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Mathematical Modeling of Processes in the Ionosphere Concerned with Radiowave Propagation

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Abstract—Models of the D, E, and F-regions of the ionosphere, the mesosphere, and the lower thermosphere are considered, together with the model of electromagnetic wave propagation in the Earth's ionosphere. It is shown that the calculated parameters of the ionospheric plasma can be used in the radiowave propagation range. The results of the calculation of the ionosphere's parameters are compared with the experimental data.

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INTRODUCTION

One of the main purposes of the mathematical modeling of ionospheric plasma is the numerical calculation of the spatial-temporal fields of electron concentrations. These fields obtained through computational experiments are employed in the tasks of electromagnetic wave propagation.

In this study, the models of the *E*, *D*, and *F*-regions of the ionosphere are briefly presented, together with a model of electromagnetic wave propagation. The possibility of using the calculated results in the radiowave propagation problem is investigated.

MODELS OF THE *D* AND *E*-REGIONS

The model proposed makes it possible to calculate the spatial-temporal distributions of the following components

$$N_2, O, O_2, O_3, O(^1D), O(^1S), O_2(^1\Delta_g), O_2(^1\Sigma_g^+), H, H_2, OH, H_2O, H_2O_2, N(^4S), N(^2D), NO, NO_2,$$

$$CO, CO_2, O^+, O_2^+, NO^+, O_4^+, N^+, N_2^+, N_e, O_2^-, Y^+, Y^-,$$

where Y^+ is the total concentration of positive binder ions and Y^- is that of negative ions.

The continuity equation for ions is written in the form:

$$\frac{\partial n_j}{\partial t} = \frac{\partial}{\partial z}\left(D_j\frac{\partial n_j}{\partial z} + \bar{D}_j n_j\right) - L_j n_j + P_j, \tag{1}$$

where $D_j = \kappa T(\sum_{j\neq k}\mu_{jk}\nu_{jk}\sin^2 I)^{-1}$ is the ion diffusion coefficient and

$$\bar{D}_j = \sum_{k\neq j}\mu_{jk}\nu_{jk}V_{kz}\sin^2 I/\sum_{k\neq j}\mu_{jk}\nu_{jk} + D_j\left(\frac{1}{N_e}\frac{\partial N_e}{\partial z} + \frac{2}{T}\frac{\partial T}{\partial z} + \frac{1}{H_j}\right).$$

For neutral components the continuity equation is as follows:

$$\frac{\partial n_k}{\partial t} = \frac{\partial}{\partial z}\left(D_k\frac{\partial n_j}{\partial z} + \bar{D}_k n_k\right) - L_k n_k + P_k, \tag{2}$$

where $D_k = D_{km} + D_T$, $D_{km} = \kappa T(m_k\sum_{k\neq j}S_{kj}n_j)^{-1}$ is the molecular diffusion coefficient for component k, $\bar{D}_k = \frac{D_{km}}{H_k} + \frac{D_T}{H} + \frac{(D_{km}+D_T)}{T}\frac{\partial T}{\partial z} - \bar{V}_k$, $\bar{V}_k = \sum_{k\neq j}S_{kj}n_jV_{jm}/\sum_{k\neq j}n_jS_{kj}$, and D_T is the turbulent diffusion coefficient which is empirically specified, depending on the season.

The total concentration of the positive and negative binder ions is calculated from the equation

$$\frac{d[Y^{\pm}]}{dt} = P^{\pm} - \alpha[Y^{\pm}]. \tag{3}$$

In this equation the following positive ion formation rate and loss coefficient are adopted

$$P^{+} = B_{\mathrm{NO}^+}[\mathrm{NO}^+] + B_{\mathrm{O}_2^+}[\mathrm{O}_2^+], \quad \alpha^{+} = \alpha_{Y^+}[N_e] + \alpha^*[Y^-],$$

where $\alpha_{Y^+} = 10^{-5}$ cm^{-3} s^{-1} and $\alpha = 10^{-7}$ sm^{-3} s^{-1}.

