ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2016, Vol. 37, No. 6, pp. 787–793. © Pleiades Publishing, Ltd., 2016.
Original Russian Text © L.U. Sultanov, 2015, published in Uchenye Zapiski Kazanskogo Universiteta, Seriya Fiziko-Matematicheskie Nauki, 2015, Vol. 157, No. 4, pp. 158–165.

Analysis of Finite Elasto-Plastic Strains. Medium Kinematics and Constitutive Equations

L. U. Sultanov*

Kazan (Volga Region) Federal University, Kremlevskaya ul. 18, Kazan, Tatarstan, 420008 Russia

Received August 4, 2015

Abstract—The paper puts forwards principal kinematic relations and constitutive equations, which can be applied in designing numerical methods of study of finite elasto-plastic strains. The medium kinematics is considered under the multiplicative decomposition of the total deformation gradient. The constitutive equations are deduced using the theory of flow and the second law of thermodynamics. As a result, we find the dependence of the stress tensor rate on the free energy function and on the yield function.

DOI: 10.1134/S1995080216060032

Keywords and phrases: *nonlinear elasticity, finite strains, plasticity.*

INTRODUCTION

An increasingly popular technique is to use the multiplicative decomposition of the gradient of deformations in the study if finite elasto-plastic, visco-elastic or visco-plastic strains. With such expansion there are several ways of construction of the study algorithm. First of all, this is the choice problem of a basis relative to which the process of strain will be studied. In the present paper the basic relations are written with respect to the actual (deformed) state. Note that numerical algorithms for the study of solids for hyperelastic material were constructed in [1–4] and methods of solution of elasto-plastic problems with additive expansion of the strain rates were constructed in [5–8].

1. KINEMATICS

We shall assume that the medium under consideration admits finite strains and is in an elasto-plastic state. The gradient of strains [9, 10] is used as the principal kinematic characteristic; the tensor of the gradient of total deformations is represented as the product of the gradient of elastic and plastic deformations:

$$\mathbf{F} = \mathbf{F}_e \cdot \mathbf{F}_p. \tag{1}$$

Such representation is widely useful in solving nonlinear problems of mechanics of deformable bodies and is considered as fairly (mathematically) rigorous. As an example we mention the papers [11–15].

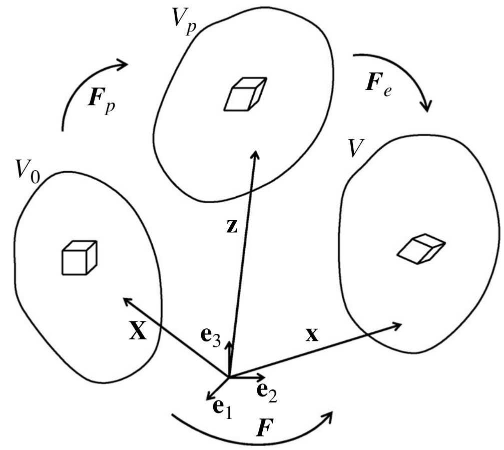
Using expansion (1) means introduction of some deformed state, which is admissible in the medium microvolume near a material point under consideration when removing the stresses. In other words, this is a singling out of the residual strains appeared due to plastic (viscous) flow. A geometric illustration is given in the figure, which shows the radius vector of the material point in the undeformed state, to which there corresponds the volume V_0,

$$\mathbf{X} = X_i\mathbf{e}_i,$$

the radius vector of the same material point in the actual configuration, which corresponds to the volume V,

$$\mathbf{x} = x_i\mathbf{e}_i,$$

*E-mail: Lenar.Sultanov@kpfu.ru



Kinematics of finite strains.

the radius vector of the material point in the unloaded condition,

$$\mathbf{z} = z_i \mathbf{e}_i.$$

With the use of this parameters one may write expansion (1) in the form

$$\mathbf{F} = \frac{\partial x_i}{\partial X_k} (\mathbf{e}_i \mathbf{e}_k) = (\mathbf{e}_i \mathbf{e}_k) \frac{\partial x_i}{\partial z_j} \frac{\partial z_j}{\partial X_k} = \frac{\partial x_i}{\partial z_j} (\mathbf{e}_i \mathbf{e}_j) \cdot (\mathbf{e}_m \mathbf{e}_k) \frac{\partial z_m}{\partial X_m} = \mathbf{F}_e \cdot \mathbf{F}_p.$$

