ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2022, Vol. 43, No. 5, pp. 1110–1115. © Pleiades Publishing, Ltd., 2022.

Resonance Oscillations of Gas in a Closed Tube in the Presence of a Heterogeneous Temperature Field

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Received March 4, 2022; revised March 29, 2022; accepted April 6, 2022

Abstract—A numerical modeling of non-linear oscillations of gas in a closed tube in consideration of a heterogeneous temperature field was done. A dispersion of the resonance frequency of gas under various values of the heat source was found. Radial and axial distributions of oscillation amplitudes of gas velocity in a tube were obtained. It was demonstrated that at a set amplitude of the piston displacement, an increase of the temperature results in a growth of intensity of the gas oscillations.

DOI: 10.1134/S1995080222080145

Keywords and phrases: *non-linear oscillations, resonance, temperature field, closed tube.*

1. INTRODUCTION

The interest in the research of non-linear oscillations of a continuous medium is due to their wide spreading in a variety of fields of science and technology. Components of engines and power units comprise volumes constricted with solid walls, in which volumes phenomena occur that are accompanied with established oscillations of velocity, pressure, and temperature of gaseous combustion products. The fluctuating flow promotes intensification of a variety of processes of heat and mass transfer: an increase of the heat density in operating volume of power units [1], intensification of the heat exchange between particles and the plasma jet in the process of plasma coating [2], change of structure of the glow discharge [3]. There is a great interest in the thermosacoustic prospects of caloric engines and refrigerators that depend on strong oscillations of pressure and velocity of flow in presence of a temperature gradient. Such engines with an external heat delivery, where the heat energy is converted to acoustic energy, is one of methods to extract energy efficiently [4] due to a thermodynamic cycle that occurs.The paper [5] describes thermoacoustic effects in the channel caused by heated surfaces. An analysis of the temperature field in the resonator performed in [6, 7] showed that the rate of heat exchange is more dependent on second-order flows rather than on primary acoustic oscillations. However, within Rayleigh flows in resonators in consideration of heat conductivity and dependency of viscosity to the temperature, heat effects influence the acoustic flow insignificantly [8]. The effect of oscillations of the gas parameters on transfer processes only occurs in the near-wall area [9]. A high increase of the axial gradient of the temperature also leads to a disturbance of the shape of eddy currents [11]. The temperature gradient supported by the resonator wall also greatly changes the pattern of flow due to inertial effects [12]. In case of even relatively weak radial temperature gradients, a significant distortion of the acoustic flow profile and effect of the temperature gradient are observed to be much stronger than the effect of the liquid inertia on the flow [13]. As a part of the analytic theory, a structure of forced acoustic oscillations in presence of homogeneity of gas parameters is reviewed in the paper [14]. A velocity field in consideration of a heterogeneous gas heating was obtained.

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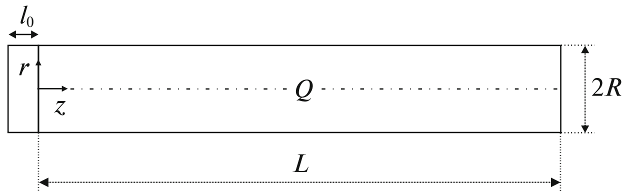

Fig. 1. Geometry of the problem.

Authors [15] showed that a prevailing strong temperature homogeneity throughout the radius of the tube is responsible for occurrence of acoustic currents affecting the transfer processes. Such processes are typical for acoustic oscillations in the plasma of a diffuse glow discharge lit in a cylindrical tube, where gas features a parabolic radial temperature field [16] under a low energy output. The results can be used in the design of thermoacoustic engines, exhaust ducts, combustion chambers, creating heat exchangers, cooling electronic components, creating gas lasers, manufacturing highly efficient resonators for acoustic compressors and developing methods for plasma application of functional coats. A paper [17] demonstrated a thermoacoustic effect under resonance oscillations of gas in a closed tube. It was identified that the highest gas temperature is observed in the area of pressure node, while the maximum cooling occurs in the central part of the tube in the pressure node. The tube is a device with a high efficiency of the energy transformation from an external source to a heat flow [18]. The operation of the source has a negative value in case of dissipation of the acoustic power, while a positive value means generation of the acoustic power. Within the linear acoustic theory, the dissipation of power is explained with a superficial attenuation on walls of the tube due to viscosity and heat conductivity of the gas [19]. The temperature difference is a key parameter in terms of cooling, as a greater heat drop is required in certain applications. Such property is used in the course of development of thermoacoustic refrigerators and is of a great significance in industry [20, 21]. According to research results, to avoid disintegration of certain parts of thermoacoustic units and undesired energy losses, it is necessary to optimize geometric and acoustic parameters of the heat-exchange systems. Therefore, temperature gradients exist in some resonators, which gradients emerge due to non-linear wave processes (thermoacoustic effect), chemical reactions (burning), or electricity flow through ionized gas (gas discharge). Examination of resonance gas oscillations under such conditions is a relevant objective being of an important practical significance. Taking above into consideration, more detailed researches of gas oscillations in a closed tube with a heat source are required.

