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Up-to-Third-Order Determination of Time Constants of Models of Avionics Thermocouples in Gas Temperature Control Loop of Automatic Control System of Gas Turbine Engine

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Abstract—Ensuring the necessary accuracy of measurement of unsteady temperature of gas in an aircraft gas turbine engine (GTE) is a topical problem. The use of thermocouples in the gas temperature control loop of the automatic control system (ACS) of a GTE is complicated by the need of reducing the thermocouple inertia, which varies significantly in dependence on the GTE operation regimes. The existing methods and means for compensating the inertia of aircraft thermocouples in the gas temperature control loop of the GTE ACS are based solely on the use of a mathematical model of thermocouple in the form of a first-order inertia element. This mathematical description of avionics thermocouples with wire sensors is very approximate. An avionics thermocouple is described more accurately with a second-order mathematical model and in some cases with a third-order one in accordance with OST 1 00334-79 "Temperature Sensors. Dynamic characteristics." The difficulty with the use of second- and third-order thermocouple models is associated with the need to establish the dependence of all time constants of a selected model on the changing operating regimes of GTE. No such dependencies have been determined yet for practical use. The purpose of this work is to find out the functional dependence of all time constants occurring in mathematical models up to the third-order inclusive on actual operating parameters of GTE. The time constants calculated from the established dependencies can be used for continuous correction of the gas temperature control loop of the GTE ACS to ensure optimal correction of the dynamic characteristics of thermocouples.

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1. INTRODUCTION

In measurement of the temperature of air and gas flows in aircraft GTEs, thermocouples with wire sensors of various diameters are widely used. In accordance with GOST 1790-2016 "Wire made of chromel T, alumel, copel, and constantan alloys for thermoelectrodes of thermoelectric converters," thermoelectrode wires for thermocouples are manufactured with diameter of 0.2 to 5.0 mm. For avionics thermocouples with open sensors or those placed in a braking chamber, thermoelectrodes with diameter of 1.2 mm are mainly used.

For thermocouples in the gas temperature control loop of the GTE ACS, there is a need for continuous correction of their inertia under changing operating conditions of the GTE. Some issues of designing and research of various correcting devices and methods of correction in the GTE ACS and in other objects where measurement of unsteady gas temperatures is required are described in detail in works [1–9].

Optimal and continuous correction of the dynamic characteristics of thermocouples necessitates establishment with a required accuracy of the structure of mathematical models of the thermocouples used, subject to changes in their parameters under varying operating regimes of GTE. Some issues of identification of the dynamic characteristics of various measuring instruments that are described by mathematical models of the first and higher orders and can be applied to avionics thermocouples are published in [10–17].

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Currently, compensation for the inertia of thermocouples in the gas temperature control loop of the GTE ACS is performed with exclusively the first-order mathematical model, the transfer function of which has the form

$$W(p) = \frac{k}{Tp+1}, \tag{1}$$

where k is the static conversion coefficient of the thermocouple, mV/°C; T is the time constant, or thermal inertia indicator, which depends on the influencing parameters of gas-air flows, s; p is the Laplace variable, 1/s.

The existing proposals for the use of self-adjusting meters in the gas temperature control loop of the GTE ACS also involve the first-order mathematical model of thermocouple [7–9].

For taking into account the change in the time constant T in (1) in dependence on the influencing parameters of gas-air flows, there are several different relationships.

In Technical Guides 1594-79 "Measurement of unsteady temperature of air flow during bench tests of GTE. Thermometers," the following relation is given:

$$\varepsilon_1 = \varepsilon_0 \left(\frac{V_0\, p_{0\ st}}{V_1\, p_{1\ st}}\right)^n, \tag{2}$$

where ε_0 is the indicator of the thermometer thermal inertia at the velocity V_0 and static pressure p_0; ε_1 is the indicator of the thermometer thermal inertia at the velocity V_1 and static pressure $p_{0\,st}$; $n = 0.488$ is the empirical coefficient for an open cross-flow sensor of the thermometer.

