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Thermal Processes in Electronic Equipment at Uncertainty

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Abstract—At present, electronic systems are thermally designed on the basis of the assumption that all the parameters and factors that determine the thermal processes are fully known and unambiguously determined, id est, that they are determinate. However, the practice of creation and operation of real electronic systems shows that the real values of determining parameters and factors, as well as the thermal processes and temperature distributions, are uncertain and can take any values within some intervals of their variation with an equal probability. The disregard for the interval stochastic character of the thermal processes leads to design errors and development of uncompetitive electronic systems. This article elaborates a method that permits modeling non-stationary interval stochastic thermal processes in an electronic system at interval uncertainty of input factors and parameters. The method is based on obtaining equations for non-stationary statistical measures (mathematical expectations, variances, mean square deviations, and covariances) of thermal processes at specified statistical measures of input data. The article gives an example of applying the elaborated method to thermal processes in a real electronic system that consists of electronic modules with printed circuit boards, as well as integrated microcircuits, resistors, and other electronic components installed on them.

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1. INTRODUCTION

At the present time, electronic systems (ESs) are thermally designed on the basis of the assumption that all the parameters and factors that determine the thermal processes are fully and unambiguously known and determined, id est, that they are determinate. However, as shown in [10–12], the real practice of development and operation of ESs shows that parameters determining thermal processes in ESs are not determinate at all. For example, if parameters that determine thermal processes in different specimens of theoretically identical ESs are compared, they as a rule significantly differ one from another. For example, measurements of most important parameters of integral microcircuits (ICs) such as the junction-casing (R_{jc}) and casing-environment (R_{ca}) thermal resistances, as well as the consumed powers (P), for different specimens of ICs of identical type show that their values have a significant statistical spread within some intervals. The boundaries of these intervals are determined both by the technology of making the ICs and by the further output control over the ICs.

In fact, different specimens of theoretically identical ESs contain lots of interchangeable electronic components (microprocessors, ICs, and other electronic components), and the exact values of parameters of components installed in every particular specimen of an ES are unknown a priori. It is only known that these parameters vary within some intervals, the boundaries of which are determined by the statistical technological spread of manufacturing the components. Within these intervals, the particular values of the parameters can take arbitrary values with an equal probability. That is why it may be supposed that the indeterminate parameters of the components are random values evenly distributed within their intervals. The interval uncertainty of the parameters and factors that determine the thermal processes causes in turn interval uncertainty of the thermal processes and temperature distributions in the ESs. The interval uncertainty of the factors and processes the values of which are evenly distributed within their variation intervals is called interval stochastic uncertainty.

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Disregard for the stochastic nature of thermal processes in an ES and considering them as exclusively determined with point values of temperatures are not adequate to the real practice and hampers reliable calculation of the component temperatures and temperature distributions in the ES, which leads to design errors and creation of uncompetitive ESs.

The interval stochastic uncertainty of parameters and factors that determine interval stochastic thermal processes in an ES occurs because of three groups of interval stochastic factors:

(1) the statistical technological spread of the manufacturing, assembling, and mounting parameters of the ES components;

(2) the random character of the factors that arise while the ES is functioning, id est, the electricity supply currents and voltages, the powers consumed by the microprocessors, ICs, resistors, and other components of the ES, and the temperature and velocity of the cooling medium flows inside the ES;

(3) the randomness of the environmental parameters such as the temperature, pressure, and flow velocities.

The interval stochastic uncertainty of factors and parameters that determine the thermal processes in ESs is unavoidable and inherent to any technology of making and mounting ES components. That is why the problem of mathematical and computer modeling of interval stochastic thermal processes subject to interval stochastic factors and parameters is topical for adequate thermal design of ESs. Despite the importance and topicality of this problem, there are only few works dedicated to the said problematics [2–4, 7–12]. The methods that they consider are ad hoc and cannot be used in the practice of modeling and thermally designing complex ESs, which consist of a large number of diverse components, which interact one with another and with the environment.

This article considers a method for mathematical and computer modeling of non-stationary non-linear thermal processes under the conditions of interval stochastic uncertainty of factors and parameters that determine thermal processes in ESs. The method is based on obtaining equations that describe non-stationary statistical measures (mathematical expectations, variances, mean square deviations, and covariances) of thermal processes at specified statistical measures of input interval stochastic factors and parameters. Application of the devised method is exemplified by modeling interval stochastic thermal processes in a particular real ES.

