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Numerical Simulations of Subcooled Boiling Flow in Vertical Pipe at High Pressure

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Abstract—The results of numerical modeling of hydrodynamics and heat and mass transfer at boiling of subcooled liquids in conditions of forced flow in vertical heated pipes are presented. The mathematical model is based on the use of Euler's description of mass, motion and energy conservation for liquid and gas phases, recorded within the framework of the theory of interacting continua. The turbulent characteristics of the fluid are calculated using a modified model of transfer of components of the Reynolds stress tensor, taking into account the presence of the gaseous phase in the medium. For an approximate calculation of the heat transfer coefficient for bubble boiling of a liquid near the heat-generating wall, generalized empirical dependences are used, taking into account the various mechanisms of heat transfer in a two-phase vapor-liquid medium. Comparison of the results of numerical modeling with experimental data has shown that the proposed approach allows to simulate bubble boiling modes in a wide range of pressure values, mass flow rates, heating modes of subcooled liquids during forced turbulent fluid flow in vertical pipes.

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1. INTRODUCTION

Subcooled flow boiling denotes the process of evaporation of liquid flowing near a heated surface (usually channel wall), when the bulk flow temperature is lower than the local saturation temperature. The liquid temperature near the heated surface exceeds the saturation temperature and then gradually decreases (below saturation temperature) as the distance from the surface increases. Although subcooled boiling may appear in the form of different boiling regimes, the present work deals only with nucleate subcooled boiling, in which liquid evaporates in the form of vapour bubbles and a two-phase layer occurs near the heated surface. Among many industrial systems, subcooled flow boiling is important in water-cooled nuclear reactors, where the presence of vapour influences the system reactivity. The increased interest to investigate subcooled boiling, which has appeared in recent years, is mainly due to the need to perform safety analyses in the nuclear and chemical industry [1, 2]. Boiling fluids allow large heat fluxes to be dissipated at low temperature heads, and the need to intensify heat transfer during boiling occurred much later, than when using single-phase coolants. However, the desire to optimize the heat dissipation process, to increase the reliability and efficiency of heat exchangers (and, first of all, of power steam generators) forced to engage in a comprehensive study of heat transfer during boiling water and a number of other liquids [3–5].

Many experiments on subcooled boiling flow in channels have been performed over the past decades. On the basis of modern ideas about the boiling mode, a fairly extensive number of papers were published in which various criterion dependences were obtained, which allowed to describe as hydraulic resistances, and heat transfer during bubble boiling [6, 7]. The intensity of steam condensation in the

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flow of subcooled to the state of saturation of the liquid is one of the determining parameters in the design of heat-exchanging devices of mixing type, the analysis of processes occurring in the descending sections of natural circulation circuits, etc. In [8] the vapor volume fraction at boiling of subcooled liquid in a 1.4 m pipe with step heating was measured. Steam generated in the heated section of the experimental channel up to a height of 1 m was condensed in the downstream adiabatic section. Questions of physical and theoretical modeling of boiling up various liquids and heat-transfer processes in two-phase bubbly flows are devoted to [9, 10]. An experimental and numerical study of the effect of channel inclination on the change in friction voltage on the wall and heat transfer in a two-phase bubble flow in a rectangular channel was carried out in [9]. The similarity of the behavior of friction and heat transfer with a change in the angle of inclination of the channel and the volumetric gas content is obtained. In [11], the results of an experimental study and mathematical modeling of the process of boiling up of subcooled water and alcohol under pulsed heat release in a heater wall are given.

For mathematical modeling of two-phase gas-liquid flows with phase transitions, different approaches are used on the euler representation of each phase, including the description of heat and mass transfer between the phases [12–14]. Among multidimensional theoretical descriptions of subcooled boiling, the most widely used approach so far appears to be two-fluid modelling. In [15] the models for gas-liquid flow, which take into account the difference in pressure in the liquid and gas phases in the modeling of the processes of interphase interaction are presented. In this paper, the model proposed in [17] is used, where it is shown that the two-phase bubble flow can be considered as a gas-liquid system with one modified pressure. Namely, although the generic features of the two-fluid model are the same, many closure relations describing mass, momentum and energy exchange at the gasЏliquid interface do not apply to both high-pressure and low-pressure conditions.