In works [4, 9], relations of the following form are given:

$$T_T = T_{CT} \left(\frac{G_{CG}}{G_G}\right)^{0,5}, \tag{3}$$

where T_{CT} is the calculated time constant of thermocouple at the calculated flow rate G_{CG} of the gas flowing around it; T_T is the actual (expected) time constant of thermocouple at the actual (expected) flow rate G_G of the gas flowing around it.

A relation of the following form is also known:

$$\tau = \tau_0 \left(\frac{\rho_0 V_0}{\rho_i V_i}\right)^n \frac{A(\rho_i V_i)^n + 1}{A(\rho_0 V_0)^n + 1}, \tag{4}$$

where τ is the actual (expected) time constant of the thermocouple; ρ and V is the density and speed, respectively, of the air flowing around the thermocouple; the index 0 and i refers to the experimental and flight conditions, respectively; $n = 0.6\ldots0.85$ and $A = 0\ldots0.05$ are constants governed by the design features of the thermocouple.

In relations (2)–(4), the parameters V_1, G_G, ρ_i, and V_i relate to the calculated parameters of the gas flow in the GTE, and the parameter $p_{1\,st}$ in relation (2) can be directly measured in the area of the thermocouple by means of pressure sensors and therefore is a measured parameter of the gas flow of the GTE.

Analysis of relations (2)–(4) showed that they are applicable only for thermocouples described by the first-order mathematical model. It should be noted that to use these relationships, it is necessary first to determine the ε_0 value on a certified installation [18] from the experimental transient characteristics in (2) at known V_0 and $p_{0\,st}$, T_{CT} in (3) at a known G_{CG}, and τ_0 in (4) at known ρ_0 and V_0.

A theoretical justification for the dependence of all time constants of thermocouple in mathematical models of up to the third order inclusive on the expected operating conditions is given in [19–21].

In [19–21], a hyperbolic dependence of the time constants on the heat transfer coefficient was established:

$$T_i(\alpha) = \frac{1}{\alpha\,\Psi_{T_i}} + T_{i\infty}, \tag{5}$$

where α is the coefficient of heat transfer between the thermocouple sensitive element and the medium to measure W/(m^{2}K); Ψ_{T_i} is the ith time constant criterion, characterizing the uneven distribution of temperatures in the thermocouple and depending on the geometrical and physical features of the thermocouple and the conditions of its heat transfer; $T_{i\infty}$ is value of the ith time constant at $\alpha \to \infty$.

As known, the coefficient of heat transfer between the thermocouple sensitive element and the medium to measure is a complex multiparameter expression, including the diameter of the thermocouple thermoelectrodes, gas velocity, and thermophysical characteristics of the gas, which depend on the temperature and pressure of the gas. Direct calculation of the actual coefficient of heat transfer between the thermocouple sensitive element and the medium to measure during GTE operation is a challenge because in addition to calculating the gas velocity, it is necessary to have a database on the dependence of the thermophysical characteristics of the gas on the temperature and pressure.

The purpose of this work is to establish the functional dependence of all time constants in thermocouple mathematical models of up to the third order inclusive on the actual operating parameters of the GTE, without access to the database on the dependence of the thermophysical characteristics of gas on the temperature and pressure.

The time constants calculated from the established dependencies can be used for continuous correction of the gas temperature control loop of the GTE ACS for ensuring optimal correction of the dynamic characteristics of thermocouples.

2. MATHEMATICAL JUSTIFICATION

Works [19, 20, 22] show that form (5) of each time constant of the selected mathematical model can be established from results of testing thermocouples on certified air setups, which are supposed to be used in the gas temperature control loop of the GTE ACS.

