

Monte Carlo Method for Numerical Simulation of Solar Energy Radiation Transfer in Crystal Clouds

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Abstract—The paper deals with numerical simulations related to radiation transfer in ice clouds. A mathematical model of crystal particles of irregular shape and an algorithm for modeling such particles based on constructing a convex hull of a set of random points are considered. Two approaches to simulating radiation transfer in optically anisotropic clouds are studied. One approach uses pre-calculated scattering phase functions for crystals of various shapes and orientations. In the other approach, no knowledge of phase functions is required; the radiation scattering angle is simulated directly at interaction of a photon with faces of crystal. This approach enables simple adjustment of the input parameters of the problem to changing microphysical characteristics of the environment, including the shape, orientation, and transparency of particles and roughness of their boundaries, and does not require time-consuming pre-calculations. The impact of flutter on the radiation transfer by the cloud layer and angular distributions of the reflected and transmitted radiation are studied.

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INTRODUCTION

Ice crystal clouds (cirrus, cirrostratus, and cirrocumulus ones) regularly cover about 20–30% of the Earth's surface and have a significant effect on the radiative heat transfer in the atmosphere, altering the albedo of the climatic system and the flux of atmospheric heat emission [1–3]. These circumstances have led meteorologists, climatologists, and developers of common atmospheric circulation models to remain interested in the problem of constructing a radiative simulation for cirrus clouds. Development of such numerical simulation obviously requires, first of all, knowledge of the optical characteristics of crystal clouds, such as scattering phase functions and cross-sections of radiation attenuation by the particles residing in the clouds. Calculation of these characteristics, in turn, requires detailed information on the structure of the microphysical parameters of clouds: the concentrations and distributions in the composition, size, and spatial orientation of non-spheric particles. Because of the extreme variability of these characteristics in time and space, microphysical and optical models of crystal clouds can only be adequately described in terms of random functions, which, in turn, enables not only making statistically averaged radiative simulations, but also evaluating the spatial and temporal variation of parameters of radiation fields. The only way to solve this difficult problem is through the Monte Carlo method.

According to experimental data, most particles of ice clouds have an irregular and highly variable form [4]. In particular, in [5] it is shown that only 3% of ice crystals of Arctic clouds have perfectly correct shapes (hexagonal prisms, plates, needles, and dendrites), and up to 90% of atmospheric ice particles in cirrus clouds may have an irregular shape in all temperature intervals [6]. These

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studies explain why halo and other optical phenomena that are characteristic of regular polyhedral ice crystals are rarely met in observation of cirrus clouds. Most numerical calculations of the optical characteristics of ice crystals are made for regular-shape particles [4, 7–9]. However, in [10], results of calculation of scattering phase functions for rough-surface particles and random-shape particles with the angle between the faces randomly varying are presented. In [11], scattering phase functions are given for crystal models obtained from regular particles by cutting off their parts by randomly inclined planes.

This paper briefly describes a new algorithm of computer construction of random realizations of three-dimensional convex bodies the geometric parameters of which obey given laws of probability distribution. Such models present ice cloud particles of random size and irregular shapes. This algorithm is described in more detail in [12]. In the cirrus cloud optics, hexagonal prisms are most commonly used as ice crystal models. The proposed class of particle models complements the variety of already studied regular [7, 13] and random [10, 11] models of crystalline particles.

In Section 2, the algorithm of the Monte Carlo method for estimating scattering phase functions and attenuation cross-sections for crystalline particles is described in brief.

Due to the large variety of microphysical properties of real crystalline particles, the problem of light scattering by crystals of imperfect shapes turns out to be extremely complex. Its solution requires huge amounts of initial data that take into account the shape, size, and orientation of particles.

Section 3 introduces a new Monte Carlo method algorithm for simulation of the optical radiation transfer in crystal clouds, which does not require pre-calculation of anisotropic distributions of scattering. In this algorithm, the shape and orientation of a given particle is chosen directly when a photon collides with the particle, and the scattering angle is simulated in accordance with the laws of reflection and refraction of geometric optics. Section 4 presents the results of numerical test calculations for the problem of radiation transfer in ice clouds.

1. MODELS OF RANDOM-SHAPE CRYSTALLINE PARTICLES

As models of irregular-shape crystalline particles, convex polyhedra with random vertex positions, i.e., convex hulls of a random set of points located in some limited three-dimensional subspace, are constructed [14]. The vertices of polyhedron can be defined deterministically or modeled randomly with a given distribution law so that, on average, the particle-typical ratios of the length, width, and height are reproduced. The number of vertices of polyhedron is either set by the user or chosen randomly according to some distribution law with a given mathematical expectation and variance.

The convex hull is constructed by the incremental algorithm [15]. A limited space area in which the simulation takes place is specified. The initial body is a tetrahedron with vertices whose coordinates are modeled in a given volume in accordance with some distribution law. Further construction of the polyhedron is done by modeling of the next random point and testing whether this point is internal to the constructed polyhedron. If the point lies outside the constructed body, it becomes another vertex. The modeling is performed until the number of vertices reaches the required value.

Based on the algorithm described in [12, 15], the program “Convex Hull” has been developed. It yields a given or random number of the faces of a constructed convex polyhedron and coordinates of its vertices and records ordered vertex sets for each face. They are ordered so that the vector product of the resulting edge vectors corresponds to the rule of the right hand and defines the direction of the external normal to the face. Such ordered queues are useful for calculation of radiation scattering matrices by the ray tracing method. The program is written in C++ and implements the function of visualization of polyhedra using the OpenGL library.

Figure 1 presents examples of constructed convex polyhedra for which the random points used in the polyhedron construction are modeled uniformly in a cube.