Despite the large number of works in this field, the average characteristics of the turbulent vapor-liquid flow in various column apparatuses of chemical production taking into account turbulent pulsations of the carrier medium, as well as the processes of crushing, merging, condensation and evaporation, typical for vapor-liquid media, are not fully studied. This applies, above all, to the influence of a large number of hydrodynamic parameters on the flow structures of the carrier and dispersed phases in the multiphase flow. This work is devoted to the verification of the developed algorithm for the calculation of gas-liquid flows on the example of numerical study of the turbulent boiling flow of subcooled liquid in a vertical pipe, taking into account the processes of condensation and evaporation.

2. MODEL DESCRIPTION

For mathematical description of gas-liquid medium motion Euler's two-liquid approach based on the model of interpenetrating and interacting continua is used, which is reduced to describing the conditions of the separated phase motion and defining the values describing the interphase interaction. The system of Reynolds of the Navier–Stokes equations for the flow of gas-liquid flow, taking into account the influence of the gas phase on the processes of transfer in the liquid phase, looks like [2, 17, 18]

$$\frac{\partial}{\partial t}(\rho_k\alpha_k) + \nabla\cdot(\rho_k\alpha_k\mathbf{u}_k) = \dot{m}_{lg} - \dot{m}_{gl}, \quad \alpha_l + \alpha_g = 1, \quad k = l, g, \tag{1}$$

$$\begin{aligned}\frac{\partial}{\partial t}(\rho_k\alpha_k\mathbf{u}_k) + \nabla\cdot(\rho_k\alpha_k\mathbf{u}_k\mathbf{u}_k) = -\alpha_k\nabla P + \alpha_k\rho_k\mathbf{g} + \nabla\cdot\left(\alpha_k\mu_{eff,k}\left[\nabla\mathbf{u}_k + (\nabla\mathbf{u}_k)^T\right]\right)\\ + \mathbf{M}_{lg} + \dot{m}_{lg}\mathbf{u}_l - \dot{m}_{gl}\mathbf{u}_g,\end{aligned} \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_k\alpha_k h_k) + \nabla\cdot(\rho_k\alpha_k\mathbf{u}_k h_k) = \nabla\cdot[\alpha_k\lambda_k\nabla T_k] + \dot{m}_{lg}h_l - \dot{m}_{gl}h_g + Q_{lg}. \tag{3}$$

In the equation system (1)–(3) t is time, ϱ_l and ϱ_g are continuous and dispersed phase densities respectively, α_l, α_g are volumetric concentrations of gas and bearing phase, λ_k is heat conductivity coefficient of k phase, $\mathbf{g}$ is gravity acceleration vector, $\mathbf{u}_k = u_{1k}i + u_{2k}j + u_{3k}k$ is velocity vector k-phases, P is carrying phase pressure, $\mu_{eff,k}$ is effective dynamic the viscosity of the k phase, $\mathbf{M}_{lg}$ is vector of interphase interaction force, h_k is enthalpy of k phase, T_k is temperature of k phase, Q_{lg} is heat transfer between phases, $\dot{m}_{lg}$ is mass transfer rate between the phases of l and g.

To simulate turbulence, the Reynolds stress transfer model is used, which includes the effective viscosity of the medium $\mu_{eff,l}$, determined by the ratio $\mu_{eff,l} = \mu_{lam,l} + \mu_{t,l} + \mu_{BI,l}$. To describe the additional dissipation of the kinetic energy of turbulence by the pulsation of the bubbles, the viscosity of $\mu_{BI,l}$ is entered [2, 18]. For calculation of $\mu_{eff,l}$ applies the Kolmogorov formula, and for $\mu_{BI,l}$ the ratio from [2] is applied

$$\mu_{t,l} = \frac{C_\mu \varrho_l k_l^2}{\varepsilon_l}, \quad \mu_{BI,l} = C_{\mu b}\rho_l \alpha_l d_s \left|\mathbf{u}_g - \mathbf{u}_l\right|, \quad \mu_{\text{eff},g} = \frac{\varrho_g}{\varrho_l}\mu_{\text{eff},l}, \quad C_{\mu b} = 0.6. \tag{4}$$

