angles) are taken and the point-by-point relative convergence of $DSC^{P,S}$ in the taken angles is checked until the assigned error (0.5%) is attained [11]:

$$\tau = \max_{l=\overline{1,12}}\left(\left|DSC_{M+1}^{P,S}(\theta_0,\theta_l,\varphi = 0.90^\circ) - DSC_M^{P,S}(\theta_0,\theta_l,\varphi = 0.90^\circ)\right| \Big/ DSC_{M+1}^{P,S}(\theta_0,\theta_l,\varphi = 0.90^\circ)\right) \le 0.05.$$

Using this approach, one can significantly decrease the number of harmonics compared to the value of M taken in Section 1 (as a rule, by 35–40%). In this respect, the determination of $DSC^{P,S}$ is significantly speeded up for all angels of incidence.

(iv) The total scattering cross section of the absorption is determined analytically with the use of recurrence formulas (15) and (16).

Figure 2 presents the dependences of the residual of $DSC^{P,S}$ τ on the number of a harmonic m for a flattened spheroid having the ratio of axes $a/b = 6$, wave number $ka = 50$, and various refraction indices. As seen from the figure, $M = 29$ ensures the fulfillment of the convergence criterion, whereas the initial value of M was taken as 51. Figure 3 shows the analogous results for the spheroid with $ka = 60$. In this case,

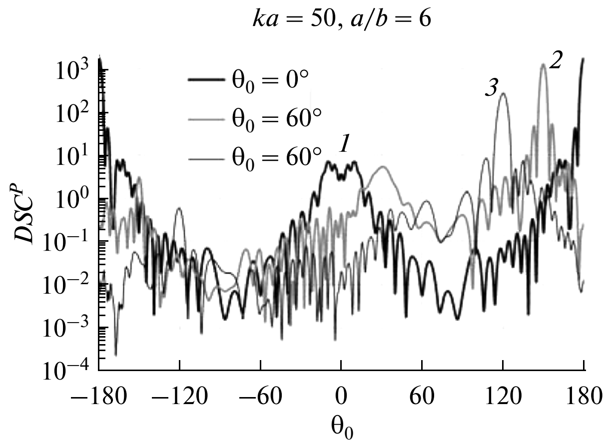

Fig. 4. The results of DSC^P in relation to the angle θ for $a/b = 6$, $ka = 50$, and $n_i = 2.5 - 0.02j$. For curve *1*, $\theta_0 = 0°$; for curve *2*, $\theta_0 = 30°$; and for curve *3*, $\theta_0 = 60°$.

$M = 35$ ensures the fulfillment of the convergence criterion, while initially $M = 59$. Hence, by using the new concept, one can significantly decrease the number of computations that need to be carried out.

Figure 4 presents the computational results of DSC^P for $ka = 50$, $n_i = 2.5 - j0.02$, and various angles of incidence of the plane wave ($\theta_0 = 0°$, 30°, 60°). As noted above, the intensity has a large number of oscillations over the considered range of observation angles.

CONCLUSIONS

We propose and implement a new concept of the DSM; by applying this concept, one can investigate scatterers with large wave parameters and a high refraction index. By checking the convergence by a scattering pattern, one can significantly reduce the number of computations that need to be carried out. In addition, it is shown that the total scattering cross section can be determined analytically.

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