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A Method of Accelerated Statistical Simulation and Its Application in the Problems with Inherent Error

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Abstract—We study some available methods for solving the stochastic problems on the basis of statistical testing procedure (the Monte Carlo method). In order to perform comparative analysis of the effectiveness of these methods, we solve several problems in the theory of technical systems with inaccurately specified and random parameters and characteristics.

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INTRODUCTION

In system simulation, it is often necessary to take into account the error in the initial data [1, 2]. Among the appropriate methods, the method of a small parameter (or perturbation method) is often used. Its accuracy depends on the error in initial data. Another shortcoming of this method is the lack of guaranty of the reliability of estimates. A relatively new approach is the use of two-sided methods [3–7]; but these methods are effective only for a small error in the model. The approach described in [5] is based on the use of a variational principle. It can be used both in the case of the systems with lumped and distributed parameters, and does not impose any significant restrictions on the value of the inherent error. The above methods allow us to solve only one of the problems in the theory of tolerances; namely, the problem of calculating the so-called worst case. In addition, the two-sided methods are ineffective in solving the stochastic problems in which the random parameters have large variances. The Monte Carlo methods [1, 3, 8, 9] have wide capabilities. However, their use often leads to large computational costs since we need to solve one and the same problem many times for different values of the initial data [1, 8].

To improve the effectiveness of statistical methods, some approaches are proposed: the method of stratified sampling, the method of essential sampling, the method of correlated processes, the sampling method with a "ratio estimate," the method of random quadrature formulas, and the method of selecting the leading part [3, 8, 9]. In general, an increase in efficiency is achieved by decreasing the variance of the statistical estimate.

In the method of stratified sampling, the accuracy is increased due to selecting the largest number of random variables from those regions of the space of their values, where the simulation results have the greatest dispersion. When implementing stratified sampling, the space of values of random variables is divided into several pairwise disjoint subsets (layers), and a simple random sample of fixed size is taken from each layer. Using a special kind of function to calculate the probability characteristic allows us to ensure the unbiasedness of its estimation, and the sample sizes in the layers are selected in such a way as to ensure that the variance of the estimate is smaller than for a simple random sample. The method of essential sampling is based on a similar idea, but here the choice of points is regulated not by setting the number of points in the regions, but by means of a special function of probability density. One of the universal approaches to reducing the variance estimates is the method of random quadrature formulas (a generalization of the Monte Carlo method). The sampling method with a "ratio estimate" as well as

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the method of correlation sample uses a function that approximates well the operator of the problem with known values of the probability characteristics. In many respects, this is also applicable to the method of stratified sampling.

Thus, the common methods of accelerating the statistical simulation involve the construction of a simplified model that approximate the operator of the problem under consideration. This circumstance requires taking into account the specificities of applied problems. We consider the technology of the method of correlated processes as applied to the problems of simulation of the oscillatory systems and the electrical circuits with inaccurately specified or random parameters.

1. DESCRIPTION OF THE METHOD OF CORRELATED PROCESSES

Let λ denote the sought-for n-dimensional vector of the probabilistic characteristics of the original system, and let μ stand for the m-dimensional vector of probabilistic characteristics of the simplified system: $\lambda = M[R]$ and $\mu = M[S]$. Here $M[\dots]$ is the mathematical expectation of the quantity in parentheses; while R and S are the n- and m-dimensional vectors whose components are some functions of the process values in the original and simplified systems, respectively. Suppose that N independent experiments according to the Monte Carlo method are carried out under the same conditions for the initial and simplified systems. The statistical values λ^* and μ^* of vectors λ and μ obtained from these N experiments are as follows:

$$\lambda^* = \frac{1}{N}\sum_{j=1}^{N} R_j, \tag{1}$$

$$\mu^* = \frac{1}{N}\sum_{j=1}^{N} S_j. \tag{2}$$

Without loss of generality, we assume that the experiments with the same external influences on the original and simplified systems have the same number; i.e., the values of R_j and S_j are obtained under the same influences. It is assumed that the exact value of the vector μ of the probabilistic characteristics of the simplified system can be found analytically or by a suitable approximate method but with high accuracy (for example, using a one-factor model of the original system). The method of correlated processes is based on the optimal estimate λ_0 of the vector λ that uses the values of the vectors λ^*, μ^*, and μ; to that end, the estimates of the probabilistic characteristics of the original system are sought using the statistical values of the probabilistic characteristics of the original and simplified systems. Thus, the results of analytical study of the simplified problem are used in statistical simulation. The sought-for estimates have the form [9]