The basic formula for calculation of the average coefficient of convective heat transfer between the thermocouple sensitive element and the gas flow for practical calculations has the form

$$\alpha = \lambda\,\frac{C\,\mathrm{Re}^n\,\mathrm{Pr}^m}{d} = \lambda\,\frac{C\left(\frac{V\,d\,\rho}{\eta}\right)^n\left(\frac{\eta\,c_p}{\lambda}\right)^m}{d}, \tag{6}$$

where $C = 0.5$, $n = 0.5$, $m = 0.38$ at Re of 5 to 10^3; $C = 0.25$, $n = 0.6$, $m = 0.48$ at Re of 10^3 to 2×10^5; $C = 0.023$, $n = 0.8$, $m = 0.37$ at Re of 2×10^5 to 2×10^9; d is the thermoelectrode diameter, m; λ is the gas thermal conductivity coefficient, W/(m·K); η is the coefficient of dynamic viscosity of the gas, Pa s; c_p is the thermal capacity of the gas at a constant pressure, J/(kg·K); $\mathrm{Re} = V\,d/\nu$ is the Reynolds number; ν is the coefficient of kinematic viscosity of the gas, m/s^2; Pr is the Prandtl number.

Formula (6) can be used for thermocouples with open cross-flow wire sensors.

If wire sensors of thermocouples are placed in braking chambers, then the velocity V in formula (6) shall be replaced by the velocity in the braking chamber V_{Gbc}. In [20], it is suggested to calculate the latter velocity with the following expression:

$$V_{Gbc} = V\sqrt{\frac{1-\xi_{bc}}{1-\xi_{se}}},$$

where ξ_{bc} is the recovery coefficient of thermocouple with the braking chamber; ξ_{se} is the recovery coefficient of the thermocouple sensor; V is the velocity of the gas flowing around the thermocouple.

The ξ_{bc} and ξ_{se} values for specific types of thermocouples are determined experimentally on certified setups [18].

As seen, formula (6) does not include the gas temperature and pressure p, which can be measured in the area where the thermocouple is located. The gas velocity in (6) is a variable parameter, and it is not measured, but is a calculated GTE parameter.

It is proposed to achieve the goal by replacing the calculation of the heat transfer coefficient according to (6) with a calculation using an empirical expression containing the complex of the product of the gas velocity V by the gas pressure p, i.e., pV.

3. RESEARCH METHODOLOGY

The study was performed with thermoelectrodes with diameter of 1.2 mm. The medium to measure was air flow, which is close in terms of thermophysical characteristics to the gas flow behind the GTE turbine.

The studies were carried out with variation of the air flow velocity V from 50 to 200 m/s, pressure p from 0.1 to 5 MPa, and temperature t from 527 to 927°C.

Table 1 presents the results of the calculation of the heat transfer coefficient α according to (6) versus the complex pV at different air flow temperatures.

Figure 1 shows graphs of the dependence of the heat transfer coefficient α on the complex pV according to the data from Table 1.

From Table 1 and Fig. 1, it is clear that the heat transfer coefficient α is relatively weakly dependent on the air temperature in the temperature range under study, which can be a basis for deriving a simplified empirical relationship.

The empirical relation was sought for by means of the regression analysis, where the regression function was presented by an expression of the following form:

$$\alpha = B(pV)^n,$$

where B and n are the required parameters of the regression function.

Table 2 presents the established regression functions at five air flow temperatures according to the Table 1 data with assessment of the accuracy of the regression analysis by the R-square criterion.

Table 1. Heat transfer coefficient α versus complex pV at different temperatures

pV, MPa m/s	α, W/(m^{2}K)				
	No. 1: 527°C (800 K)	No. 2: 627°C (900 K)	No. 3: 727°C (1000 K)	No. 4: 827°C (1100 K)	No. 5: 927°C (1200 K)
5	1212	1139	1117	1103	1078
10	1837	1726	1693	1671	1633
15	2343	2201	2159	2132	2083
20	2785	2616	2566	2533	2476
50	4602	4511	4428	4360	4285
100	6976	6837	6712	6608	6495
150	8897	8720	8560	8428	8284
200	10570	10360	10170	10020	9845
250	12070	11810	11610	11400	11230
500	18300	17910	17590	17280	17020
750	23340	22840	22440	22040	21700
1000	27730	27140	26660	26200	25790