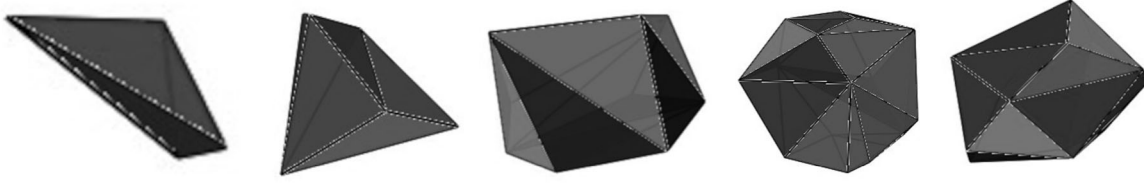


Fig. 1. Models of random-shape crystalline particles.

2. MONTE CARLO CALCULATION OF PHASE FUNCTIONS AND CROSS-SECTIONS OF LIGHT WAVE SCATTERING BY ORIENTED CRYSTALS

To solve the problem of radiation transfer through a flat layer of a crystal cloud, it is necessary to calculate the scattering phase functions $G(\omega', \omega, r)$, which depend on the incoming radiation direction ω' and outgoing radiation direction ω , and the cross-sections $\Sigma_e(\omega, r)$ and $\Sigma_s(\omega, r)$ of radiation attenuation and scattering, respectively, by crystalline particles. If the crystal size significantly exceeds the radiation wavelength, that is, $D/\lambda > 100$, where λ is the wavelength and D is the typical particle diameter, the geometric-optics approximation and ray tracing algorithm are fairly accurate and least-time-consuming ways to determine the macrophysical parameters of scattering (attenuation and scattering cross-sections and scattering phase functions). In this algorithm, the optical radiation trajectory is represented as a polyline, the vertices of which are the points of collisions with the faces of the crystal or with irregularities within the crystal. The description of the interaction of radiation and a flat face of crystal corresponds to a simulation in which the reflection and refraction of light on a face of crystal occur according to the Snellius and Fresnel laws.

Let \mathbf{n} be the vector of external normal to the surface of the crystal face at the point where a light quantum falls onto this face, expressed in the global coordinate system. In this case, when a ray with the direction ω falls onto the face surface, mirror reflection in the direction $\omega_{\text{refl}} = \omega - 2(\omega, \mathbf{n})\mathbf{n}$ occurs with the probability $R(\omega, \mathbf{n})$, and refraction in the direction $\omega_{\text{refr}} = \nu\omega - D\mathbf{n}$ takes place with the probability $1 - R(\omega, \mathbf{n})$. Here $R(\omega, \mathbf{n})$ is the Fresnel reflection coefficient, which can be presented in the following form for ease of calculation [16]:

$$R(\omega, \mathbf{n}) = \frac{(|A| - B)^2(A^2B^2 + C^2)}{(|A| + B)^2(|A|B + C)^2}, \quad (1)$$

$$A = (\omega, \mathbf{n}), \quad B = \sqrt{1/\nu^2 - 1 + A^2}, \quad C = 1 - A^2, \quad D = A - \text{sign}(A)B,$$

$$\nu = \begin{cases} 1/n_{\text{ice}}, & (\omega, \mathbf{n}) \leq 0, \\ n_{\text{ice}}, & (\omega, \mathbf{n}) > \sqrt{1 - 1/n_{\text{ice}}^2}. \end{cases}$$

Here n_{ice} denotes the real part of the complex index $\nu_{\text{ice}} = n_{\text{ice}} + i\kappa_{\text{ice}}$ of refraction of ice relative to air. At $0 \leq A \leq \sqrt{1 - 1/n_{\text{ice}}^2}$, by the total internal reflection law, we take $R(\omega, \mathbf{n}) = 1$. The real part of the refraction index is responsible for the reflection and refraction of light on the crystal faces, and the imaginary part κ_{ice} is responsible for the beam attenuation due to absorption inside the crystal.

If the light falls normally to the crystal face, i.e., $\omega = -\mathbf{n}$,

$$R = \left(\frac{n_{\text{ice}} - 1}{n_{\text{ice}} + 1} \right)^2. \quad (2)$$

For calculation of the scattering phase functions $G(\omega', \omega, r)$, we fix a set of incoming radiation directions $\omega'_{ij} = (\cos \varphi'_i \sin \theta'_j, \sin \varphi'_i \sin \theta'_j, \cos \theta'_j)$ given in the polar coordinate system, where θ'_j and

φ'_i are the zenith and azimuthal angles of the vector ω'_{ij} , $\theta'_j \in [0, \pi]$, $\varphi'_i \in [0, 2\pi]$, $j = 1, \dots, N_{\theta'}$, $i = 1, \dots, N_{\varphi'}$. For each ω'_{ij} , the probabilities G_{ijkl} of scattering into the solid angle element $\Omega_{kl} = \{\omega = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta) : \theta \in [\theta_k, \theta_{k+1}], \varphi \in [\varphi_l, \varphi_{l+1}]\}$ are estimated by the Monte Carlo method. Here $k = 1, \dots, N_{\theta}$ and $l = 1, \dots, N_{\varphi}$. Therefore, the four-dimensional ‘‘matrices’’ G_{ijkl} are an approximation of the scattering phase function $G(\omega', \omega, r)$ [17].