2. THERMAL AND MATHEMATICAL MODELS OF NON-STATIONARY NON-LINEAR INTERVAL STOCHASTIC THERMAL PROCESSES IN ELECTRONIC SYSTEMS

Let us examine rather a generalized structure of an ES enclosed in a casing inside which diverse electronic components and commutating, fastening, and other structural parts are installed [10, 11]. The components used in the ES are divided into active and passive ones. The active components (microprocessors, ICs, resistors, and heat-releasing electronic components) consume electricity from electric power sources and convert it into heat, which warms up both the components and the whole ES. The passive components (heatsinks, electrical connectors, condensers, and fastening and structural components) do not consume electricity and are not heat sources. The fluid medium (air and drop liquid) inside the ES casing is formed by the flows that enter through the ES casing inlets and exit through its outlets. All the active and passive components of the ES thermally interact one with another, with the fluid medium inside the ES casing, and the environment outside the ES casing.

The thermal model of ES developed in the works [5, 9, 10, 11] is a system of $N+1$ isothermal components that is obtained after discretization of the ES structure into its constituents.

The ES thermal scheme (Fig. 1) that corresponds to the ES thermal model and is built on the basis of electrothermal analogy [5, 9–11] is a graph consisting of $N+1$ nodes, $M+1$ lines, $N-1$ independent sources of heat flows (Φ_i, $i = 1, 2, \ldots, N-1$), and two nodes with a priori known and specified temperature T_e of the environment and temperature $T_{a,in}$ of the fluid flow at the ES inlet (Fig. 1). According to the electrothermal analogy, the thermal scheme nodes model the isothermal components of the thermal model; the potentials in the nodes are the temperatures of the isothermal components; the currents in the lines are the thermal flows; the independent sources of the currents are the active component heat sources (powers consumed by them); the independent potentials in the nodes are the specified temperatures; the lines with the condensers are the total volume heat capacities of the thermal model components.

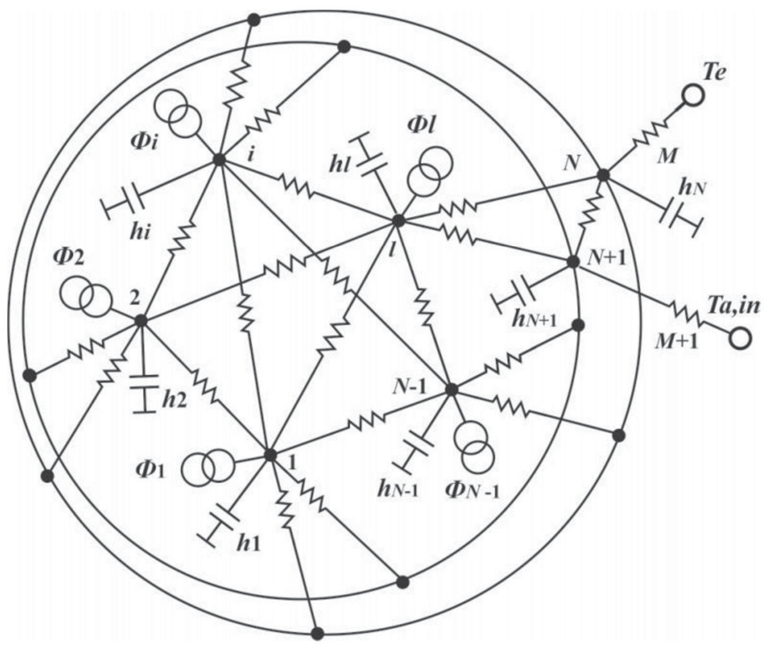

Fig. 1. A thermal scheme that corresponds to the ES's thermal model.

The mathematical model that describes the interval stochastic thermal processes in the ES is built on the basis of the thermal scheme graph (Fig. 1).

A thermal process in the ES is unambiguously defined by the following determining factors:

—the shape, dimensions, and spatial arrangement of the components in the ES, as well as the ES structure;

—the power consumed by the active components;

—the boundary conditions on the surfaces of the ES components and casing;

—the heat exchange character, determined by the convection (forced or natural), radiation, and conduction;

—the temperature and velocity of the fluid medium inside the ES casing;

—the environment temperature;

—conditions at the initial moment of time;

—the thermophysical characteristics (density, heat conductivity, heat capacity, and emissivity factor) of the materials of the components, fluid medium, and environment.

If the factors and parameters that determine the thermal processes in the ES are determinate, the thermal process will also be determinate, and the mathematical model that describes it will be determinate, too. If at least one of the parameters or factors of the thermal process is interval stochastic, the thermal process itself that takes place in the ES will have an interval stochastic character and will be described by a stochastic mathematical model. In order to ensure the adequacy, the mathematical model must reflect the character of the real thermal processes in the ES, namely: non-stationarity, non-linearity, and interval stochastic uncertainty.