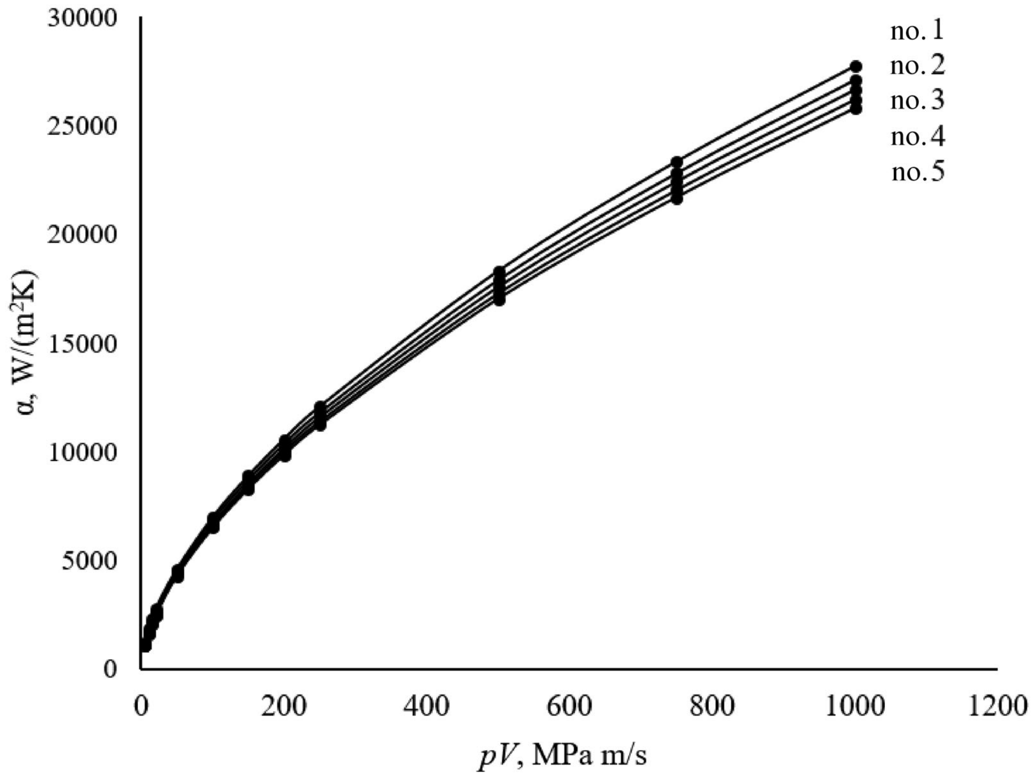

Fig. 1. Heat transfer coefficient α vs. complex pV at different temperatures: no. 1—527°C; no. 2—627°C; no. 3—727°C; no. 4—827°C; no. 5—927°C.

Table 2. Determined regression functions at five temperatures of air flow

No.	α, W/(m^{2}K)	R-square
1	$448.5(pV)^{0.598}$	1.0
2	$433.8(pV)^{0.598}$	1.0
3	$425.5(pV)^{0.598}$	1.0
4	$420.4(pV)^{0.598}$	1.0
5	$411.7(pV)^{0.598}$	1.0
Note. Degrees of regression functions determined with accuracy of ± 0.001.		

From Table 2 it can be seen that the regression functions found have the same degrees in the air temperature range from 527 to 927°C. Differences are observed in the multipliers B of the functions, the temperature dependence of which is shown in Fig. 2.