For negative binder ions $P^- = \bar{x}[\mathrm{O}_2^-]$ and $\alpha^- = [Y^+] \times 10^{-7} + \gamma$. The electron concentration is calculated from the equation

$$\frac{dN_e}{dt} = P_e - \alpha_{\mathrm{eff}} N_e \tag{4}$$

with a checking on the fulfillment of the quasineutrality condition

$$[N_e] + [Y^-] = [Y^+] + \sum_{i=1}^{6} [N_i],$$

where $\sum_{i=1}^{6}[N_i]$ is the total concentration of positive ions.

The parameters B_{NO^+}, $B_{\mathrm{O}_2^+}$, $\bar{x}$, and α_{eff} are calculated in accordance with the chosen photochemical schemes [1].

MODEL OF THE *F*-REGION

The altitudinal-temporal distributions of the O^+ and H^+ concentrations were calculated on the basis of the following equations:
the continuity equation for the components

$$\frac{\partial n_i}{\partial t} + B\frac{\partial}{\partial s}\left(\frac{1}{B}n_i V_i\right) = F_i - \alpha_i n_i, \tag{5}$$

the equation of motion

$$n_i m_i\left(\frac{\partial V_i}{\partial t} + V_i\frac{\partial V_i}{\partial s}\right) + \frac{\partial P}{\partial s} = -n_i m_i g \sin I + n_i \sum_j S_{ij}(V_j - V_i) + n_i R_i(V_{nx}\cos I - V_i) - \frac{n_i}{N_e}\frac{\partial P_e}{\partial s}, \tag{6}$$

and the equation of energy

$$\frac{3}{2}kN_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial t} = B\frac{\partial}{\partial s}\left(\frac{1}{B}\lambda_{\mathrm{e}}\frac{\partial T_{\mathrm{e}}}{\partial s}\right) + \sum_i \frac{3m_{\mathrm{e}}N_{\mathrm{e}}}{mi}\nu_{\mathrm{e}i}k(T_i - T_{\mathrm{e}}) + Q_{\mathrm{e}} - L_{\mathrm{e}n}, \tag{7}$$

$$\frac{3}{2}kn_i\frac{\partial T_i}{\partial t} = B\frac{\partial}{\partial s}\left(\frac{1}{B}\lambda_i\frac{\partial T_i}{\partial s}\right) + 3n_i\nu_{i\mathrm{e}}k(T_{\mathrm{e}} - T_i) + \sum_n \frac{3m_i n_i}{m_i + m_n}\nu_{in}k(T_n - T_i) + Q_i - L_i, \tag{8}$$

written in a coordinate system fitted with a magnetic field line.

The system of equations governing the meridional V_{nx} and zonal V_{ny} components of a neutral wind along a geomagnetic field line is written in the form:

$$\frac{\partial V_{nx}}{\partial t} = \frac{\xi}{\sin^2 I}\frac{\partial^2 V_{nx}}{\partial s^2} - \frac{1}{\rho_n}\sum_j n_j R_j(V_{nx} - V_j\cos I) + 2\Omega\sin\varphi V_{ny} - \frac{1}{\rho_n}\frac{\partial P_n}{\partial x}, \tag{9}$$

$$\frac{\partial V_{ny}}{\partial t} = \frac{\xi}{\sin^2 I}\frac{\partial^2 V_{ny}}{\partial s^2} - \frac{1}{\rho_n}\sum_j n_j R_j V_{ny} - 2\Omega \sin\varphi V_{ny} - \frac{1}{\rho_n}\frac{\partial P_n}{\partial y}. \quad (10)$$

The neutral component concentrations, temperatures, and pressure gradients were calculated in accordance with the model [4], while those of the molecular ions were calculated by numerically solving the continuity equation (5) in the absence of transport processes. Since the molecular ions are present only at low altitudes (100 to 200 km), for them the coordinate s can be replaced by the altitude z.