2. THEORETICAL MODEL

Let us examine resonance oscillations of gas in a closed tube with the length of $L = 0.938$ m and the radius of $R = 0.05$ m (Fig. 1). Harmonic oscillations are excited by means of a flat piston located in one end of the tube, where the radius equals to the internal radius of the tube, and the other end of the tube is closed with a flat cover. As this problem is stated, gas is heated throughout the volume of the tube by means of an internal heat source Q.

Gas oscillations in resonator can be described with a linearized Navier–Stokes' combined equations [22, 23]. If we assume that the medium is an ideal gas, let us record equations of conservation of mass, momentum, and energy

$$i\omega\rho_1 + \nabla \cdot (\rho_0 \mathbf{u_1}) = 0, \tag{1}$$

$$i\omega\rho_0\mathbf{u_1} = \nabla \cdot \Bigg\{ -p_1\mathbf{I} + \mu[\nabla\mathbf{u_1} + (\nabla\mathbf{u_1})^{\mathsf{T}}] - \left(\frac{2\mu}{3} - \zeta\right)(\nabla \cdot \mathbf{u_1})\mathbf{I}, \tag{2}$$

$$\rho_0 c_p(i\omega T_1 + (\mathbf{u_1} \cdot \nabla)T_0) - (i\omega p_1 + (\mathbf{u_1} \cdot \nabla)p_0) = \nabla \cdot (\lambda\nabla T_1), \tag{3}$$

where ρ is the liquid density, $\mathbf{u}$ is the velocity vector, T is the temperature, p is the pressure, $\mathbf{I}$ is the identity matrix, c_p is the specific heat capacity at constant pressure, μ is the coefficient of dynamic viscosity, ζ is the coefficient of volume viscosity, λ is the thermal conductivity coefficient. Index 0 corresponds to the gas at rest, index 1 corresponds to the first (acoustic) approximation, symbol T in

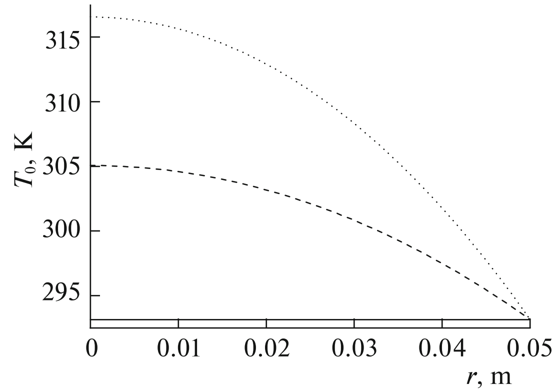

Fig. 2. Radial temperature distribution in a closed tube ($z = 0.5L$). The solid line means no heat source, the dash line means $Q = 0.5$ kW/m^3, dotted line means $Q = 1$ kW/m^3.

the upper index is the transposition operation. The equation of the ideal gas results in a dependence $\rho_1(p_1, T_1)$ as follows

$$\rho_1 = \rho_0 \left(\frac{p_1}{p_0} - \frac{T_1}{T_0} \right). \tag{4}$$

The heat inhomogenuity in the tube is considered by means of a steady energy equation with the heat source Q, which is recorded as follows

$$\nabla \cdot (\lambda \nabla T_0) = Q. \tag{5}$$

Navier–Stokes' combined equations (1)–(4) and the steady energy equation (5) are solved in a two-dimension statement of the problem by means of finite elements. The number of elements of the computational grid is 40149. At the first stage, to obtain the temperature field in a closed tube $T_0 = T_0(r, z)$, the equation (5) is solved with a first-kind boundary condition $T_0 = T_w$ at the piston ($z = 0$), the end wall ($z = L$) and the side wall ($r = R$). At the second stage, combined equations (1)–(4) are solved with boundary conditions $\mathbf{u_1} = 0$ and $T_1 = 0$ at the end wall ($z = L$) and the side wall ($r = R$). At the piston ($z = 0$), the boundary condition is set for the axial velocity component $u_{1z} = \omega l_0$, where l_0 is the piston displacement amplitude. It is assumed that gas oscillations do not affect the steady temperature field due to a short piston displacement amplitude.

3. RESULTS AND THEIR INTERPRETATION

Figure 2 illustrates the radial gas temperature distribution in the tube in the cross section $z = 0.5L$. In absence of a heat source, the temperature field is homogeneous. In case of $Q > 0$, a parabolic temperature field is observed. Dependence of the temperature T on the axis of the tube on the volume density of the heat flow Q is close to linear in the specified power range. For volume densities of the heat flow from 0.5 kW/m^3 to 1 kW/m^3, the temperature on the axis of the tube increases by 12 K and 23.4 K, respectively.

Figure 3 shows amplitude-frequency response (AFR) of gas oscillations in the section $z = 0$ obtained in the numeric experiment for the closed tube with a variety of heat sources Q. In all cases reviewed, the piston displacement amplitude l_0 equals to 0.1 mm. In absence of a heat source, the resonance frequency amounts to $f = 182.5$ Hz. Along with a temperature increase, a dispersion of the resonance frequency is observed, i.e. a shift towards an increase compared to the resonance frequency obtained from the calculation in lack of a heat source. It is specific for a case under review, as the resonance frequency is proportional to the speed of sound and, as a consequence, to the quadratic root of the temperature $\omega \sim c_0 \sim \sqrt{T_0}$.