$$\lambda_{i0} = \lambda_i^* - K_{R_iS}K_{SS}^{-1}(\mu^* - \mu), \tag{3}$$

where

$$\lambda_i^* = \frac{1}{N}\sum_{j=1}^{N} R_{ij}, \qquad K_{R_iS} = M[R_i(S-\mu)^T], \qquad K_{SS} = M[(S-\mu)(S-\mu)^T].$$

Here, λ_{i0} and λ_i^* are the ith components of vectors λ_0 and λ^*, respectively, while R_{ij} is the ith component of R_j. The difference $\mu^* - \mu$ is equal to the statistical error in the determination of the vector μ which can be found by the available exact or practically exact value μ of this vector. The quantity $K_{R_iS}K_{SS}^{-1}(\mu^* - \mu)$ is the value of the error recalculated for the quantity λ_i^*, taking into account the correlation relation between λ_i^* and μ^*. To obtain the estimate λ_{i0}, this recalculated error value is subtracted from the statistical value λ_i^* of λ_i. In result, the estimate λ_{i0} turns out to be more accurate than the statistical value λ_i^*. The evaluation of each of the components of vector λ can be performed independently of the evaluation of other components. To use formula (3), we need to know the correlation row matrix K_{R_iS} and the matrix K_{SS}. Since the simplified system can be studied by an analytical or, in many cases, an economical approximate method (at least, for the determination of vector μ); therefore, in principle, it is possible to economically calculate with high accuracy the values of the correlation matrix K_{SS} by an analytical or an approximate method. However, in some cases the determination of K_{SS} may turn out

Table 1.

N	δ, %	g_1	g_2
50	0.65	80	20
200	0.3	80	30
800	0.16	100	20

to be difficult. Thus, instead of K_{SS}, we can find its statistical value K^*_{SS} calculated from the same N experiments from which λ^* and μ^* were obtained, i.e. by the formula

$$K^*_{SS} = \frac{1}{N}\sum_{j=1}^{N}(S_j - \mu)(S_j - \mu)^T.$$

The original system cannot be studied analytically; therefore, instead of the correlation row matrix K_{R_iS}, we can only find its statistical value $K^*_{R_iS}$ according to the formula

$$K^*_{R_iS} = \frac{1}{N}\sum_{j=1}^{N}(R_{ij} - \mu)(S_j - \mu)^T.$$

The construction of an approximate model is an important problem. As a rule, this is achieved by linearizing or decreasing the level of details, the number of factors, and so forth of the original model. An important advantage of the method is the following: Even if the approximate model is very inaccurate, the accuracy of the statistical method will not be worse than when using the usual Monte Carlo method. To increase the calculation accuracy, it is necessary that there be a correlation between the estimated parameters of the original and approximate models. The presence of a systematic error in the statistical evaluation of the parameters is indicated in [8] as the main shortcoming the method.

2. EXAMPLES OF APPLICATION OF THE METHOD OF CORRELATED PROCESSES

2.1. Comparison of Efficiency Between the Method of the Leading Part Selection and the Method of Correlated Sampling in the Computation of Integrals

Suppose that we need to calculate the integral $J = \int_\Omega \Psi(u)\,du$. Assume known the function $\Psi_1(u) \approx \Psi(u)$ and the value of the integral $I = \int_\Omega \Psi_1(u)\,du$. Let ξ be some random variable (scalar or vector) uniformly distributed in the domain of integration Ω whose volume, without loss of generality, is set equal to unity. In this case,

$$R = \Psi(\xi), \qquad S = \Psi_1(\xi), \qquad \lambda = M\Psi(\xi),$$
$$\mu = M\Psi_1(\xi), \qquad K_{SS} = D\Psi_1, \qquad K_{RS} = \operatorname{cov}(\Psi(\xi)\Psi_1(\xi)).$$