The design multipliers B were approximated with the following linear dependence on temperature with an R-square estimate of 0.9672:

$$B = 491.1 - 0.08668\ t.$$

A more accurate quadratic approximating function with an R-square estimate of 0.9872 has been established:

$$B = 544.8 - 0.2405\, t + 0.0001057\, t^2.$$

So, the following empirical expressions were obtained for calculation of the actual heat transfer coefficient from the complexes pV:

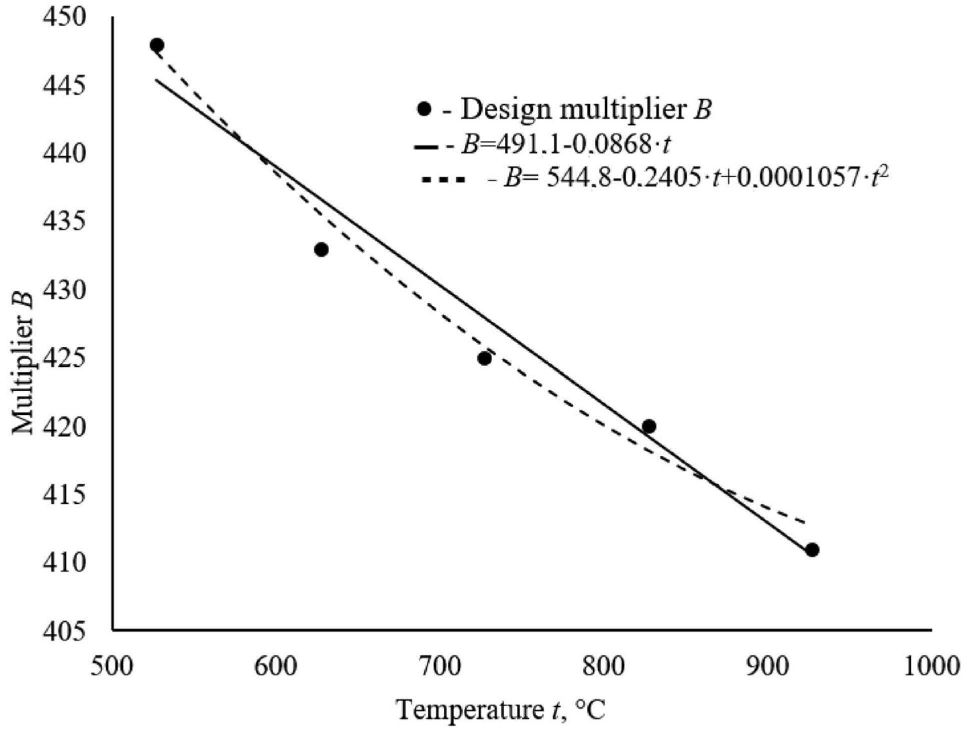

Fig. 2. Multiplier B vs. temperature.

$$\alpha = (491.1 - 0.0868\,t)(pV)^{0.598}\ \mathrm{W(m^2K)}; \tag{7}$$

$$\alpha = (444.8 - 0.2405\,t + 0.0001057\,t^2)(pV)^{0.598}\ \mathrm{W(m^2K)}, \tag{8}$$

where t is the air temperature, °C; p is the air pressure, MPa; V is the air speed, m/s.

4. CONCLUSIONS

It is proposed to calculate the actual time constants of the selected thermocouple model by the following expression:

$$T_i(\alpha) = \frac{1}{\alpha\,\Psi_{T_i}} + T_{i\infty},$$

where Ψ_{T_i} and $T_{i\infty}$ are the parameters of the corresponding time constant $T_i(\alpha)$, which are determined by the rules given in [19–21]; α is the heat transfer coefficient of the thermocouple sensitive element made of thermoelectrodes with diameter of 1.2 mm and calculated by (7) or (8).

The derived empirical expressions (7) and (8) yield the heat transfer coefficient α with varying accuracy. A comparison of the heat transfer coefficients calculated by (7) and (8) with the heat transfer coefficients calculated by (6) with the complex pV changing from 5 to 1000 MPa·m/s and the air temperature t from 527 to 927°C has shown that the calculation error according to (7) and (8) does not exceed ±1%.