For each state, we introduce the corresponding tensors of the strain measures:
—the right Cauchy–Green tensor (the Cauchy–Green strain measure)

$$\mathbf{C} = \mathbf{F}^T \cdot \mathbf{F}, \quad \mathbf{C}_e = \mathbf{F}_e{}^T \cdot \mathbf{F}_e, \quad \mathbf{C}_p = \mathbf{F}_p{}^T \cdot \mathbf{F}_p;$$

—the left Cauchy–Green tensor (the Cauchy–Green strain measure)

$$\mathbf{B} = \mathbf{F} \cdot \mathbf{F}^T, \quad \mathbf{B}_e = \mathbf{F}_e \cdot \mathbf{F}_e{}^T, \quad \mathbf{B}_p = \mathbf{F}_p \cdot \mathbf{F}_p{}^T;$$

—the left Piola tensor (the Almansi strain measure):

$$\mathbf{B}^{-1} = \mathbf{F}^{-T} \cdot \mathbf{F}^{-1}, \quad \mathbf{B}_e^{-1} = \mathbf{F}_e^{-T} \cdot \mathbf{F}_e^{-1}, \quad \mathbf{B}_p^{-1} = \mathbf{F}_p^{-T} \cdot \mathbf{F}_p^{-1};$$

—the right Piola tensor:

$$\mathbf{C}^{-1} = \mathbf{F}^{-1} \cdot \mathbf{F}^{-T}, \quad \mathbf{C}_e^{-1} = \mathbf{F}_e^{-1} \cdot \mathbf{F}_e^{-T}, \quad \mathbf{C}_p^{-1} = \mathbf{F}_p^{-1} \cdot \mathbf{F}_p^{-T}.$$

There are certain relations between the above tensors; we give only those that will be used later:

$$\mathbf{B}_e = \mathbf{F} \cdot \mathbf{C}_p^{-1} \cdot \mathbf{F}^T, \quad \mathbf{C} = \mathbf{F}_p{}^T \cdot \mathbf{C}_e \cdot \mathbf{F}_p.$$

In what follows we shall adopt the theory of plastic flow, which is traditionally described in terms of time derivatives. So, we first introduce the principal tensors employed in the description of the kinematics of flow for a continuous medium. The tensor of spatial velocity gradient is basic one; it is written in the form

$$\mathbf{h} = \dot{\mathbf{F}} \cdot \mathbf{F}^{-1}. \tag{2}$$

The symmetrical part of this tensor is called the deformation rate tensor

$$\mathbf{d} = \frac{1}{2} \left[\mathbf{h} + \mathbf{h}^T \right]; \tag{3}$$

the skew-symmetric part is known as the spin rate tensor

$$\boldsymbol{\omega} = \frac{1}{2} \left[\mathbf{h} - \mathbf{h}^T \right]. \tag{4}$$

The following relations hold:

$$\dot{\mathbf{C}} = 2\mathbf{F}^T \cdot \mathbf{d} \cdot \mathbf{F}, \quad \dot{\mathbf{B}} = \mathbf{h} \cdot \mathbf{B} + \mathbf{B} \cdot \mathbf{h}^T,$$

$$\mathbf{d} = -\frac{1}{2}\mathbf{F} \cdot \dot{\mathbf{C}}^{-1} \cdot \mathbf{F}^T, \quad \dot{\mathbf{B}}^{-1} = -\mathbf{B}^{-1} \cdot \mathbf{h} - \mathbf{h}^T \cdot \mathbf{B}^{-1}.$$