To determine the kinetic energy of the turbulence of k_l and the dissipation rate of ε_l of the liquid phase, a two-parameter model of turbulence of $k-\varepsilon$ is applied, modified for two-phase media [2, 18]:

$$\frac{\partial\left(\rho_l \alpha_l k_l\right)}{\partial t} + \nabla\cdot\left(\rho_l \alpha_l k_l \mathbf{u}_l\right) = -\nabla\left(\alpha_l \frac{\mu_{eff,l}}{\sigma_k}\nabla k_l\right) + \alpha_l\left(G_{k,l} - \rho_l \varepsilon_l\right), \tag{5}$$

$$\frac{\partial\left(\rho_l \alpha_l \varepsilon_l\right)}{\partial t} + \nabla\cdot\left(\rho_l \alpha_l \varepsilon_l \mathbf{u}_l\right) = -\nabla\left(\alpha_l \frac{\mu_{eff,l}}{\sigma_k}\nabla \varepsilon_l\right) + \alpha_l \frac{\varepsilon_l}{k_l}\left(C_{\varepsilon 1} G_l - C_{\varepsilon 2}\rho_l \varepsilon_l\right), \tag{6}$$

$$G_l = \mu_{\text{eff},l}\nabla\mathbf{u}_l\left[\nabla\mathbf{u}_l + (\nabla\mathbf{u}_l)^T\right] - \frac{2}{3}\nabla\mathbf{u}_l\left(\mu_{\text{eff},l}\nabla\mathbf{u}_l + \rho_k k_l\right).$$

To determine the value of the turbulent viscosity of the gas phase, the next ratio is used

$$\mu_{t,g} = \frac{\rho_g}{\rho_l}\frac{\mu_{t,l}}{\mathrm{Pr}_\sigma}. \tag{7}$$

Constant turbulence models have the following values:

$$C_\mu = 0.09, \quad C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.92, \quad \sigma_k = 1.0, \quad Pr_\sigma = 0.9. \tag{8}$$

The total interphase interaction force $\mathbf{M}_{lg}$ plays a very important role when modeling multiphase flows [2]. In the present paper, it is the sum of the resistance force $\mathbf{M}_{lg}^D$, lifting force $\mathbf{M}_{lg}^L$, wall force $\mathbf{M}_{lg}^W$, attached force $\mathbf{M}_{lg}^{VM}$:

$$M_{li} = -M_{gi} = -\left(M_{gi}^D + M_{gi}^L + M_{gi}^W + M_{gi}^{VM}\right). \tag{9}$$

In equation (1)–(3) the term $\dot{m}_{lg}$ determines the speed of mass transfer from the phase l to the phase g, while the term $\dot{m}_{gl}$ expresses the speed the mass transfer from g to l. Inside the flow rate of mass transfer between the two phases depends on the temperature of the liquid. When the liquid is underheated, there is a volumetric condensation from the gas phase into the liquid, in turn, when the liquid is overheated, there is a volumetric evaporation of the liquid into the vapor. Mass transfer from liquid to vapor on the wall due to evaporation occurs directly due to the heat transfer of Q_e due to evaporation. Then the intensity of both these processes can be expressed through the interphase coefficient of heat transfer ξ_{lg} and the latent heat of the phase transition of the medium L_{fg} in the form

$$\dot{m}_{lg} = \frac{\xi_{lg} A_{lg}(T_{sat} - T_l)}{L_{fg}} + \frac{Q_e}{L_{lg} + C_{p,l}(T_{sat} - T_l)}. \tag{10}$$

In (10) the term A_{lg} is the interface area. For definition of ξ_{lg} the expressions received in works [2], when the heat transfer coefficient is determined by the correlation ratio for the Nusselt number by evaporating water bubbles into the surrounding dry air through

$$\xi_{lg} = Nu_b \frac{\lambda_l}{d_b} = \frac{\lambda_l}{d_b}(2 + 0.6 Re_b^{0.5} Pr^{0.33}),$$

where dimensionless parameters of Reynolds' number Re_b, Prandtl Pr are defined as $Re_b = \mu_l \rho_l |\mathbf{u}_l|/d_b$, $Pr = \mu_l C p_l/\lambda_l$, d_b is average bubble diameter. The interface area of A_{lg} is define with $A_{lg} = 6\alpha_g/d_b$.