If such error in the calculation of the coefficient α of the heat transfer between the thermocouple and the gas flow in the GTE is acceptable for the developers of the ACS gas temperature control loop, then the derived empirical expressions (7) and (8) can be used in practice in calculation of the actual time constants of the selected thermocouple model.

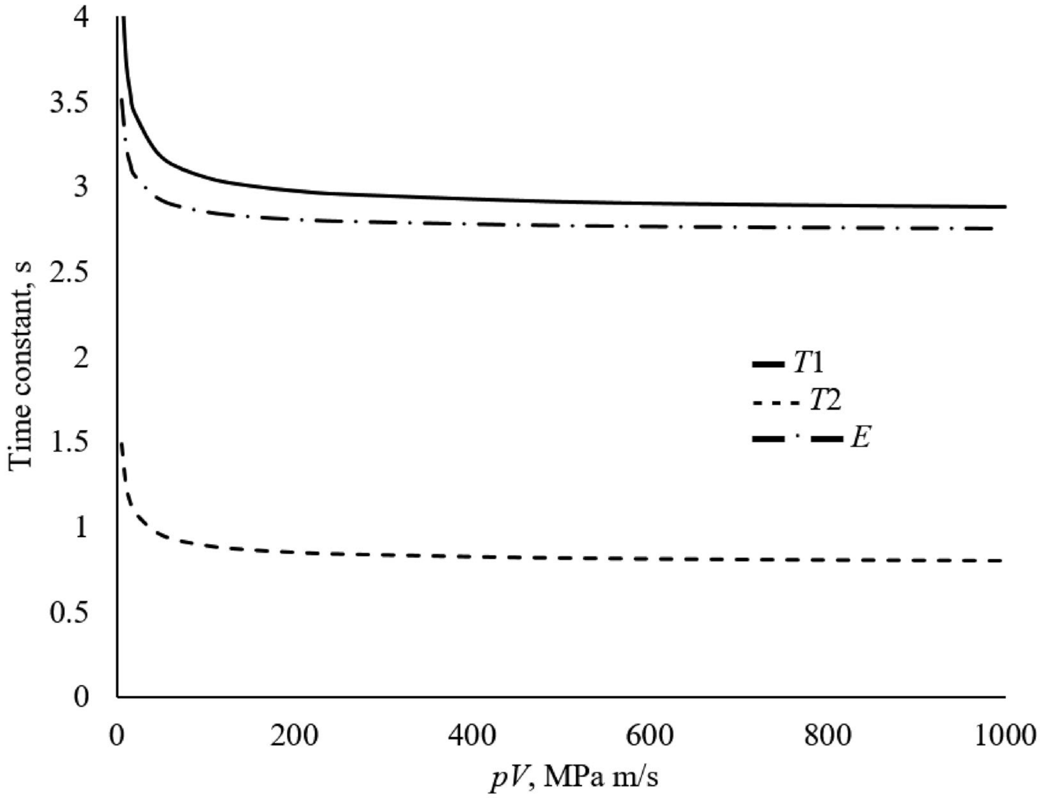

Fig. 3. Time constants of second-order model of thermocouple vs. complex pV.

The obtained relations (7) and (8) were used for the data from [19] for determination of the time constants (varying during operation) of an experimental thermocouple described by a second-order model of the form

$$W(p) = \frac{k\,[E(\alpha)p + 1]}{[T_1(\alpha)p + 1]\,[T_1(\alpha)p + 1]},$$

where $T_1(\alpha) = \frac{1}{\alpha \cdot 0.000666} + 2.825071$ [s]; $T_2(\alpha) = \frac{1}{\alpha \cdot 0.001305} + 0.774012$ [s];

$$E(\alpha) = \frac{1}{\alpha \cdot 0.001178} + 2.720597 \text{ [s]}.$$

Figure 3 shows the dependences of the time constants $T_1(\alpha)$, $T_2(\alpha)$, and $E(\alpha)$ on the complex pV with the heat transfer coefficient α calculated according to (8) at $t = 927°\text{C}$.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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