In accordance with expansion (1) we introduce the analogues of the tensors (2)–(4) for elastic and plastic rates of strains $\mathbf{h}_e$, $\mathbf{d}_e$, $\boldsymbol{\omega}_e$, $\mathbf{h}_p$, $\mathbf{d}_p$, $\boldsymbol{\omega}_p$. The above tensors are related as follows:

$$\mathbf{h} = \left[\dot{\mathbf{F}}_e \cdot \mathbf{F}_p + \mathbf{F}_e \cdot \dot{\mathbf{F}}_p\right] \cdot \left[\mathbf{F}_p^{-1} \cdot \mathbf{F}_e^{-1}\right] = \mathbf{h}_e + \mathbf{F}_e \cdot \mathbf{h}_p \cdot \mathbf{F}_e^{-1},$$

$$\mathbf{F} \cdot \dot{\mathbf{C}}_p^{-1} \cdot \mathbf{F}^T = -\mathbf{F} \cdot \mathbf{C}^{-1} \cdot \dot{\mathbf{C}}_p \cdot \mathbf{C}_p^{-1} \cdot \mathbf{F}^T$$

$$= -\mathbf{F}_e \cdot \mathbf{F}_p \cdot \mathbf{F}_p^{-1} \cdot \mathbf{F}_p^{-T} \cdot \left[\dot{\mathbf{F}}_p^T \cdot \mathbf{F}_p + \mathbf{F}_p^T \cdot \dot{\mathbf{F}}_p\right] \cdot \mathbf{F}_p^{-1} \cdot \mathbf{F}_p^{-T} \cdot \mathbf{F}_p^T \cdot \mathbf{F}_e^T$$

$$= -\mathbf{F}_e \cdot \mathbf{h}_p^T \cdot \mathbf{F}_e^T - \mathbf{F}_e \cdot \mathbf{h}_p \cdot \mathbf{F}_e^T = -2\mathbf{F}_e \cdot \mathbf{d}_p \cdot \mathbf{F}_e^T,$$

$$\dot{\mathbf{B}}_e = \dot{\mathbf{F}} \cdot \mathbf{C}_p^{-1} \cdot \mathbf{F}^T + \mathbf{F} \cdot \dot{\mathbf{C}}_p^{-1} \cdot \mathbf{F}^T + \mathbf{F} \cdot \mathbf{C}_p^{-1} \cdot \dot{\mathbf{F}}^T = \mathbf{h} \cdot \mathbf{B}_e + \mathbf{B}_e \cdot \mathbf{h}^T - 2\mathbf{F}_e \cdot \mathbf{d}_p \cdot \mathbf{F}_e^T.$$

The generalized derivative

$$\overset{\nabla}{\mathbf{B}}_e = \dot{\mathbf{B}}_e - \mathbf{h} \cdot \mathbf{B}_e - \mathbf{B}_e \cdot \mathbf{h}^T \tag{5}$$

satisfies the following relations

$$\overset{\nabla}{\mathbf{B}}_e = \mathbf{F} \cdot \dot{\mathbf{C}}_p^{-1} \cdot \mathbf{F}^T = -2\mathbf{F}_e \cdot \mathbf{d}_p \cdot {\mathbf{F}_e}^T.$$

Let us give some comments on the above relations. First, one should test the membership of the tensors to the corresponding basis. Here we have three bases constricted in the original, actual and intermediate states. In the initial configuration we have the tensors $\mathbf{C}$, $\mathbf{C}_p$, $\mathbf{C}^{-1}$, $\mathbf{C}_p^{-1}$, $\dot{\mathbf{C}}$, $\dot{\mathbf{C}}_p$, $\dot{\mathbf{C}}_p^{-1}$, which are invariant tensors. In the basis of actual configuration we have the indifferent tensors $\mathbf{B}$, $\mathbf{B}^{-1}$, $\mathbf{B}_e$, $\mathbf{B}_e^{-1}$, $\overset{\nabla}{\mathbf{B}}_e$, $\mathbf{d}$ and the tensors $\mathbf{h}$, $\boldsymbol{\omega}$, $\dot{\mathbf{B}}_e$. In the intermediate (unloaded) basis we have the tensors $\mathbf{C}_e$, $\mathbf{B}_p$, $\mathbf{h}_p$. Second, the tensors in one basis are united by addition–multiplication operations. This fact should be also taken into account below in stating the corresponding state equations. Third, a transition from one basis into another one is effected by the respective multiplication from the left and right by the tensors of the strain gradient $\mathbf{F}$, $\mathbf{F}_e$, $\mathbf{F}_p$ and the place gradient $\mathbf{F}^T$, $\mathbf{F}_e^T$, $\mathbf{F}_p^T$ and by the inverse tensors. Fourth, the rate of plastic strains in the initial configuration is governed by the tensor $\dot{\mathbf{C}}_p$, in the intermediate configuration, by $\mathbf{d}_p$, and in the actual configuration, by the tensor $\overset{\nabla}{\mathbf{B}}_e$.