Estimation of the integral value in the method of correlated sampling is carried out by the formulas (1)–(3). In the case of the usual Monte Carlo method, $J \approx \lambda^*$. When using the widely applied method of selecting the leading part (also called the controlled variable method [8]), the value of the integral is estimated by the formula $J \approx \lambda^* - (\mu^* - \mu)$. As a numerical example, we consider the case when $\Omega = [0, 1]$, $\Psi(u) = u + \varepsilon u^2$, and $\Psi_1(u) = u$. The results of calculations are presented in Table 1 (for $\varepsilon = 0.5$) and Table 2 (for $\varepsilon = 0.1$). In the tables we let δ to be the relative error of the method of correlated sampling, while g_1 and g_2 are the gain in accuracy in comparison with the Monte Carlo methods and the method of selecting the leading part, respectively. For $\varepsilon = 0.01$, the gain in accuracy of the method of correlated processes in comparison with the Monte Carlo method is already about 2500; for $\varepsilon = 0.001$, it is about 20 000. These regularities remain valid also for other more complicated variants of integrands.

Table 2.

N	δ, %	g_1	g_2
50	0.16	200	25
200	0.08	300	30
800	0.04	300	25

2.2. Comparison of Efficiency Between the Methods under Consideration in Simulation of the Nonlinear Oscillations under Random Influences

The equations of a nonlinear oscillatory circuit can be written in normalized form:

$$\frac{d^2x}{dt^2} = -x - kx^3 + f(t), \qquad t > 0;$$
$$x(0) = x_0, \qquad \frac{dx}{dt}(0) = x_1, \tag{4}$$

where $k = \text{const}$ is the nonlinearity coefficient, whereas $f(t)$ is some random function.

It is required to estimate the mathematical expectation of the solution for $t = t_F$. When using the Monte Carlo method, we have to solve N times the problem (4) with the corresponding realizations $f_1(t), \ldots, f_N(t)$ of the random process $f(t)$. As an approximate solution, we take the arithmetic mean of the obtained values $x(t_F)$: $M[x(t_F)] \approx x^*(t_F)$. In the method of correlated processes,

$$M[x(t_F)] \approx x^*(t_F) - (y^*(t_F) - M[y(t_F)])\,\mathrm{cov}(x(t_F), y(t_F))/D[y(t_F)],$$

where $y(t)$ is the solution of the linearized problem

$$\frac{d^2y}{dt^2} = -y + f(t), \quad t > 0; \qquad y(0) = x_0, \qquad \frac{dy}{dt}(0) = x_1,$$

calculated for the same realizations $f_1(t), \ldots, f_N(t)$ of the random process $f(t)$. The mathematical expectation and variance of $y(t)$ can be found analytically or numerically, the value of $\mathrm{cov}(x(t_F), y(t_F))$ is calculated approximately using the appropriate statistical estimate. On the computer, the test Cauchy problem (4) was solved by the method of finite differences for the specific case

$$f(t) = a^* \sin(\omega t)(1 - \omega^2 + k^* a^2 \sin^2(\omega t)),$$

where $\omega = \pi/2$, and a is a random variable uniformly distributed on the interval $(0, 1)$. For the initial conditions $x(0) = 0$ and $\dfrac{dx}{dt}(0) = a\omega$, the Cauchy problem (4) has the solution $x(t) = a^* \sin(\omega t)$. In the calculations, the following parameter values were taken: $k = 0.01 \div 1$ and $N = 50 \div 1000$. The relative accuracy of statistical estimation by the method of correlated processes was about 0.005–0.5 %. The accuracy gain in comparison with the usual Monte Carlo method was 100–1000 times. As a rule, with decrease of the coefficient of nonlinearity, the accuracy of the method of correlated processes increased.

When choosing a method for solving more complicated stochastic problems, some caution is needed since the costs of analyzing the auxiliary problem in the methods of the type of correlated or stratified sampling may not pay off. The Monte Carlo method seems most reliable in general, although not always optimal. The two-sided method on the basis of the maximum principle can also be evaluated positively [5]. These conclusions are obtained by using the analysis of the results of the next subsection.

2.3. Comparison Between the Statistical and Two-Sided Methods in Calculation of the Nonlinear Transient Process Taking into Account the Inherent Error

Consider an electrical circuit whose diagram is depicted in Fig. 1.