The system of equations (1)–(10) and the numerical methods for its solution are described in detail in [1–3].

RADIOWAVE PROPAGATION MODEL

On the basis of a geometric optics approximation [5], the wave equation that describes the interaction between an electromagnetic wave and the ionosphere is reduced to a system of equations for the field phase (eikonal) and amplitude (transport equation). The characteristic system of equations takes the form:

$$\frac{d\bar{x}}{d(ct)} = \frac{c}{\omega}\bar{k},$$

$$\frac{d\bar{k}}{d(ct)} = \frac{2\pi e^2}{m\omega^2 c}\nabla N, \quad (11)$$

$$\frac{d\varphi}{d(ct)} = \frac{\omega}{c}\left(1 - \frac{4\pi e^2}{m\omega^2 N}\right),$$

where $\bar{k}$ is the wave vector, $\bar{x}$ is a current point on the trajectory, ω is the angular frequency, φ is the phase, c is the speed of light in a vacuum, e and m are the electron charge and mass, and N is the electron concentration.

For numerically solving system (11), which determines the ray trajectory, an optimal interpolation of N and its gradients N_r, N_φ, and N_θ with respect to r, φ, and θ should be chosen. One of the approaches to the interpolation is given by mathematical models.

In [6] the basic requirements to the theoretical models of the ionosphere were developed. These are as follows: the characteristic dimensions of ionosphere inhomogeneities at the altitude L_h, the longitude L_φ, and the latitude must be greater than the corresponding distances between the grid points at the altitude Δh, the longitude, and the latitude, that is, the inequalities $L_h \geq \Delta h$ and $L_\varphi \geq \Delta\varphi$ must be fulfilled and the integration step S of the characteristic system of equations must satisfy the inequalities $S \leq L_h$ and L_φ.

The calculations of the parameters listed above were performed under calm geophysical conditions, for the mean solar activity, and at mid latitudes (45° N).

RESULTS OF THE COMPUTATIONAL EXPERIMENT

To check the quality of the prediction of the short-wave radio signal propagation it is necessary to predict the ionosphere's state on the basis of a dynamic model, to calculate paths, and to compare the results of the path calculations with experimental data. The quality of the prediction of the ionosphere's state can be checked using ionograms.

This check was preliminarily made on the basis of the experimental data on noncoherent scattering without a comparison with the experimental data on radiowave propagation for June 9, 1969. From the data [7] the electron concentration profiles along the Sverdlovsk–Kaliningrad path were constructed with a step in the longitude $\Delta\varphi = 7.5°$, which corresponds to a time step of 0.5 h. In relation to the altitude, the profiles were approximated by a spline on a nonuniform grid, with a lower boundary of 100 km. In Fig. 1 these profiles are presented as the curves with points, with the first profile corresponding to 18 h above Kaliningrad and the last to 21 h above Sverdlovsk, after one hour. In the same figure, the profiles with crosses were calculated by means of a dynamic model [2], which makes it possible to calculate the ionospheric plasma parameters along a field tube starting from an altitude of 120 km. The profiles were calculated for the same conditions, as in [1], but starting from an altitude of 50 km. From Fig. 1 it can be seen that the difference between the experimental and predicted profiles amounts to about 20%.