2. CONSTITUTIVE EQUATIONS

As a basic one we take the Cauchy stress tensor $\boldsymbol{\Sigma}$, which is also called the hydrodynamic stress tensor. This tensor is defined in the actual basis; it is indifferent and conjugate in power to the velocity strain tensor (3). So, the power of internal forces is given as

$$N = \int\limits_V \boldsymbol{\Sigma} : \mathbf{d} dV.$$

To find the power as an integral with respect to work in the initial configuration use is made of two tensors: the Kirchhoff stress tensor

$$\boldsymbol{\tau} = J\boldsymbol{\Sigma} \tag{6}$$

(J is the relative variation of the volume) and the second Piola–Kirchhoff stress tensor

$$\mathbf{S} = J\mathbf{F}^{-1} \cdot \boldsymbol{\Sigma} \cdot \mathbf{F}^{-T} = \mathbf{F}^{-1} \cdot \boldsymbol{\tau} \cdot \mathbf{F}^{-T}.$$

In this case, the power is given as

$$N = \int\limits_{V_0} \frac{1}{2}\mathbf{S} : \dot{\mathbf{C}} dV_0 = \int\limits_{V_0} \boldsymbol{\tau} : \mathbf{d} dV_0.$$

Physical relations will be obtained from the equation of the second law of thermodynamics for an isothermic process. According to [11, 12], we have the inequality

$$\boldsymbol{\tau} : \mathbf{d} - \rho_0 \dot{\psi} \geq 0, \tag{7}$$

where ρ_0 is the original density and ψ is the free energy. It is assumed that the free energy depends only on elastic strains, because the plastic strain results in expenditure (dissipation) of energy. As the elastic strains tensor, whose components are arguments of the energy function, one usually chooses either $\mathbf{F}_e$ (see [13]) or $\mathbf{C}_e$ (see [11, 12]) or $\mathbf{B}_e$ (see [14, 15]). There also exist some other variants (the distortion tensors, the logarithmic measures of strains, the Cauchy–Green and Almansi strains tensors, etc. [16–20]). In the present paper, we take the following variant as a basic one:

$$\psi = \psi\left(\mathbf{B}_e\right).$$

Substituting this in (7) and using the expression for the generalized derivative (5), we find that

$$\left\{\boldsymbol{\tau} - \rho_0 \left[\mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \frac{\partial \psi}{\partial \mathbf{B}_e} \cdot \mathbf{B}_e\right]\right\} : \mathbf{d}$$
$$- \rho_0 \left[\mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} - \frac{\partial \psi}{\partial \mathbf{B}_e} \cdot \mathbf{B}_e\right] : \boldsymbol{\omega} - \rho_0 \frac{\partial \psi}{\partial \mathbf{B}_e} : \overset{\nabla}{\mathbf{B}}_e \geq 0. \tag{8}$$

This gives us the constitutive equation

$$\boldsymbol{\tau} = 2\rho_0 \mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e}, \tag{9}$$

the symmetry equation

$$\mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} = \frac{\partial \psi}{\partial \mathbf{B}_e} \cdot \mathbf{B}_e$$

and the dissipative inequality

$$\rho_0 \frac{\partial \psi}{\partial \mathbf{B}_e} : \overset{\nabla}{\mathbf{B}}_e \leq 0.$$

As the plastic flow condition we take

$$\Phi_p\left(\boldsymbol{\Sigma}, \chi\right) \leq 0, \tag{10}$$

where Φ_p is the yield function and χ is the hardening parameter.