Heat flow to the saturated liquid and realized in the process transformations of liquid into vapor, it is possible to present as the sum of $Q_w = Q_q + Q_e + Q_c$ [19]. Here Q_c is heat dissipated due to forced convection at the surface elements, where the zone was not formed yet with bubbles of steam. From the surface elements underneath the steam bubbles, the heat flow is mainly disposed of with steam bubbles (in the form of evaporation heat Q_e), as well as in the form of excess enthalpy of superheated liquid pushed

Table 1. Characteristics of mode parameters of experiments [8]

	P, MPa	G_l, kg/m^2 s	q_W, MW/m^2	T_{sat}, K	$T_{2,IN}$, K	ΔT, K
B3	6.89	1500.0	0.8	558	519	39
B7	6.89	1000.0	0.8	558	534	24

out of the wall layer by steam bubbles during their growth and at the distance from the heat-emitting surface of Q_q. To determine Q_q, Q_e, Q_c the ratios from [19, 20] are used.

The system of equations (1)–(10) describe the hydrodynamics and heat and mass transfer in the movement of steam-gas-liquid mixture in thermal power engineering apparatuses of chemical technology.

3. RESULTS AND DISCUSSION

The verification of the presented mathematical model was carried out by comparing the results of calculations with experimental data. The data of experiments presented in [8] were chosen, in which the boiling ascending bubble flow of subcooled liquid at high pressure in a tube with step heating (diameter of the pipe D = 12.0 mm, height H = 1400 mm, height of the heated section 1000 mm) was studied. In the table the basic characteristics of gas-liquid flow at the inlet of the pipe are presented: G_l is mass flow rate of liquid at the inlet of the pipe, P is pressure, q_W is heat flux on the wall of the pipe, T_{sat} is saturation temperature of liquid, $T_{2,\,IN}$ is water temperature at the inlet, ΔT is degree of subcooling of liquid. Two-phase flow in a vertical pipe is assumed to be axisymmetric, therefore for numerical simulation the calculated the area consisting of a circular sector with a radius of $r_0 = d/2 = 6.0 \times 10^{-3}$ m, a length of $L = 1.4$ m and a solution angle of 5 degrees. Numerical calculations were performed on finite volume grids consisting of $N_e = 32000$, 84000, 160000 nodes of the computational grid. In the section of the plane x_1x_2, the number of partitions by coordinates along the pipe axis and by length for different grids is $N_1 = 20 \times 200$, $N_2 = 40 \times 400$, $N_3 =$ 60x600, respectively. The numerical solution on the N_3 grid differs from N_2 less than by 2%, so all the above numerical results are performed using a grid of finite volumes N_2. Input and output boundary conditions are set at the input part and the output circular sector, the conditions of adhesion on the channel wall for the liquid and gas phase, heat flux on the part of the outer wall boundary, symmetry conditions for both phases on the side walls of the sector.

The following parameters of the water-steam two-phase components are set: density $\rho_l =$ 770 kg/m^3, $\rho_g = 61.0$ kg/m^3, dynamic water viscosity coefficient $\mu_l = 9.9 \times 10^{-5}$ Pa $\cdot$ s, vapour $\mu_g = 2.0 \times 10^{-5}$ Pa $\cdot$ s, the latent heat of vaporization is $L_{lg} = 1.3 \times 10^6$ J/kg, constant pressure heat capacity for liquid and gas $C_{pl} = 4990$ J/kg K, $C_{pg} = 7000$ J/kg K, coefficients of heat conductivity of medias $k_l = 0.59$ W/m K, $k_g = 0.08$ W/m K. At boiling of subcooled water in pipes on regularities of boiling in the conditions of free movement influence of movement of the directional flow of the vapor-liquid mixture is imposed. Due to the subcooling of the liquid at the inlet and the presence of heat supply to the walls, the water is heated to boiling point at the initial section of the pipe. You can distinguish the section of the pipe where the visible boiling starts and the distance from the entrance of the pipe before this section we select as a heating zone. At the boiling zone (from the boiling point to the output section) due to the heat supply there is a continuous increase in the steam content of the vapor-liquid mixture moving up to maximum values in the upper section of the pipe. The liquid is accelerated in the two-phase flow of the vapor-liquid system. The acceleration depends on the heat load and the pressure in the pipe. At high loads or low pressures, the acceleration of the liquid can be very significant. Figure 1 shows the distribution of the average volume steam content of $\langle \alpha_g \rangle$ along the pipe in comparison with the experimental the results of the work [8]. This graph illustrates the nature of the changes in the steam bubbles after separation from the heating surface in depending on the degree of subcooling of the liquid at the inlet.

