

On the Topological Structure of a Mathematical Model of Human Unconscious*

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Received March 20, 2016

Abstract—On the basis of two our previous works, in this paper, following Jacques Lacan psychoanalytic theory, we wish to outline some further remarks on the topological structure of a mathematical model of human unconscious.

DOI: 10.1134/S2070046617010071

Key words: *Lacan's psychoanalysis, topology, chirality, symmetry breaking.*

1. ON THE TOPOLOGY OF UNCONSCIOUS

Topological structures are considered innate, while all the other mathematical structures (algebraic, ordered, linear, and so forth), as meant in the *architecture des mathématiques* in the sense of N. Bourbaki, are considered acquired. This follows from previous studies and researches on mathematical skills and capabilities of blind mathematicians, who in particular excel in geometry with respect to the other mathematicians [19, 22]. Therefore, the study of the formal structures (in particular the topological ones) of human unconscious, seems to be particularly appreciated and useful. In two our previous works [11, 12], we have put forward a simple formal model of human unconscious phenomenologically based on hysteresis mechanisms, and formally laid out within *p*-adic analysis framework. In describing briefly this model, we have stressed the fundamental importance of temporal dimension for the origin of human consciousness, formally related with the occurrence of a primordial ordering springing out from symmetry breaking phenomena just due to the emergence of these hysteresis phenomena. These latter take place in certain zones of unconscious which are not sharply identified by clear and fixed boundaries, contours or profiles, because psychoanalysis says us that there is a kind of fluidity, undulation, evanescence in these latter which entail a certain, so to speak, topological indeterminacy subsisting from deep and primordial zones of unconscious realm to preconscious and consciousness-nearer ones. The key point of our view is just the occurrence of a some primary form of ordering in the basic set of natural numbers built up from its spectrum of primes arising just from the emergence of hysteresis phenomena as discussed in [12]. Therefore, just for these reasons, we think that *p*-adic analysis, with *p* prime, is one of the most suitable mathematical frameworks with which working out formal models of human psyche, above all in regard to its unconscious aspects.

In this setting, we wish further argue on other possible formal features of this model, especially on the topological side, starting from some basic points of the well-known Jacques Lacan psychoanalytic theory whose slogan is the celebrated sentence according to which "unconscious is structured like a language". To be precise, Lacan [15] states that the unconscious has not to be considered as an interior put against an exterior. For Lacan, the unconscious is characterized by a confine topological

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structure which allows the psychic drives to lean on bodily parts with boundary. This is coherent with what we have said in [12] about the primary relevance of boundary conditions on the diffusion of psychic energy as grounding phenomenology lying at the early bases of consciousness. The above mentioned Lacan topology of unconscious, is molded on that of Möbius band: indeed, the irruptions of unconscious formations into the effective discourse does not require the overcoming of any boundary, but rather they follow a continuous route like the forehand and backhand of a Möbius strip. With this emblematic topological model, Lacan has wished to describe the main interrelations between conscious and unconscious, and the related interpretation of the latter. Indeed, although locally the Möbius band seems to have two distinct faces, it globally has instead a unique face. On the other hand, on the basis of an original rereading of classical Freudian theory, the crucial question of unconscious structure may be roughly solved, in Lacan terms, considering it as an "inscription" into a place which is *other* with respect to the conscious discourse which is however enunciated on the same versus of the former, and, since unconscious may always intervene in every point of conscious discourse, Lacan intuitively grasped the idea for which all this may be elegantly formalized by the topology of a Möbius band that, as it has a unique global face, lends very well to explain this crucial intertwining unconscious-conscious [2].

2. ON THE POSSIBLE ROLE PLAYED BY CHIRALITY

Roughly speaking, Möbius strip is the most elementary example of a non-trivial fiber bundle whose base space is the circle and whose fiber is the real line [3]. Lacan has been the first psychoanalyst to have identified such basic fiber bundle structure of unconscious, and this gives credit to that possible further formal characterization of unconscious structure in terms of fibre bundles as briefly discussed in [10]. So, we are encouraged to carry on along this line. Möbius band is the prototype of non-orientable surfaces as every non-orientable surface contains a submanifold which is a Möbius band [8]. So, we may consider the topology of the Möbius two-dimensional manifold as the most typical formal feature characterizing unconscious structure which therefore turns out to be the non-orientability. This agrees with the typical traits of unconscious according to Freud, in which any form of order cannot exist, hence any orientation. Therefore, the rising of a primordial form of order might be correlated with the occurrence of some form of orientation, that is to say, with the passage from non-orientable structures to oriented ones. An orientation is always related with an order because the former establishes the basic dual pair $\{-1, 1\}$ (spin down-spin up) from which to deduce the line \mathbb{Z} with its two-values orientation. This last fact, in turn, may be equivalently related with the distinction between clockwise and anticlockwise rotations [7, 17].