According to the Kirchhoff laws, the system of equations of the circuit state can be written as

$$\frac{d\Psi}{dt} + i_0 r_0 + i_1 r_1 = E(t), \qquad \frac{C du_C}{dt} = i_0 - i_1, \qquad i_1 r_1 - u_C = 0,$$

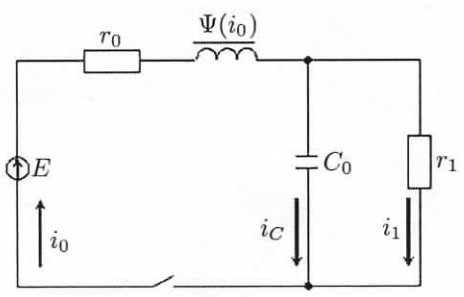

Fig. 1. Diagram of an electrical circuit.

where $\Psi = \Psi(i_0)$ is the flux linkage of the coil with the steel magnetic core, C is the capacity, r_0 and r_1 are the resistances, i_0 and i_1 are the currents in the corresponding branches, u_C is the voltage across the capacitor, and $E(t)$ is a nonsinusoidal periodic EMF. The nonlinear dependence of the flux linkage on the current is expressed by the formula $\Psi(i) = k_1 k_2^{-1/2} \arctan\left(i k_2^{-1/2}\right)$ [4, 5]. Consequently, the incremental inductance equals $L_0(i) = k_1/(1 + k_2 i^2)$ [4, 5]. The EMF $E(t)$ acting in the circuit is represented by the sum of the two harmonic components

$$E(t) = E_{1m} \sin(\omega_1 t + \phi_1) + E_{4m} \sin(\omega_4 t + \phi_4),$$

where E_{1m} and E_{4m} are the amplitudes, ω_1 is the fundamental frequency, $\omega_4 = 4\omega_1$, while φ_1 and φ_4 are the initial phases.

The parameters and characteristics of the equivalent electrical circuit are given with some relative errors:

$$r_0 = M(r_0)(1 + \varepsilon(r_0)), \qquad r_1 = M(r_1)(1 + \varepsilon(r_1)), \qquad C = M(C)(1 + \varepsilon(C)),$$
$$E_{1m} = M(E_{1m})(1 + \varepsilon(E_{1m})), \qquad E_{4m} = M(E_{4m})(1 + \varepsilon(E_{4m})),$$
$$k_1 = M(k_1)(1 + \varepsilon(k_1)), \qquad k_2 = M(k_2)(1 + \varepsilon(k_2)),$$

where $M\xi$ is the given average value of the parameter ξ,

$$\varepsilon(r_0) \le \varepsilon_0(r_0), \qquad \varepsilon(r_1) \le \varepsilon_0(r_1), \qquad \varepsilon(C) \le \varepsilon_0(C), \qquad \varepsilon(E_{1m}) \le \varepsilon_0(E_{1m}),$$
$$|\varepsilon(r_0)| \le \varepsilon_0(r_0), \qquad |\varepsilon(r_1)| \le \varepsilon_0(r_1), \qquad |\varepsilon(C)| \le \varepsilon_0(C),$$
$$|\varepsilon(E_{1m})| \le \varepsilon_0(E_{1m}), \qquad |\varepsilon(E_{4m})| \le \varepsilon_0(E_{4m}), \qquad |\varepsilon(k_1)| \le \varepsilon_0(k_1), \qquad |\varepsilon(k_2)| \le \varepsilon_0(k_2),$$

$\varepsilon_0(\xi)$ is the given value of the relative error of the parameter ξ. When such a circuit is closed, a phenomenon is observed called a surge of current, whose magnitude may be of practical interest. It is required to calculate the transient process in the circuit under consideration after the key is closed, taking into account the error of the initial data of the problem.

The system of equations of state of the circuit in the normal form is as follows:

$$\frac{dx_i}{dt} = f_i(x, t), \qquad i = 1, 2, \qquad t \in (0, t_F), \qquad x_1(0) = x_2(0) = 0,$$

where $x_1 = i_0$, $x_2 = i_1$, $f_1 = (E(t) - x_1 r_0 - x_2 r_1)/L_0(x_1)$, and $f_2 = (x_1 - x_2)/L(C r_1)$.