This figure shows that the liquid at the inlet is more underheated, so most of the liquids in the central part of the pipe are underheated over the entire height of the pipe. After detachment and demolition into bubbles in the main underheated stream, vapor condensation will take place on the surface of the liquid enclosing the bubbles. The greatest condensation of steam occurs in the upper an unheated part of the

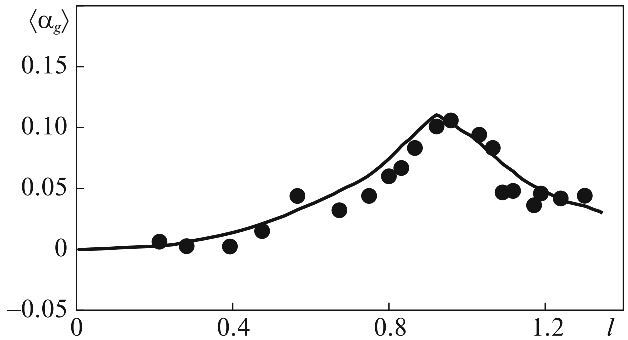

Fig. 1. Comparison of the distribution of the average volumetric gas content $\langle \alpha_g \rangle$ along the length of the pipe for B3 mode (line—calculation, experiment [8]—symbols).

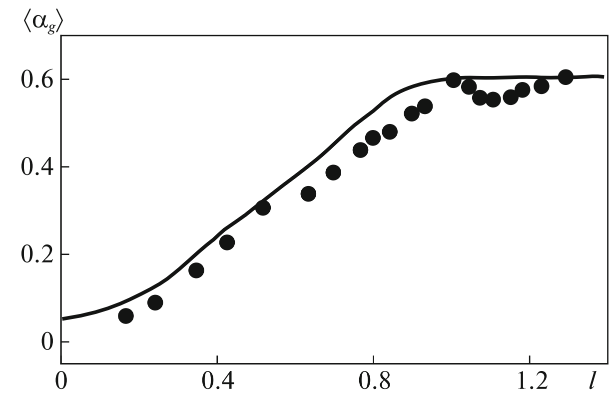

Fig. 2. Comparison of the distribution of the average volumetric gas content $\langle \alpha_g \rangle$ along the length of the pipe for B7 mode (line—calculation, experiment [8]—symbols).

pipe, resulting in a monotonous reduction in this section. Figure 2 shows a similar distribution at similar flow regime parameters, but at a lower degree of underheating of the liquid at the inlet. Since the liquid heats up to the saturation temperature at a lower pipe height, the carried bubbles continue to grow due to evaporation from the surface of the bubble volume. And since in the upper unheated part of the channel, the surrounding conditions are close to adiabatic, the steam bubbles do not change in size. This leads to the fact that $\langle \alpha_g \rangle$ practically does not change in height in the outlet part of the pipe.

4. CONCLUSION

The Euler's two-liquid model for numerical modeling of hydrodynamics and heat and mass transfer at boiling of subcooled liquids in conditions of forced flow in vertical heated pipes is presented. The proposed mathematical model allows to carry out calculations for the velocity and pressure fields of gas-liquid mixture, to determine the distribution of the volume phase of vapour and the average diameter of bubbles in the entire flow region. The analysis of the influence of distribution of average volume gas content on the pipe height is carried out. The specific features are revealed and the patterns of behavior of the steam-liquid medium for different degrees of heating are obtained. On the basis of comparison of the results of the numerical model with the experimental data at bubble boiling of the unheated liquid in a vertical pipe with a stepwise heating, applicability of the proposed model for numerical modeling of the boiling steam-liquid flows to a wide range of values of the parameters determining the movement of the medium is shown.

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