On the other hand, from a physical viewpoint, the occurrence of the distinction between clockwise and counterclockwise rotations may be induced, for example, by symmetry breaking of chirality as for instance induced by a chiral anomaly, in turn due to a \mathbb{Z}_2 -symmetry breaking ([1], Chapter 6, Section 6.3.2). On the other hand, chirality plays a very important role in physics, chemistry and, above all, in biology [5, 17] where many works assume that, at a certain point in the early Earth history, physical and/or chemical processes have triggered symmetry breaking phenomena, which occurred in an inanimate environment and allowed the initiation of life processes; therefore, a strong mirror symmetry breaking phenomenon, springing out from a relatively modest asymmetry in mutual (auto)catalytic activity (in which D-enantiomers more likely catalyze other D-enantiomers than L-enantiomers, while L-enantiomers preferentially catalyze their L-brothers and sisters), is put at the early bases of the origin of life [13]. Furthermore, fluctuations are known to promote changing in the nature of phase transitions or even destroy long-range order. It has been observed as well that fluctuations may induce symmetry breaking transitions in systems where such symmetry breaking would not have occurred in its absence. Studying autocatalytic production of chiral enantiomers from achiral reactants in reaction-diffusion systems whose mean field steady state is chiral symmetric, has shown that spatiotemporal fluctuations induce a novel chiral ordering and sharp phase transitions. This implies that an arbitrary perturbation from the chiral symmetric fixed point generated by the slightest stereo-preference, will remain as is (i.e., unamplified) and will not give rise to a global chiral symmetry breaking. In short, it has been identified a new role for fluctuations in generating a symmetry broken state in systems which are perfectly symmetric in the absence of fluctuations. This phenomenon has been illustrated in the context

of autocatalytic reaction-diffusion systems, where spatiotemporal fluctuations drive chemical systems to a chiral symmetry broken steady state [6].

Indeed, just in relation to our model based on hysteresis, there are many physics studies in which chiral symmetry breaking, hence breakdown of inversion symmetry, in the charge-density wave¹ transition of certain materials (like NbSe_3), gives rise to drastic hysteresis effects [9, 16, 21]. Moreover, there are also important relations between chirality and hysteresis in biology [14]. Therefore, a chirality symmetry breaking, which is a phenomenon contemplated by biology, might induce just those hysteresis phenomena involved in our model, and both however inducing an ordering. In any case, the effects of the occurrence of both chirality phenomena and hysteresis ones are independent of each other because, for instance, in superconductivity context, the realization of recent experiments in hole-doped iron arsenides has revealed a magnetically ordered ground state that preserves tetragonal symmetry, which is likely either a checkerboard charge-and-spin density wave (CSDW) or a spin-vortex crystal (SVC), in which either of these phases can also melt in two stages, resulting in an intermediate phase with vestigial order, namely, a charge density-wave (CDW) for a CSDW ground state, and a remarkable spin-vorticity density-wave (SVDW) for a SVC ground state. Now, while the CDW has an Ising-like order parameter, the SVDW has a vector chiral order parameter that breaks inversion without breaking time-reversal symmetry, giving rise to Goldstone modes [4]. Thus, the effects entailed by both chiral and hysteresis symmetry breaking phenomena are independent of each other, so that both orders of effects are possible and equally probable to occur together. Finally, also curvature effects may induce chirality symmetry breaking [18]. There are also interesting relations between breakdown of chiral symmetry and oscillatory chemical reactions (*chiral oscillations*) [20].

3. CONCLUSIONS

From an epistemological viewpoint, the proper quantitative and numerical methods of experimental physics clearly fail in probing unconscious structure, due to its basic non-directly observable nature as usually meant in physics. At most, we are able to handle qualitative methods of mathematics to try to descry, even indirectly, what typical formal features human unconscious may have. From this standpoint, since topological structures are the most innate ones [19, 22], we are inclined to think that, just qualitatively, the unconscious structure of human psyche might be formally identified through topological methods and its applications. Here, we have begun to outline some very basic considerations of this type as regard unconscious structure, starting from an original revisiting of Freudian thought by Jacques Lacan, the first thinker who has majorly looked at the formal aspects of psychoanalysis. Lacan says us that unconscious has mainly a non-orientable feature, a qualitative formal trait owned by non-orientable manifolds, hence a \mathbb{Z}_2 -symmetry. The other higher psychic functions, from preconscious zones to consciousness as formally characterized by other mathematical structures worked out upon the basic topological ones, seem therefore to spring out from a kind of breaking of this symmetry, so giving rise to an orientation, hence the basic dual pair $\{-1, 1\}$ from which to deduce \mathbb{Z} thought as an ordered algebraic system. We have called into question the breaking of chirality, which is one of the most typical phenomenon occurring in biology of human evolution, trying to coherently implement it into a wider framework which also includes an our previous model of human psyche based on hysteresis phenomena reformulated in the formal language of p -adic analysis [11, 12]. Therefore, a p -adic geometry for unconscious urges. For instance, we are inclined to think that a p -adic knot theory might turn out to be also useful in describing, formally, the phenomenological dynamics of m -adic trees of unconscious and human psyche, in general.

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¹A periodic electronic charge modulation, often called *charge-density wave* (CDW), is roughly an ordering physical phenomenon which is accompanied by a distortion of the underlying lattice with the same periodicity, and therefore often linked with the so-called *Jahn-Teller effect* [9].

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