Because of the absence of circular symmetry of crystals, a medium consisting of crystalline particles with a given orientation is anisotropic, and therefore the attenuation cross-section $\Sigma_e(\omega, r) = \Sigma_s(\omega, r) + \Sigma_a(\omega, r)$; the cross-sections of scattering $\Sigma_s(\omega, r)$ and absorption $\Sigma_a(\omega, r)$ depend on the light quantum direction vector ω . The dependence of $\Sigma_e(\omega, r)$ and $\Sigma_s(\omega, r)$ on the position vector r in the global coordinate system means that the medium may be nonuniform in the spatial coordinates. For transparent particles (pure ice crystals for visible range radiation), the imaginary part of the refractive index $\kappa = 0$, there is no absorption, and $\Sigma_e(\omega, r) = \Sigma_s(\omega, r)$. If $\kappa \neq 0$, then some radiation is absorbed inside the crystal. In contrast to the simulation of photon absorption by aerosol particles or water droplets, in which absorption occurs in a collision, in this case, the beam of radiation is attenuated on the entire photon trajectory inside the particle. For a fixed direction of the incoming radiation ω' , the absorption cross-section in collision with a particle is assessed by the Monte Carlo method:

$$\Sigma_a(\omega') = \sigma_{a,cr} L(\omega').$$

Here $L(\omega')$ is the average length of photon trajectories inside the crystal for the incoming radiation direction ω' and a fixed crystal orientation; $\sigma_{a,cr} = 4\pi\kappa/\lambda$ is the absorption coefficient of the crystalline matter for radiation wavelength λ (see [18, pp. 99/100]).

The ray tracing procedure allows one to evaluate the matrices $\Sigma_{ij}(r)$, which are approximations for $\Sigma_s(\omega, r)$ if $\omega = (\cos \varphi \sin \theta, \sin \varphi \sin \theta, \cos \theta)$, $\theta \in [\theta_i, \theta_{i+1}]$, $\varphi \in [\varphi_j, \varphi_{j+1}]$, simultaneously with the calculation of the elements of the matrix G_{ijkl} . For this purpose, for each crystal and direction ω'_{ij} , the radiation component of scattering is estimated, which is equal to the average area S_{ij} of the crystal projection to a plane perpendicular to the direction ω'_{ij} . Let the crystal be inside a ball of radius R , and let the plane perpendicular to the vector ω'_{ij} cross the ball over the circle C_{ij} . Then

$$S_{ij} = \pi R^2 N_{ij}^r / N_{ij},$$

where N_{ij} is the number of modelled points, uniformly distributed within the circle C_{ij} , and N_{ij}^r is the number of points that are inside the crystal. For some particles of regular shape, S_{ij} values are known for different directions ω'_{ij} . For example, the area of projection of a regular hexagonal prism with base diameter d and lateral edge height h to a plane perpendicular to the vector $(\cos \nu \sin \theta, \sin \nu \sin \theta, \cos \theta)$ is calculated by the formula [7]

$$S(\theta, \nu) = \frac{3\sqrt{3}d^2}{8} \sin \theta + dh \cos \theta \cos(\pi/6 - \nu).$$

For chaotically oriented hexagonal prisms, with the integration over all incoming directions taken into account, the average projection area is

$$S_{\text{mean}} = \frac{3}{8} \left(\sqrt{3}d^2/4 + dh \right).$$

According to the optical extinction theorem [7], the diffraction component of scattering is equal to the ray component S_{ij} . Therefore, the diffraction component taken into account, the scattering cross-section $\Sigma_{ij} = 2S_{ij}$ in the direction ω_{ij} .

Suppose that a unit volume contains a set of crystals of m arrays of particles, grouped by form, and the total number of particles in the unit volume is $M = \sum_{\nu=1}^m m_{\nu}$, where m_{ν} is the calculated

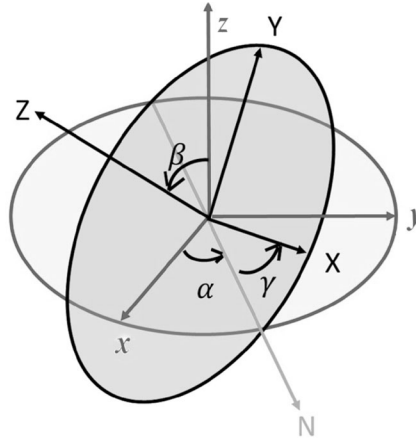


Fig. 2. Determination of Euler angular coordinates: XYZ coordinate system is obtained by rotation of global coordinate system xyz through angles α, β, γ .

concentration of particles of the ν th type. All particles in each group can be of different sizes, which are random quantities distributed with the density $P_\nu(\rho)$, $\nu = \overline{1, m}$. As the non-spheric particle size ρ , we take the form factor, i.e., the ratio of the length of the preferred main axis of the particle to the preferred effective diameter of the base. Random orientations of crystals in a three-dimensional space, given by probability distribution densities $P_{\Theta, \nu}(\alpha, \beta, \gamma)$, are described with the system of the Euler angular coordinates α, β, γ , which define the rotation of particles relative to the global Cartesian coordinate system [20] (see Fig. 2).

Then the scattering cross-section $\Sigma_s(\omega, r)$, attenuation cross-section $\Sigma_a(\omega, r)$, and the scattering phase function $G(\omega', \omega, r)$ averaged over the distributions of the crystal sizes, shapes, and orientations are calculated by the following formulas:

$$\begin{aligned} \Sigma_s(\omega', r) &= \sum_{\nu=1}^m m_\nu \int_0^{2\pi} \int_0^\pi \int_0^{2\pi} \int_{\rho_{\min}}^{\rho_{\max}} P_{\Theta, \nu}(\alpha, \beta, \gamma) P_\nu(\rho_\nu) \Sigma_{s, \nu}(\omega'(\alpha, \beta, \gamma), r, \rho_\nu) d\rho_\nu d\alpha d\beta d\gamma, \\ \Sigma_a(\omega', r) &= \sum_{\nu=1}^m m_\nu \int_0^{2\pi} \int_0^\pi \int_0^{2\pi} \int_{\rho_{\min}}^{\rho_{\max}} P_{\Theta, \nu}(\alpha, \beta, \gamma) P_\nu(\rho_\nu) \Sigma_{a, \nu}(\omega'(\alpha, \beta, \gamma), r, \rho_\nu) d\rho_\nu d\alpha d\beta d\gamma, \quad (3) \\ G(\omega', \omega, r) &= \sum_{\nu=1}^m m_\nu \int_{\rho_{\min}}^{\rho_{\max}} P_\nu(\rho_\nu) \int_0^{2\pi} \int_0^\pi \int_0^{2\pi} P_{\Theta, \nu}(\alpha, \beta, \gamma) G_\nu(\omega'(\alpha, \beta, \gamma), \omega, r, \rho_\nu) d\alpha d\beta d\gamma d\rho_\nu. \end{aligned}$$

For particles shown in Fig. 3, the elements of the matrix G_{ijkl} of anisotropic distributions of scattering and the scattering cross-section $\Sigma_s(\omega')$ were calculated by the ray tracing method. For these particle models, because of the particle rotation about the horizontal axis, the scattering phase function $G(\omega', \omega)$ and the attenuation cross-section are independent of the azimuthal angle of incoming radiation; for these functions, we use the same designations $G(\theta', \theta, \varphi)$ and $\Sigma_s(\theta')$.

Figure 4 presents the coefficient $\Sigma_s(\mu')$ of radiation scattering by horizontally-oriented hexagonal ice columns and hexagonal columns the axis of which is randomly tilted relative to the horizontal plane with an angle in the interval $[0, \pi/12]$ versus the cosine of the angle μ' between the incoming radiation direction and the axis z .

3. TWO METHODS OF SIMULATING RADIATION TRANSFER IN CRYSTALLINE MEDIUM

Let us consider the problem of optical radiation transmission through a flat layer $0 \leq z \leq H$ of scattering and absorbing matter. The interaction of photons with the matter will be characterized

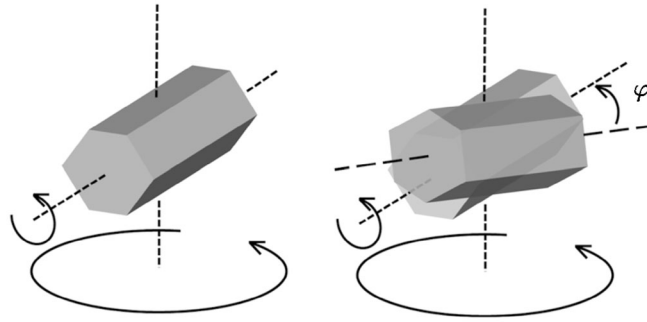


Fig. 3. Position and shape of ice particles: horizontal column (left) and column with predominantly horizontal orientation and flutter angle φ (right).

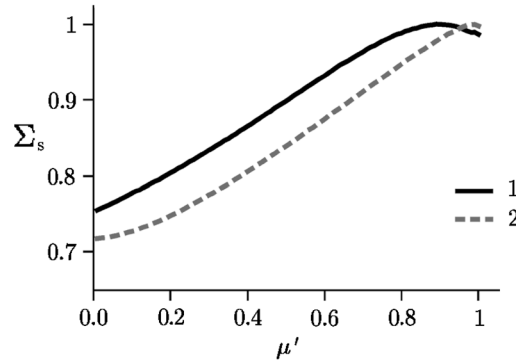


Fig. 4. Scattering coefficient $\Sigma_s(\mu')$ vs. cosine of incoming radiation zenith angle μ' for mostly horizontally oriented columns with random flutter angle within $[0, 15^\circ]$ (solid line 1) and horizontally oriented columns (bar line 2).

by the coefficients of attenuation $\Sigma_e(\omega', r)$, scattering $\Sigma_s(\omega', r)$, and absorption $\Sigma_a(\omega', r)$, as well as by the scattering phase function $G(\omega', \omega, r')$ normalized as follows:

$$\int_{\Omega} G(\omega', \omega, r') d\omega = 1.$$

Here ω' and ω is the unit vector of the photon motion direction before and after scattering, respectively. The boundary $z = 0$ of the flat layer $0 < z < H$, filled by crystalline particles, is illuminated by an infinitely wide light flux in the direction ω_0 . It is required to calculate the radiation absorption, transmission, and integral albedo by the cloud layer.

Photon propagation can be described by the integral transfer equation for the collision density $f(\omega, r)$ [16]:

$$f(\omega, r) = \int_X k(\omega', r', \omega, r) f(\omega', r') d\omega' dr' + \psi(\omega, r), \tag{4}$$

where $\psi(\omega, r)$ is the initial collision density at the point r in the direction ω . The kernel in Eq. (4) looks as follows:

$$k(\omega', r', \omega, r) = \frac{\Sigma_s(\omega', r')}{\Sigma_e(\omega', r')} G(\omega', \omega, r') \frac{\Sigma_e(\omega, r) e^{-\tau(r', r)}}{|r - r'|^2} \delta\left(\omega - \frac{r - r'}{|r - r'|}\right) \tag{5}$$

and is the density of probability of transition from the point (ω', r') to the point (ω, r) ; it defines the Markov chain of photon collisions with particles of the matter. In (5), $\tau(r', r)$ is the optical distance

from the point r' to the point r ; $\delta(\omega)$ is the Dirac delta function. For the case of a uniform optically anisotropic scattering layer ($\Sigma_e(\omega, r) = \Sigma_e(\omega)$), the optical distance $\tau(r', r) = \Sigma_e \left(\frac{r-r'}{|r-r'|} \right) |r-r'|$ depends not only on the distance between the points r and r' , but also on the direction $\omega = \frac{r-r'}{|r-r'|}$.