Let us compose the generalized dissipation functional

$$\rho \boldsymbol{\Sigma} : \left[\frac{1}{2} \overset{\nabla}{\mathbf{B}}_e \cdot \mathbf{B}_e^{-1}\right] + \dot{\lambda} \Phi_p\left(\boldsymbol{\Sigma}, \chi\right) \leq 0. \tag{11}$$

Here we used the relations (6) and (9), in terms of which the gradient of the functional $\dfrac{\partial \psi}{\partial \mathbf{B}_e}$ is expressed via the stress tensor $\boldsymbol{\Sigma}$. Next, in accordance with [11, 12, 14, 15], we write down the condition for an extremum of the functional (11) with respect to possible fields of stresses. As a result, we get the relation

$$-\frac{1}{2} \overset{\nabla}{\mathbf{B}}_e \cdot \mathbf{B}_e^{-1} = \dot{\lambda} \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}}. \tag{12}$$

This equation is an analogue of the equation of the associated law of plastic flows in the theory of small elasto-plastic strains (the flow theory). Here we also have the parameter $\dot{\lambda}$, which controls the value of the plasticity strain rate. To obtain the linearized constitutive relation we differentiate equation (9) in time, which gives

$$\dot{\boldsymbol{\tau}} = 2\rho_0 \left[\dot{\mathbf{B}}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \mathbf{B}_e \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} : \dot{\mathbf{B}}_e \right].$$

In view of (6) we write the result as the sum of two terms

$$\dot{\boldsymbol{\tau}} = \dot{\boldsymbol{\tau}}_e + \dot{\boldsymbol{\tau}}_p,$$

where

$$\dot{\boldsymbol{\tau}}_e = 2\rho_0 \left\{ \left[\mathbf{h} \cdot \mathbf{B}_e + \mathbf{B}_e \cdot \mathbf{h}^T \right] \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \mathbf{B}_e \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} : \left[\mathbf{h} \cdot \mathbf{B}_e + \mathbf{B}_e \cdot \mathbf{h}^T \right] \right\}, \tag{13}$$

$$\dot{\boldsymbol{\tau}}_p = 2\rho_0 \left\{ \overset{\nabla}{\mathbf{B}}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \mathbf{B} \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} : \overset{\nabla}{\mathbf{B}}_e \right\}. \tag{14}$$

In [21] it was shown that a dependence of form (13) can be written as

$$\dot{\boldsymbol{\tau}}_e = 4\rho_0 \left[\mathbf{B}_e \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} \cdot \mathbf{B}_e \right] : \mathbf{d} + \mathbf{h} \cdot \boldsymbol{\tau} + \boldsymbol{\tau} \cdot \mathbf{h}^T.$$

Expressing the generalized derivative $\overset{\nabla}{\mathbf{B}}_e$ from (12) in terms of the unknown parameter $\dot{\lambda}$ and substituting into (14), this gives

$$(\dot{\boldsymbol{\tau}}) = 4\rho_0 \boldsymbol{\Lambda}_e : \mathbf{d} + \mathbf{h}\boldsymbol{\tau} + \boldsymbol{\tau}\mathbf{h}^T - \dot{\lambda} 4\rho_0 \left\{ \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} \cdot \mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \mathbf{B}_e \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} : \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} \mathbf{B}_e \right\}, \tag{15}$$

where

$$\boldsymbol{\Lambda}_e = \mathbf{B}_e \cdot \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} \cdot \mathbf{B}_e. \tag{16}$$

The rate of plastic strains is determined from the equation of stationarity (matching equation), which is obtained by differentiating equation (6):

$$\frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \dot{\boldsymbol{\Sigma}} - H\dot{\lambda} = 0; \tag{17}$$

here we introduced the hardening function $H = \dfrac{\partial \Phi_p}{\partial \chi} \dfrac{\partial \chi}{\partial \lambda}$.

The derivative of the tensor $\boldsymbol{\Sigma}$ is expressed in terms of the tensor $\boldsymbol{\tau}$ by the formula

$$\dot{\boldsymbol{\Sigma}} = \frac{1}{J} \dot{\boldsymbol{\tau}} - \boldsymbol{\Sigma} \left[\mathbf{I} : \mathbf{d} \right], \tag{18}$$

where we used the available kinematic relation $\dfrac{\dot{J}}{J} = I_{1\mathbf{d}} = \mathbf{I} : \mathbf{d}$.