2.3.1. Solution by the Two-Sided Method [5]. Suppose that we need to compute an estimate from above for the value $x_1(t_F)$ on the set of all possible values of the parameters of the circuit. The calculation of the transient process was carried out by a numerical two-sided method [5] with the following initial data:

$$M(E_{1m}) = 75, \qquad \varepsilon_0(E_{1m}) = 0.01 \div 0.1, \qquad M(E_{4m}) = 5, \qquad \varepsilon_0(E_{4m}) = 0.01 \div 0.1,$$
$$\omega = 100\pi, \qquad \phi_1 = 0, \qquad \phi_4 = 0, \qquad M(r_0) = 1, \qquad \varepsilon_0(r_0) = 0.01, \qquad M(r_1) = 1,$$
$$\varepsilon_0(r_1) = 0.01 \div 0.1, \qquad M(k_1) = 0.1, \qquad \varepsilon_0(k_1) = 0.01 \div 0.1, \qquad M(k_2) = 0.25,$$
$$\varepsilon_0(k_2) = 0.01 \div 0.1, \qquad M(C) = 10^{-4}, \qquad \varepsilon_0(C) = 0.01 \div 0.1, \qquad t_F = 0.04, \qquad h \le 10^{-4},$$

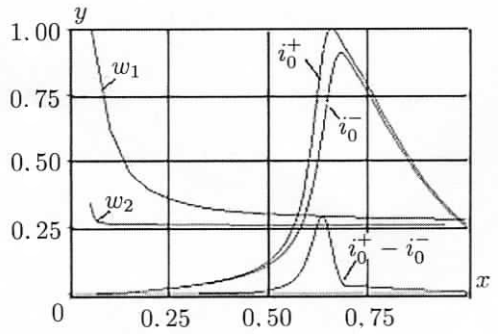

Fig. 2. Two-sided estimates of the magnitude of current $i_0(t)$.

where h is the step of numerical integration of the equations of the problem.

The graphs of two-sided estimates of the magnitude of the current $i_0(t)$ for the case of the relative error of the initial data being 1 % are presented in Fig. 2, which shows the graphs of envelopes near the first maximum of the current, where the sensitivity of the solution of the problem to perturbations in the initial data is the highest, which is confirmed by the graphs of the width of two-sided solution $i_0^+(t) - i_0^-(t)$. The graphs of the curves are normalized by $\max i_0^+(t) = 36.8$.

The graphs of w_1 and w_2 illustrate the dependence of the width of approximate interval estimates of functions $x_1(t) = i_0$ and $x_2(t) = i_1$ on the values of $1/h$. A specific feature of this problem is high sensitivity of the maximum value of the current to the change in the input parameters, which, in the example, vary by only one percent, while the current values vary by up to 30 %. Another important advantage of the two-sided method [5] is manifested, when the circuit equations contain a large number of approximately given parameters. In the methods of sensitivity theory [2], it is required to integrate an ODE system with the dimension corresponding to the product of the dimension of the original problem and the number of approximately given parameters. Thus, in the example under consideration, in addition to the two components of the zero-approximation vector x_i^0, $i = 1, 2$, it would be necessary to calculate the values of the 14 sensitivity functions $\dfrac{\partial x_i}{\partial \alpha_j}$, $i = 1, 2$, $j = 1, \ldots, 7$, where α_j is the vector of parameters of the problem $\{r_0, r_1, C, E_{1m}, E_{4m}, k_1, k_2\}$.

In this way, the dimension of the Cauchy problem will be 16, while in the two-sided method under consideration, it is required to integrate the main and conjugate ODE systems, each of which has the dimension of only 2. In the problems typical for modern electronics, where the number of elements in the circuit may be hundreds or more, this advantage is very significant. The advantage of the method [5] is also that, for a broad class of problems and at moderate cost, it allows to find the boundaries of the solution absolutely reliably and with high accuracy. For this reason, this approach can be used to control the results obtained by statistical and other methods. In principle, this two-sided method can also be applied in the method of stratified sampling to obtain the solution boundaries in the layers.