Figure 4 presents radial distributions of oscillation velocity amplitude in the closed tube, in the core of the flow (cross section $z = 0.5L$). In absence of a heat source in the core of the flow ($R - 4.6\delta$,

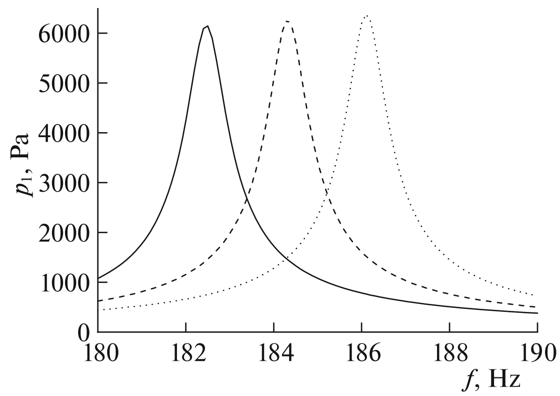

Fig. 3. AFR of the closed tube ($z = 0$). Solid line means no heat source, dashed line means $Q = 0.5$ kW/m^3, dotted line means $Q = 1$ kW/m^3.

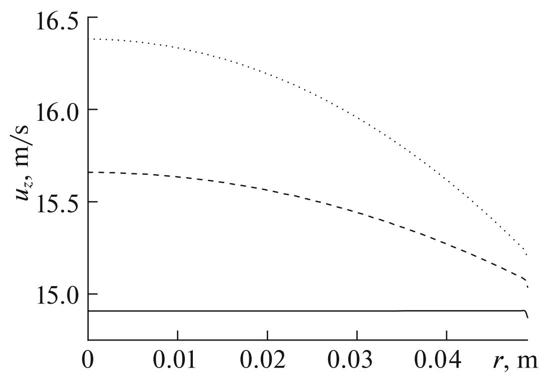

Fig. 4. Radial distribution of the amplitude of oscillations velocity in the closed tube, in the core of the flow ($z = 0.5L$). Solid line means no heat source, dashed line means $Q = 0.5$ kW/m^3, dotted line means $Q = 1$ kW/m^3.

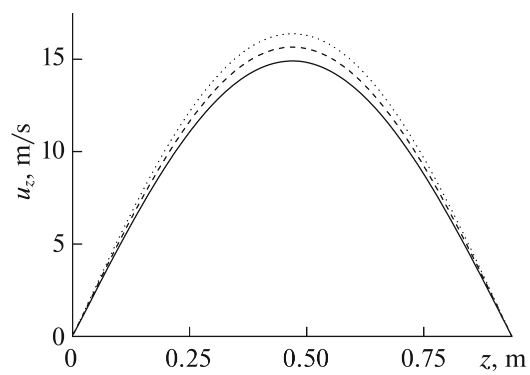

Fig. 5. Axial distribution of the amplitude of oscillations velocity in the closed tube ($r = 0$). Solid line means no heat source, dashed line means $Q = 0.5$ kW/m^3, dotted line means $Q = 1$ kW/m^3.

where $\delta = \sqrt{\frac{2\mu}{\rho_0\omega}}$ is the thickness of the acoustic boundary layer), a flat oscillation amplitude profile is observed for the axial velocity component u_z. In the case when gas oscillations are excited in the tube with an inhomogeneous heat distribution throughout the radius, the steady wave profile depends on the temperature field [14]. Due to distortion of phase planes, the velocity of longitudinal gas oscillations obtains a transversal component [15]. Due to the above reason, a parabolic radial profile of oscillations amplitude of the axial component of velocity u_z (Fig. 4) is observed in the core of the flow. Distributions of the velocity amplitude u_z along the axis of the tube are shown in (Fig. 5).

Analysis of outcomes of the numerical experiment shows (Figs. 3–5) that an increase of the heat inhomogeneity degree results in a greater intensity of gas oscillations at a set amplitude of the pistol displacement.

4. CONCLUSION

Gas oscillations in a closed tube, in an inhomogeneous temperature field near resonance excitement frequencies with a variety heat sources were examined. Amplitude-frequency responses, obtained when we solved numerically the linearized Navier–Stokes' combined equations were presented. A dispersion of the resonance frequency is observed in numerical experiments along with an increase of the volume density of the heat flow, which dispersion is conditioned with a proportionality of the resonance frequency and the speed of sound $\omega \sim c_0$. In the present a heat heat source is present in the tube, the oscillation amplitude of the axial component of the gas velocity obtains a parabolic radial profile. The calculation results demonstrate that an increase of the temperature leads to growth of intensity of gas oscillations at a set amplitude of the piston displacement.

FUNDING

This paper has been supported by the Kazan Federal University Strategic Academic Leadership Program ("PRIORITY-2030").

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