Substituting (18) in view of (15) into (17), we get the following equation $\dot{\lambda}$:

$$4\rho \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \boldsymbol{\Lambda}_e : \mathbf{d} + \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \left[\mathbf{h}\boldsymbol{\Sigma} + \boldsymbol{\Sigma}\mathbf{h}^T - \boldsymbol{\Sigma} I_{1\mathbf{d}} \right] - \dot{\lambda} 4\rho \left[\frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \mathbf{R} \right] - \dot{\lambda} H = 0, \tag{19}$$

where

$$\mathbf{R} = \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} \cdot \mathbf{B}_e \cdot \frac{\partial \psi}{\partial \mathbf{B}_e} + \mathbf{B}_e \frac{\partial^2 \psi}{\partial \mathbf{B}_e \partial \mathbf{B}_e} : \left[\frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} \cdot \mathbf{B}_e \right].$$

From (19) we find $\dot{\lambda}$:

$$\dot{\lambda} = \left[H + 4\rho \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \mathbf{R} \right]^{-1} \left\{ 4\rho \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \boldsymbol{\Lambda}_e : \mathbf{d} + \frac{\partial \Phi_p}{\partial \boldsymbol{\Sigma}} : \left[\mathbf{h} \cdot \boldsymbol{\Sigma} + \boldsymbol{\Sigma} \cdot \mathbf{h}^T - \boldsymbol{\Sigma} I_{1\mathbf{d}} \right] \right\}.$$

Next, for brevity, we set

$$4\rho\mathbf{\Lambda}_e : \mathbf{d} + \mathbf{h}\cdot\mathbf{\Sigma} + \mathbf{\Sigma}\cdot\mathbf{h}^T - \mathbf{\Sigma} I_{1\mathbf{d}} = \mathbf{G}_e : \mathbf{h}.$$

As a result, the linearized relation for $\dot{\mathbf{\Sigma}}$ is structurally written as

$$\dot{\mathbf{\Sigma}} = \mathbf{G}_e : \mathbf{h} - 4\rho\mathbf{R}\frac{\dfrac{\partial\Phi_p}{\partial\mathbf{\Sigma}} : \mathbf{G}_e : \mathbf{h}}{H + 4\rho\dfrac{\partial\Phi_p}{\partial\mathbf{\Sigma}} : \mathbf{R}}. \tag{20}$$

This equation in velocities can be looked upon as an equation in increments of the corresponding quantities. In particular, the rate of variation of the stresses $\dot{\mathbf{\Sigma}}$ should be identified with the increment of these stresses $\Delta\mathbf{\Sigma}$, while the spatial gradient of the velocity $\mathbf{h}$ should be considered as the tensor $\Delta\mathbf{h}_R$; besides,

$$\Delta\mathbf{h}_R = \frac{\partial\Delta y_i}{\partial y_j}\mathbf{e}_i\mathbf{e}_j,$$

which structurally agrees with (2). Note that changing $\mathbf{h}$ by $\Delta\mathbf{h}_R$ and replacing $\dot{\mathbf{\Sigma}}$ by $\Delta\mathbf{\Sigma}$ changes the physical dimension of relation (20). Now there is no time as a physical parameter.

CONCLUSIONS

The paper presents the principal kinematic and physical relations employed in simulation of finite elasto-plastic strains. The medium kinematics was considered with account of the multiplicative expansion of the total gradients of deformations in the elastic and inelastic components. The derivation of physical relations depends upon the equation of the second law of thermodynamics with introduction of the free energy function. On the basis of the yield condition a generalized functional is constructed, using which an analogue of the equation of the associated law of plastic flow is obtained. This eventually allowed us to construct the constitutive relations for the rates and increments of the true Cauchy stresses. These constitutive relations can be applied in constructing numerical methods of study of nonlinear strain processes.

ACKNOWLEDGMENTS

This research was carried out with the financial support of the Russian Foundation for Basic Research (grant nos. 15-01-08733 and 15-31-20602).