2.3.2. Solution of the Problem by the Monte Carlo Method and the Method of Correlated Processes. The approximating operator of the problem is obtained from the statement of the original problem by linearization. To this end, we put $L_0(i) = k_1$. The exact solution of the linearized problem is found analytically and has the form

$$y_1(t) = y_2(t) + Cr_1\frac{dy_2}{dt}, \qquad y_2(t) = \frac{E_{1m}J(t,\omega_1) + E_{4m}J(t,\omega_4)}{L_0Cr_1},$$

$$J(t,\omega) = \frac{(a-\omega_0)\sin(\omega t) - \omega\cos(\omega t) + \omega\exp(-(a-\omega_0)t)}{2\omega_0[(a-\omega_0)^2+\omega^2]} - \frac{(a+\omega_0)\sin(\omega t) - \omega\cos(\omega t) + \omega\exp(-(a+\omega_0)t)}{2\omega_0[(a+\omega_0)^2+\omega^2]},$$

$$\omega_0^2 = \frac{(1/(Cr_1) + r_0/L_0)^2}{4} - \frac{r_0/r_1 + 1}{L_0 C}.$$

The parameters of the problem $\{r_0, r_1, C, E_{1m}, E_{4m}, k_1, k_2\}$ are assumed to be random and uniformly distributed in the corresponding intervals:

$$M(\xi)(1 - \varepsilon_0(\xi)) \le \xi \le M(\xi)(1 + \varepsilon_0(\xi)),$$

where ξ is a parameter of the problem.

The original problem was solved numerically by the method of finite differences. Using the method of correlated processes, we calculated the estimates for the mathematical expectation, the second initial moment, and the variance of the solution of the original problem:

$$M[x(t_F)] \approx m[x(t_F)] = M^*[x(t_F)] - \frac{M^*[y(t_F)] - M[y(t_F)])\,\mathrm{cov}^*(x(t_F), y(t_F))}{D^*(y(t_F)},$$

$$M[x^2(t_F)] \approx m[x^2(t_F)] = M^*[x^2(t_F)] - \frac{M^*[y^2(t_F)] - M[y^2(t_F)])\,\mathrm{cov}^*(x^2(t_F), y^2(t_F)}{D^*[y^2(t_F)]},$$

while the variance was estimated in two ways using the following relations:

$$D[x(t_F)] \approx D^*[x(t_F)] - \frac{D^*[y(t_F)] - D[y(t_F)])\,\mathrm{cov}^*((x(t_F) - x^*(t_F))^2, (y(t_F) - M[y(t_F)])^2}{D^*[(y(t_F) - M[y(t_F)])^2]},$$

$$D[x(t_F)] \approx m[x^2(t_F)] - m^2[x(t_F)].$$

In the above formulas, an asterisk marks the statistical estimates of the corresponding characteristics (averages, variances, and covariances) of random variables calculated approximately by selective data of simulation modelling. The last formula for estimating the solution variance is widely used in practice, but when solving the problem under consideration it turned out to be unreliable because of significant error in the method of correlated processes. Having significantly smaller computational costs for the given problem, the Monte Carlo method turned out to be comparable in accuracy with the method of correlated estimates, which has not demonstrated convincing advantages for this problem.

The values of the boundaries of the solution $i_0(t)$, obtained by statistical methods (for $N = 1000$) and the two-sided method [5], are almost the same. For the error in the initial data of 10 % we have in the region of maximum ($t_F = 7$ ms): $10 \le i_0 \le 40$ for the Monte Carlo method and $10.7 \le i_0 \le 39.5$ for the two-sided method. In the region of current growth ($t_F = 1$ ms), we have $0.1 \le i_0 \le 0.14$ for both methods. For $M(k_2) = 0.25$ the linearization error is large and the method of correlated processes is ineffective.

The typical results of statistical simulation for $t_F = 7$ ms (near the moment of maximum current), the error of the initial data of 10 % and the nonlinearity coefficient $M(k_2) = 0.05$ have the following form: the mean value of the solution of the linear problem $M[y(t_F)] \approx 3.67$, the second initial moment $M[y^2(t_F)] \approx 13.6$; the variance $D[y(t_F)] \approx 0.08$; the standard deviation $D^{1/2}[y(t_F)] \approx 0.29$; and the coefficient of variation is about eight percent.