It is required to estimate linear functionals of solution to Eq. (4) of the form $J_\chi = (f(\omega, r), \chi(\omega, r))$, where $\chi(\omega, r) \geq 0$ is some non-negative function. For the problem stated in Eq. (4), the source function is defined by the expression $\psi(\omega, r) = \delta(z)\delta(\omega - \omega_0)$, where $r = (x, y, z)$, and the functions defining the albedo $\chi_a(\omega, r)$ and transmission $\chi_{tr}(\omega, r)$ of the crystalline layer are

$$\chi_{tr}(\omega, r) = \begin{cases} 1 & \text{at } z \geq H, \\ 0 & \text{otherwise,} \end{cases} \quad \chi_a(\omega, r) = \begin{cases} 1 & \text{at } z < 0, \\ 0 & \text{otherwise.} \end{cases}$$

Standard Monte Carlo method algorithms for solving Eq. (4) in scattering media require pre-calculation of the functions $\Sigma_e(\omega, r)$ and $G(\omega', \omega, r)$. For estimation of the functionals J_χ , photon trajectories are simulated in the form of polygonal lines with random lengths of the straight intervals and random angles of direction change at scattering points [16, 19]. The free path length $|r-r'|$ is simulated according to the probability density $\Sigma_e(\omega, r(t)) \exp\left(-\int_0^t \Sigma_e(\omega, r(t_1)) dt_1\right)$, where $r(t) = r' + t \frac{r-r'}{|r-r'|}$. If the photon has escaped from the medium, the trajectory breaks and a new trajectory is simulated. If the photon has gone into the half-spaces $z > H$ or $z < 0$, then the random value

$$W = \prod_{m=1}^M \frac{\Sigma_s(\omega'_m, r_m)}{\Sigma_e(\omega'_m, r_m)},$$

where r_M are the coordinates of the M th point of collision immediately before the escape and ω'_m is the direction of the photon motion before the m th collision, is recorded to the respective counters for the functionals $J_{\chi_{tr}}$ or J_{χ_a} . The new motion direction ω at collision with a particle provided that the previous direction of travel is equal to $\omega' = (\cos \varphi' \sin \theta', \sin \varphi' \sin \theta', \cos \theta')$ is simulated according to the distribution density, which is a linear or piecewise-constant approximation of the functions $G_{ij}(\psi, \nu)$, $G_{i+1j}(\psi, \nu)$, $G_{ij+1}(\psi, \nu)$, and $G_{i+1,j+1}(\psi, \nu)$, where $\theta' \in (\theta_i, \theta_{i+1})$ and $\varphi' \in (\varphi_j, \varphi_{j+1})$. A detailed description of this step of the algorithm is given in [21]. An algorithm for simulation of the photon motion direction after scattering in the case of the phase function $G(\omega', \omega, r)$ independent on the radiation azimuthal angle before scattering is given in [19].

The algorithm, in which the scattering angle is simulated from a phase function, has a number of limitations. The first one is the need for extensive pre-calculation of the functions $G(\omega', \omega, r)$ and $\Sigma_e(\omega, r)$ and thus significant computing resources and subsequent storage of a large data bank in the computer memory. The second limitation is that each change in the microphysical parameters of the scattering medium (for example, change in the concentration of different types of particles or consideration of different shapes, roughness of the crystal faces, or change in the radiation wavelength) necessitates recalculation of these functions.

Let us consider an alternative algorithm based on the combination of direct simulation of photon trajectories by the Monte Carlo method with the method of ray tracing in scattering angle simulation. In this algorithm, there is no need in pre-calculation of the scattering phase functions $G(\omega', \omega, r)$. The coefficients of attenuation $\Sigma_e(\omega, r)$ and scattering $\Sigma_s(\omega, r)$ are determined in advance subject to the incoming radiation direction ω for specified microphysical parameters of the scattering medium (concentration and orientation of crystals of different shapes) according to formulas (3). The free path length is simulated in accordance with the $\Sigma_e(\omega, r)$ values. At the next point of collision r_m , from the $\Sigma_s(\omega'_m, r_m)/\Sigma_e(\omega'_m, r_m)$ value, it is seen whether that was absorption or scattering of photon. In the case of scattering, the photon, which moves in the direction $\omega'_m(\theta_m, \eta_m)$, gets to the surface of the crystal. Its shape and spatial orientation are modeled according to the respective distributions $P_\nu(\rho_\nu)$ and $P_{\Theta, \nu}(\alpha, \beta, \gamma)$. The coordinates of the

intersection point of the photon path and crystal face can be determined by the following algorithm. The selected particle is placed in an inclusive sphere centered at the point r_m . The radius of the inclusive sphere R is selected such that the entire crystal resides inside it. In the plane $z = z_m$, a random point $(x'_\zeta, y'_\zeta, z'_\zeta)$ uniformly distributed in the circle $(x - x_m)^2 + (y - y_m)^2 \leq R^2$ is simulated until it gets in the crystal. Then the plane $z = z_m$ is rotated through the angle θ_m about the axis Ox and through the angle η_m about the axis Oz so that the vector ω'_m is parallel to the normal vector to the transformed plane. The new coordinates $(x_\zeta, y_\zeta, z_\zeta)$ of the points modeled are determined according to the formula

$$\begin{bmatrix} x_\zeta \\ y_\zeta \\ z_\zeta \end{bmatrix} = \begin{bmatrix} \cos \eta_m & -\sin \eta_m & 0 \\ \sin \eta_m & \cos \eta_m & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \theta_m & 0 & \sin \theta_m \\ 0 & 1 & 0 \\ -\sin \theta_m & 0 & \cos \theta_m \end{bmatrix} \begin{bmatrix} x'_\zeta \\ y'_\zeta \\ z'_\zeta \end{bmatrix}. \tag{6}$$

The point of intersection of the vector $-\omega'_m$ with a crystal face is determined in the way described in [22]. Further simulation of the direction of the photon motion inside the crystal obeys reflection and refraction laws (1) and (2) in the geometric optics approximation and is performed as by the scattering phase function algorithm [9, 17].