REFERENCES

1. A. I. Golovanov, Yu. G. Konoplev, and L. U. Sultanov, Uchen. Zap. Kazan. Univ. Ser. Fiz.-Mat. Nauki **150** (1), 29–37 (2008).
2. A. I. Golovanov, Yu. G. Konoplev, and L. U. Sultanov, Uchen. Zap. Kazan. Univ. Ser. Fiz.-Mat. Nauki **150** (3), 122–132 (2008).
3. A. I. Golovanov, Yu. G. Konoplev, and L. U. Sultanov, Uchen. Zap. Kazan. Univ. Ser. Fiz.-Mat. Nauki **151** (3), 108–120 (2009).
4. A. I. Golovanov, Yu. G. Konoplev, and L. U. Sultanov, Uchen. Zap. Kazan. Univ. Ser. Fiz.-Mat. Nauki **152** (3), 115–126 (2010).
5. A. Golovanov and L. Sultanov, Int. Appl. Mech. **41** (6), 614–620 (2005).
6. A. I. Golovanov and L. U. Sultanov, Russian Aeronautics **51** (4), 362–368 (2008).
7. R. L. Davydov and L. U. Sultanov, PNRPU Mechanics Bull., No. 1, 81–93 (2013).
8. R. Davydov and L. Sultanov, "Numerical algorithm of solving the problem of large elastic-plastic deformation by FEM," in *Proceedings of Sixth International Conference on Nonlinear Mechanics (ICNM-VI), USA, 2013* (DEStech, 2013), pp. 64–67.
9. A. I. Golovanov and L. U. Sultanov, *Mathematical Models of Computational Nonlinear Mechanics of Deformable Media* (Kazan. Gos. Univ., Kazan, 2009) [in Russian].
10. A. I. Golovanov and L. U. Sultanov, *Mathematical Theoretical Foundations of Computational Nonlinear Mechanics of Deformable Media* (Kazan. Gos. Univ., Kazan, 2008) [in Russian].

11. B. Eidel and F. Gruttmann, "Elastoplastic orthotropy at finite strains: multiplicative formulation and numerical implementation," Comput. Mater. Sci. **28** (3–4), 732–742 (2003).
12. J. Schröder, F. Gruttmann, and J. Löblein, "A simple orthotropic finite elasto–plasticity model based on generalized stress–strain measures," Comput. Mech. **30** (1), 48–64 (2002).
13. J. S. Simo, "A framework for finite strain elastoplasticity based on maximum plastic dissipation and the multiplicative decomposition: Part I. Continuum formulation," Comput. Meth. Appl. Mech. Eng. **66** (2), 199–219 (1988).
14. C. Miehe, "A theory of large-strain isotropic thermoplasticity based on metric transformation tensors," Arch. Appl. Mech. **66** (1), 45–64 (1995).
15. Y. Basar and M. Itskov,"Constitutive model and finite element formulation for large strain elasto-plastic analysis of shell," Comput. Mech. **23** (5), 466–481 (1999).
16. A. Meyers, P. Schievbe, and O. T. Bruhns, "Some comments on objective rates of symmetric Eulerian tensors with application to Eulerian strain rates," Acta Mech. **139** (1), 91–103 (2000).
17. H. Xiao, O. T. Bruhns, and A. Meyers, "A consistent finite elastoplasticity theory combining additive and multiplicative decomposition of the stretching and deformation gradient," Int. J. Plast. **16** (2), 143–177 (2000).
18. M. Asghari, R. Naghdabadi, S. Sohrabpour, "Stresses conjugate to the Jaumann rate of Eulerian strain measures," Acta. Mech. **190** (1), 45–56 (2007).
19. R. C. Lin, "Numerical study of consistency of rate constitutive equations with elasticity at finite deformation," Int. J. Numer. Meth. Eng. **55** (9), 1053–1077 (2002).
20. L.-J. Shen, "Constitutive relations for isotropic or kinematic hardening at finite elastic–plastic deformations," Int. J. Solids Struct. **43** (18–19), 5613–5627 (2006).
21. A. L. Eterovic and K.-J. Bathe, "A hyperelastic-based large strain elasto-plastic constitutive formulation with combined isotropic-kinematic hardening using the logarithmic stress and strain measures," Int. J. Numer. Meth. Eng. **30** (6), 1099–1114 (1990).