The stochastic mathematical model that describes the interval stochastic temperatures $T_i(t, \omega)$, $i = 1, 2, \ldots, N + 1$, of the thermal scheme components (Fig. 1) and allows their calculation using the computer looks as follows [9, 10]:

$$H \frac{dT(t, \omega)}{dt} + AG(T, \omega)A^T T(t, \omega) = \Phi(\omega) + AG(T, \omega)T_a(t, \omega), \qquad (1)$$

$$T(0, \omega) = T_a(0, \omega), \quad (t, \omega) \in [0, \tau] \times \Omega,$$

where $T(t,\omega) = (T_1(t,\omega), T_2(t,\omega), \dots, T_{N+1}(t,\omega))^T$ is the stochastic $(N+1)$ vector of the stochastic temperatures in the thermal scheme nodes (thermal model components, Fig. 1); A is the rectangular $(N+1)\times(M+1)$ matrix of the incidences of the thermal scheme graph, $M+1$ is the quantity of the lines; $G(t,\omega) = diag(g_1(\omega), g_2(\omega), \dots, g_{M+1}(\omega))$ is the stochastic diagonal $(M+1)\times(M+1)$ matrix of the stochastic heat transfer rates in the lines $g_k(\omega)$, $k = 1, 2, \dots, M+1$; $H = diag(h_1, h_2, \dots, h_{N+1})$, is the diagonal $(N+1)\times(N+1)$ matrix of the total volume heat capacities $h_i = \rho_i c_i V_i$ of the ES thermal model components of a volume V_i, density ρ_i and specific heat capacity c_i ($i = 1, 2, \dots, N+1$); $\Phi(\omega) = (\Phi_1(\omega), \Phi_2(\omega), \dots, \Phi_{N-1}(\omega), 0, 0)^T$ is the stochastic $(N+1)$ vector of the stochastic independent sources of heat (consumed powers) $\Phi_i(\omega)$ of the ES active components; $T(0,\omega) = (T_1(0,\omega), T_2(0,\omega), \dots, T_{N+1}(0,\omega))^T$ is the stochastic $(N+1)$ vector of the known initial temperatures in the thermal scheme nodes; $T_a(t,\omega) = (0, 0, \dots, T_e(t,\omega), T_{a,in}(t,\omega))^T$ is the stochastic $(M+1)$ vector of the known stochastic temperatures of the environment $T_e(t,\omega)$ outside the ES casing and the fluid flow $T_{a,in}(t,\omega)$ at the ES inlet; $T_a(0,\omega) = (0, 0, \dots, T_e(0,\omega), T_{a,in}(0,\omega))^T$ is the stochastic $(M+1)$ vector of the known temperatures of the environment at the initial moment of time $t = 0$; Ω is the space of elementary events in the probability space $\{\Omega, U, P\}$, where U is the σ-algebra of Ω subsets and P is the probability on U [6]; $(*)^T$ is the transposition operation.

Thus, in the thermal scheme lines (Fig. 1), the heat transfer rates g_k ($k = 1, 2, \dots, M+1$) are stochastic functions of the stochastic temperatures $T_i = T_i(t,\omega)$, $T_j = T_j(t,\omega)$, $T_e = T_e(t,\omega)$, and $T_{a,in} = T_{a,in}(t,\omega)$, namely:

—in lines $k = 1, 2, \dots, M-1$ connected to nodes i and j, $i, j = 1, 2, \dots, N+1$, the heat transfer rates are functions of the temperatures T_i, T_j, id est $g_k = g_k(T_i, T_j, \omega)$;

—in line M between node N and a node with a known and specified temperature of the environment T_e, the heat transfer rate is the function $g_M = g_M(T_N, T_e, \omega)$;

—in line $M+1$ between node $N+1$ and a node with a known and specified temperature of the fluid flow $T_{a,in}$ at the ES inlet, the heat transfer rate is the function $g_{M+1} = g_{M+1}(T_{N+1}, T_{a,in}, \omega)$.

Stochastic matrix equation (1) with the stochastic vector initial condition is a system of non-stationary, non-linear, stochastic differential equations of the first order in the ordinary derivatives; the system fully determines the sought vector of the stochastic non-stationary temperatures $T(t,\omega)$ of the ES components.