4. RESULTS OF NUMERICAL SIMULATION OF SOLAR RADIATION TRANSFER IN CRYSTAL CLOUDS

During the testing of the presented algorithms of radiation transfer in crystalline media, the following mixture of particles was considered: 20% of hexagonal columns with form factor of 10; 50% of plates—hexagonal prisms—with form factor of ~ 0.3 ; 30% of spheres. The particles were chaotically oriented.

Table 1. Probabilities of reflection R , absorption A , and transmission T of radiation by layer of optical thickness τ , calculated by standard algorithm, and their standard deviations $\varepsilon_R, \varepsilon_A$, and ε_T

τ	$R \pm \varepsilon_R$	$A \pm \varepsilon_A$	$T \pm \varepsilon_T$	$S_T \cdot 10^5$
1	0.06432 ± 0.00008	0.11364 ± 0.00010	0.822040 ± 0.00011	1.42
2	0.10822 ± 0.00009	0.23419 ± 0.00013	0.657590 ± 0.00014	4.35
3	0.13624 ± 0.00010	0.34706 ± 0.00014	0.516691 ± 0.00015	7.07
4	0.15345 ± 0.00011	0.44573 ± 0.00015	0.400818 ± 0.00015	8.69
5	0.16365 ± 0.00012	0.52830 ± 0.00015	0.308050 ± 0.00014	9.12
10	0.17721 ± 0.00012	0.74562 ± 0.00014	0.077161 ± 0.00008	4.27

Table 2. Probabilities of reflection R , absorption A , and transmission T of radiation by layer of optical thickness τ , calculated by algorithm without using scattering phase function

τ	$R \pm \varepsilon_R$	$A \pm \varepsilon_A$	$T \pm \varepsilon_T$	$S_T \cdot 10^5$
1	0.05357 ± 0.00023	0.11015 ± 0.00031	0.83628 ± 0.00037	5.45
2	0.09119 ± 0.00029	0.22567 ± 0.00042	0.68314 ± 0.00047	17.49
3	0.11606 ± 0.00032	0.33517 ± 0.00047	0.54878 ± 0.00050	29.64
4	0.13196 ± 0.00034	0.43246 ± 0.00050	0.43558 ± 0.00050	37.89
5	0.14199 ± 0.00035	0.51553 ± 0.00050	0.34247 ± 0.00047	40.83
10	0.15618 ± 0.00036	0.74803 ± 0.00043	0.09578 ± 0.00029	11.81

The radiation attenuation coefficient $\Sigma_e = 0.03 \text{ m}^{-1}$. A series of calculations was carried out. In each calculation, the optical thickness of the cloud layer was fixed and lay in the range from 1 to 10; the single scattering albedo was 0.9. The upper boundary of the layer was illuminated by a uniform steady-state radiation flux in the vertical direction. The probabilities of reflection R , transmission T , and absorption A by the cloud of ice particles were calculated in the assumption that $R + T + A = 1$. The calculations were made by the two algorithms described above.

Table 1 offers estimates of the coefficients R , A , and T and their standard deviations ε_R , ε_A , and ε_T for different optical thicknesses of the cloud layer τ , calculated by the algorithm for simulation of photon trajectories using the scattering phase function $G(\omega', \omega)$. The last column of the table shows the computation cost $S_T = t\sqrt{V\xi_T}$ of the transmission estimation. Here $V\xi_T$ is the T estimate variance and t is the average simulation time per trajectory.

Table 2 presents similar results obtained by the algorithm without using the scattering phase function. The number of photon trajectories for these calculations was equal to 10^8 . It can be seen that the computation cost of the second algorithm is several times greater than that of the algorithm using the scattering phase function. However, this does not take into account the considerable time required for calculation of the scattering phase function values with acceptable accuracy. Besides, this adds difficulties to the calculations, especially when the composition of the scattering layer is not homogeneous and depends on the geometric coordinates.

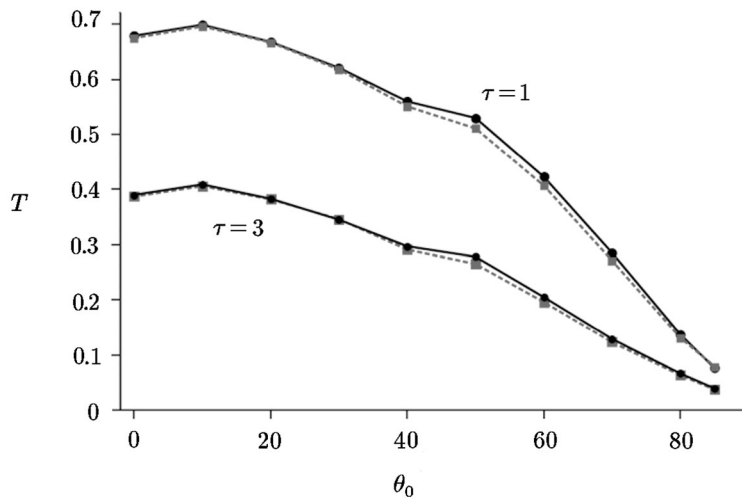


Fig. 5. Probability of radiation passing through flat scattering layers of optical thickness $\tau = 1$ and $\tau = 3$, consisting of horizontally oriented columns (solid lines) and columns with random flutter angle (bar lines) for various zenith angles θ_0 of incident radiation.