In the first experiment, ten statistical tests were performed ($N = 10$). The error of the sample mean for solving the linear problem was $M^*[y(t_F)] - M[y(t_F)] \approx 0.18$. The estimate of the mathematical expectation of the solution by the method of correlated processes was equal to $m[x(t_F)] \approx 4.69$ and 5.1, for the Monte Carlo method. The value of the ratio $\mathrm{cov}^*(x(t_F), y(t_F))/D^*[y(t_F)] \approx 2.1$. The absolute error in estimating the value of $M[x(t_F)]$ by the method of correlated processes was approximately equal to 0.02 and, when using the Monte Carlo method, it was about 0.4. The estimate of the variance of the solution by the first method for the method of correlated processes was about 0.39 and 0.74 for the Monte Carlo method (the estimates of the standard deviation being 0.62 and 0.86, respectively); the variation coefficient was of the order of 10%. The values of the estimates of the root-mean-square deviation, obtained by the second method, equaled 0.55 and 0.77, respectively.

In the second experiment, 100 statistical tests were performed ($N = 100$). The estimate of the mathematical expectation of the solution by the method of correlated processes was $m[x(t_F)] \approx 4.68$ and 4.8, for the Monte Carlo method. The value of the ratio equals

$$\mathrm{cov}^*(x(t_F), y(t_F))/D^*[y(t_F)] \approx 2.1.$$

The absolute error in estimating the value of $M[x(t_F)]$ by the method of correlated processes was approximately 0.01 and about 0.1 for the Monte Carlo method. The estimate of the variance of the solution by the first method for the method of correlated processes was about 0.39 and 0.44 for the Monte Carlo method (the estimates of the root-mean-square deviation being 0.62 and 0.66, respectively), the variation coefficient was of the order of 10 %. The values of the root-mean-square deviation estimates obtained by the second method were, respectively, 0.64 and 0.65.

CONCLUSION

In the statistical experiments under consideration, the method of correlated processes allowed us to achieve a gain in accuracy on the average by 10–20 times. However, because of the random factor, such a gain is not guaranteed; approximately in every tenth statistical experiment there was a loss in accuracy. For large values of the nonlinearity coefficient of the problem, due to the error of linearization, the method of correlated processes practically does not give any gain in accuracy. All considered methods are applicable for solving a wide class of stochastic problems with not only lumped, but also distributed parameters.

REFERENCES

1. Yu. N. Belyakov, F. A. Kurmaev, and B. V. Batalov, *Methods of Statistical Calculations of Microcircuits by Computers* (Radio i Svyaz', Moscow, 1985) [in Russian].
2. K. Gekher, *Theory of Sensitivity and Tolerances of Electronic Circuits* (Sov. Radio, Moscow, 1973) [in Russian].
3. N. S. Bakhvalov, *Numerical Methods* (Nauka, Moscow, 1973) [in Russian].
4. S. A. Nekrasov, "Efficient Interval and Bilateral Methods for Calculating Transients in Nonlinear Electric Circuits," Izv. Vyssh. Uchebn. Zaved. Elektromekh. No. 3, 38–41 (2001).
5. S. A. Nekrasov, "Integration Methods for ODEs with Interval Data Based on the Maximum Principle and Their Application for Calculation of Electrical Circuits with Allowance for an Ineradicable Error," Izv. Vyssh. Uchebn. Zaved. Elektromekh. No. 4, 18–22 (2001).
6. S. A. Nekrasov, "Interval and Two-Sided Optimization Methods for Electromagnets in the Case of Approximately Given Initial Data," Izv. Vyssh. Uchebn. Zaved. Elektromekh. No. 2, 22–25 (2001).
7. S. A. Nekrasov, "Complex Computer Applications and Techniques for Solving Problems with the Inherent Errors and Synthesis of Optimal Characteristics of Materials and Design Parameters," J. Siberian Federal Univ. Engineering and Technologies. **8** (7), 867–877 (2015).
8. D. V. Vasil'ev and O. Yu. Sabinin, *Accelerated Statistical Modeling of Control Systems* (Energoatomizdat, Leningrad, 1987) [in Russian].
9. V. N. Pugachov, *Combined Methods for Determining the Probabilistic Characteristics* (Sovet. Radio, Moscow, 1973) [in Russian].