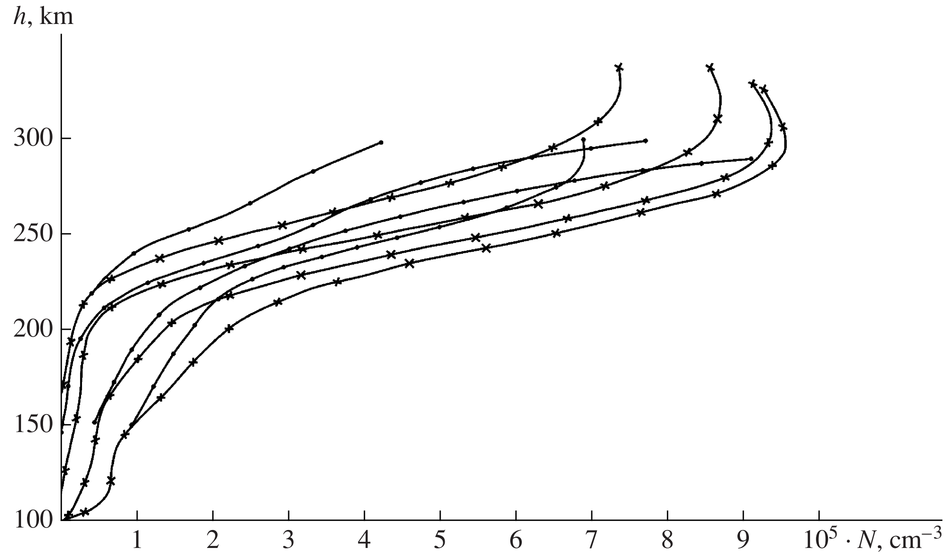

Fig. 1. Electron concentration profiles.

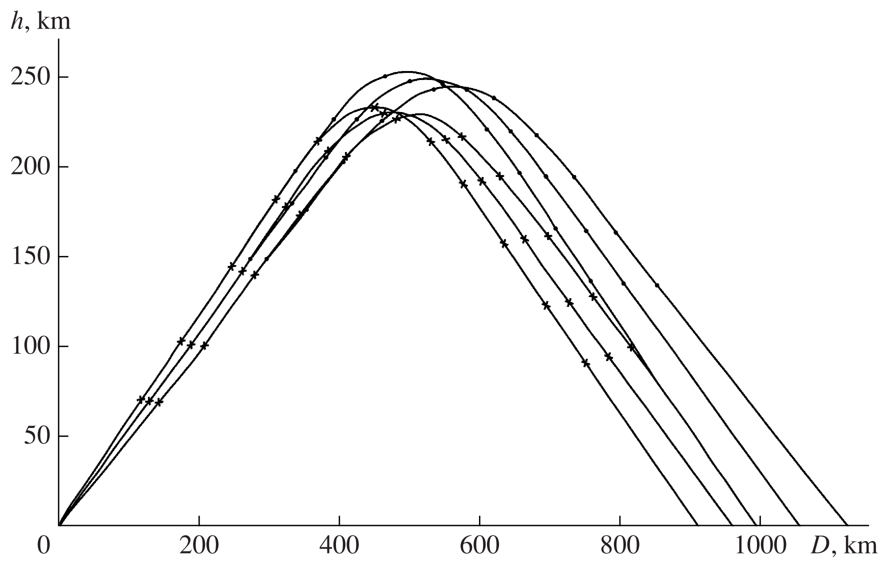

Fig. 2. Short-wave radio paths.

Figure 2 presents the results of path calculations for the data of noncoherent scattering and the dynamic ionosphere model, respectively. The paths were calculated for a frequency $f = 10$ MHz and the exit angles 25°, 27°, and 29° for the first, second, and third paths. For the same angles, the data on the signal ranges differ by 10%. As might be expected, the paths for noncoherent scattering lie higher, since here the ionosphere begins from 150 km. Thus, the ionosphere modeling error being 20%, the path calculation error is 10%.

CONCLUSIONS

The mathematical models of the mesosphere, the lower thermosphere, and the ionosphere of the Earth presented in the paper make it possible to calculate different parameters of ion and neutral composition by means of computational experiments. These data may be used in the mathematical model of radio path propagation under different conditions. It is shown that in order to calculate short-wave radio signals more accurately it is necessary to take account of the absorption of these signals in the lower layers (*D* and *E*-regions).

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