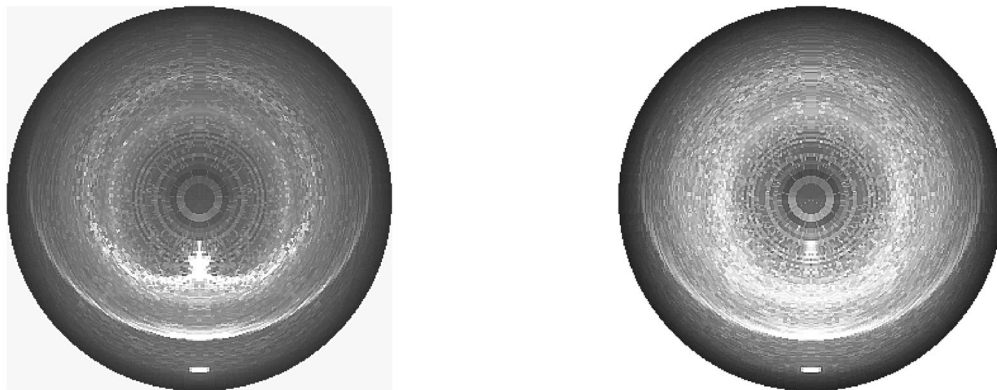


Fig. 6. Angular distribution of intensity of radiation passing through crystalline scattering layer of optical thickness $\tau = 1$. Left: Simulation 1, right: Simulation 2.

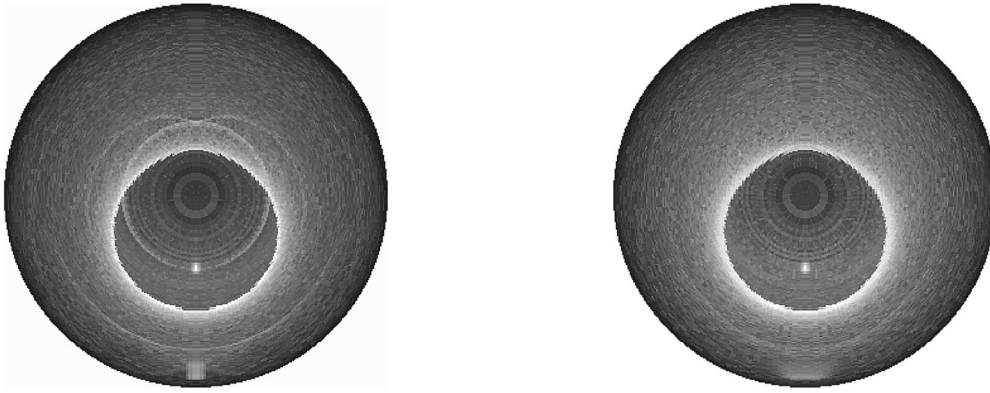


Fig. 7. Angular distribution of intensity of radiation reflected from crystalline scattering layer of optical thickness $\tau = 3$. Left: Simulation 1, right: Simulation 2.

In the following calculation series, the results for two compositions of the scattering layer are compared.

Simulation 1. The cloud consists of hexagonal prisms horizontally oriented in the space. The particle form factor is the same and equals 10.

Simulation 2. The particles from Simulation 1 are predominantly horizontally oriented, but the angle between the crystal axis and the horizontal plane is simulated randomly from a uniform distribution from the interval $[0, \pi/12]$.

For such scattering medium simulations, because of the symmetry in the horizontal plane, the scattering phase function $G(\omega', \omega)$ is independent of the azimuthal angle of the incoming radiation. It is assumed that no absorption occurs in the layer; the concentration of crystalline particles in the scattering layer is such that the maximum attenuation factor in all directions of the incoming radiation is equal to 0.005 m^{-1} ; the layer height is 200 m and 600 m. Figure 5 shows the probabilities of radiation passing through the scattering layer for different directions of the incoming radiation, which are defined by the zenith angle θ_0 . For a scattering medium simulation with flutter, the transmission is always slightly lower than that for a flutter-free simulation.

Figure 6 shows angular distributions of the intensity of radiation passing through a layer with optical thickness not exceeding $\tau = 1$. The left picture corresponds to Simulation 1, and the right one stands for Simulation 2. The direction of incoming radiation is defined by the zenith angle $\theta_0 = 80^\circ$. For the same models of ice crystals, Fig.7 shows the angular distribution of the intensity of the radiation reflected from the cloud layer. In this example, the maximum optical thickness of the layer is 3, the zenith angle of the direction toward the source $\theta_0 = 80^\circ$. In Figs. 6 and 7, it can be seen as the flutter leads to smoothing of the angular distributions of the intensity of the passing and reflected radiation.

CONCLUSIONS

The article describes an algorithm for calculation of scattering phase functions and scattering and absorption cross-sections for non-spheric particles whose sizes significantly exceed the radiation wavelength. In this case, the problem can be solved in the geometric optics approximation. Since large part of real crystal cloud particles are of irregular random shape, the authors propose a mathematical model of crystalline particles of random shape and an algorithm for modeling of such particles through construction of a convex hull of a set of random points distributed in some volume. The algorithm for calculating the attenuation cross-sections and scattering phase functions works for models of both regular and random particles.

Two approaches to simulation of radiation transfer in optically anisotropic cloud cover were considered. The first approach requires pre-computation of the scattering phase functions for crystals of different shapes and orientations. The second approach does not require knowledge of the scattering phase functions. In this case, during the construction of the photon trajectory, after simulating the free path length, the type and orientation of crystal are determined, and

scattering is simulated directly by imitation of the interaction of a photon with a particle according to the laws of geometric optics. This approach enables easy adjustment of input parameters of the problem to changes in the microphysical characteristics of the medium, including the particle shape, orientation, and opacity and roughness of their boundaries, and does not require time-consuming pre-calculations.

Simulation calculations of integral and angular characteristics of a radiation field scattered by a layer of ice clouds consisting of crystals in the form of hexagonal prisms with form factor of 10 were carried out. The results showed the effect of flutter for horizontally oriented particles on the radiation transmission by the cloud layer and the angular distribution of the reflected and transmitted radiation.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

REFERENCES

1. Volkovitskii, O.A., Pavlova, L.N., and Petrushin, A.G., *Opticheskie svoistva kristallicheskih oblakov* (Optical Properties of Crystal Clouds), Leningrad: Gidrometoizdat, 1984.
2. *Radiatsionnye svoistva peristykh oblakov* (Radiation Properties of Cirrus Clouds), Feigelson, E.M., Ed., Moscow: Nauka, 1989.
3. *Radiatsiya v oblachnoi atmosfere* (Radiation in the Cloud Atmosphere), Leningrad: Gidrometoizdat, 1981.
4. Mishchenko, M.I., Travis, L.D., and Lacis, A.A., *Scattering, Absorption, and Emission of Light by Small Particles*, Cambridge University Press, 2002.
5. Korolev, A.V., Isaac, G.A., and Hallett, J., Ice Particle Habits in Arctic Clouds, *Geophys. Res. Lett.*, 1999, vol. 26, no. 9, pp. 1299–1302.
6. Korolev, A.V., Isaac, G.A., and Hallett, J., Ice Particle Habits in Stratiform Clouds, *Quart. J. Royal Meteorol. Soc.*, 2000, vol. 126, pp. 2873–2902.
7. Liou, K.N. and Yang, P., *Light Scattering by Ice Crystals: Fundamentals and Applications*, Cambridge University Press, 2016.
8. Mishchenko, M.I., Hovenier, J.W., and Travis, L.D., *Light Scattering by Nonspherical Particles: Theory, Measurements and Geophysical Applications*, San Diego: Academic Press, 1999.
9. Macke, A., Johannes, M., and Ehrhard, R., Single Scattering Properties of Atmospheric Ice Crystals, *J. Atmosph. Sci.*, 1996, vol. 53, no. 19, pp. 2813–2825.
10. Liu, C., Panetta, R.L., and Yang, P., The Effective Equivalence of Geometric Irregularity and Surface Roughness in Determining Particle Single-Scattering Properties, *Optics Express*, 2014, vol. 22, no. 19, pp. 23620–23627; DOI:10.1364/OE.22.023620
11. Timofeev, D.N., Konoshonkin, A.V., Kustova, N.V., Shishko, V.A., and Borovoi, A.G., Estimation of the Absorption Effect on Light Scattering by Atmospheric Ice Crystals for Wavelengths Typical for Problems of Laser Sounding of the Atmosphere, *Atmosph. Oceanic Opt.*, 2019, vol. 32, no. 5, pp. 564–568.
12. Mu, Q., Kargin, B.A., and Kablukova, E.G., Computer Construction of Three-Dimensional Convex Bodies of Arbitrary Shapes, *Vych. Tekhnol.*, 2022, vol. 27, no. 2, pp. 54–61.
13. Konoshonkin, A., Kustova, N., and Borovoi, A., *Rasseyaniye sveta na geksagonal'nykh ledyanykh kristallakh peristykh oblakov* (Light Scattering by Hexagonal Ice Crystals of Cirrus Clouds), LAMBERT Academic Publishing, 2013.
14. Preparata, F.P. and Shamos, M.I., *Computational Geometry: An Introduction*, Springer Science & Business Media, 1985.
15. Berg, M., Kreveld, M., Overmars, M., and Schwarzkopf, O., *Computational Geometry: Algorithms and Applications*, Heidelberg: Springer, 1997.
16. Marchuk, G.I., Mikhailov, G.A., Nazarliev, M.A., Darbinyan, R.A., Kargin, B.A., and Elepov, B.S., *Monte Carlo Method in Atmospheric Optics*, Berlin, Springer-Verlag, 1989.

17. Kargin, B.A., Kablukova, E.G., and Mu, Q., Numerical Stochastic Simulation of Scattering of Optical Radiation by Ice Crystals of Irregular Random Shapes, *Vych. Tekhnol.*, 2022, vol. 27, no. 2, pp. 4–18.
18. Deirmendjian, D., *Electromagnetic Scattering on Spherical Polydispersions*, American Elsevier Publishing Company, 1969.
19. Prigarin, S.M., Borovoi, A.G., Grishin, I.A., and Opper, U.G., Statistical Simulation of Radiation Transfer in Optically Anisotropic Crystal Clouds, *Optika Atmosf. Okeana*, 2007, vol. 20, no. 3, pp. 205–210.
20. Korn, G.A. and Korn, T.M., *Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review*, Courier Corporation, 2000.
21. Kargin, B.A., Mu, Q., and Kablukova, E.G., Numerical Statistical Simulation of Optical Radiation Transfer in Random Crystalline Media, *Sib. El. Mat. Izv.*, 2023, vol. 20, no. 1, pp. 486–500.
22. Zhang, Z.B., Yang, P., Kattawar, G.W., et al., Geometrical-Optics Solution to Light Scattering by Droxtal Ice Crystals, *Appl. Optics*, 2004, vol. 43, no. 12, pp. 2490–2499; DOI:10.1364/AO.43.002490

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