It is known [1, 6, 13] that the laws of distribution of probabilities of all orders at all moments of time constitute an exhaustive characteristic of random processes. However, because of the extreme complexity of mathematical model equations (1), it seems to be impossible to determine the laws of distribution of the stochastic non-stationary temperatures $T(t,\omega)$ of the ES components. At the same time, no knowledge of the laws of distribution of the stochastic non-stationary temperatures of the ES components is required for modeling the stochastic thermal processes in an ES. For the engineering practice of thermal design of ESs, the most important and informative characteristics consist in the statistical measures of the ES component interval stochastic temperatures $T_i(t,\omega)$, $i = 1, 2, \dots, N+1$, such as ($E\{\cdot\}$ is the mathematical expectation operator)

—the mathematical expectations $\overline{T}_i(t) = E\{T_i(t,\omega)\}$;

—the variances $D_{Ti}(t) = E\left\{(\overset{0}{T}_i(t,\omega))^2\right\}$, where $\overset{0}{T}_i(t,\omega) = T_i(t,\omega) - \overline{T}_i(t)$ is the centered stochastic temperature with a zero mathematical expectation $E\left\{\overset{0}{T}_i(t,\omega)\right\} = 0$ and a variance equal to $D_{Ti}(t)$;

—the mean square deviations $\sigma_{Ti}(t) = \sqrt{D_{Ti}(t)}$;

—the covariances $K_{Ti,Tj}(t) = E\left\{\overset{0}{T}_i(t,\omega)\overset{0}{T}_j(t,\omega)\right\}$ between the temperatures $T_i = T_i(t,\omega)$ and $T_j = T_j(t,\omega)$ of components i and j ($i, j = 1, 2, \dots, N+1$).

From the found statistical measures of the stochastic temperatures ($\overline{T}_i(t)$, $D_{Ti}(t)$, $\sigma_{Ti}(t)$, $K_{Ti,Tj}(t)$, $i, j = 1, 2, \dots, N+1$) one can determine the boundaries of the intervals of the real values of the ES component temperatures.

3. LINEARIZED MATHEMATICAL MODEL OF STOCHASTIC THERMAL PROCESSES

Because of the non-linearity of Eqs. (1) relative to the stochastic temperatures $T_i = T_i(t,\omega)$, $T_j = T_j(t,\omega)$, $T_e = T_e(t,\omega)$, and $T_{a,in} = T_{a,in}(t,\omega)$, it seems to be impossible to obtain equations for the statistical measures of the stochastic temperatures $T_i(t,\omega)$, $i = 1,2,\ldots,N+1$. However, the random deviations of the random functions contained in the equations from their mathematical expectations are as a rule rather small in comparison with their mathematical expectations, which permits applying methods of linearization of the examined non-linear equations relative to the centered random functions. As a result of the linearization developed in works [10–12], the initial equations will preserve their non-linear character relative to the mathematical expectations of the non-stationary stochastic temperatures, but they will be linear relative to the centered stochastic temperatures. Such an approach makes it possible to find the final equations that describe the non-stationary statistical measures of the component stochastic temperatures.

Let us write the centered stochastic temperatures $\overset{0}{T}_i = \overset{0}{T}_i(t,\omega)$ $(i = 1,2,\ldots,N+1)$, $\overset{0}{T}_e = \overset{0}{T}_e(t,\omega)$, $\overset{0}{T}_{a,in} = \overset{0}{T}_{a,in}(t,\omega)$, and heat transfer rates $\overset{0}{g}_{ij} = \overset{0}{g}_{ij}(\omega)$:

$$\overset{0}{T}_i = T_i(t,\omega) - \overline{T}_i(t), \quad \overset{0}{T}_e = T_e(t,\omega) - \overline{T}_e(t), \quad \overset{0}{T}_{a,in} = T_{a,in}(t,\omega) - \overline{T}_{a,in}(t),$$

$$\overset{0}{g}_{ij} = g_{ij}(\omega) - \overline{g}_{ij},$$

where $\overline{T}_i(t)$, $\overline{T}_e(t)$, $\overline{T}_{a,in}(t)$, and $\overline{g}_{ij}$ are the mathematical expectations of the respective stochastic functions.

In practice, the random deviations of the temperatures and heat transfer rates from their mathematical expectations always meet the following conditions:

$$\left|\overset{0}{T}_i/\overline{T}_i\right| < 1, \quad \left|\overset{0}{T}_e/\overline{T}_e\right| < 1, \quad \left|\overset{0}{T}_{a,in}/\overline{T}_{a,in}\right| < 1, \quad \left|\overset{0}{g}_{ij}/\overline{g}_{ij}\right| < 1,$$

and that is why non-linear equations (1) of the mathematical model may be linearized using the method of Taylor series expansion, terms of order not higher than the first one kept. As a result, we will obtain a stochastic matrix system of equations that is non-linear relative to the non-stationary mathematical expectations of the temperatures $\overline{T}(t)$ and linear relative to the non-stationary stochastic centered temperatures $\overset{0}{T}(t,\omega)$, namely [11]:

$$\begin{aligned} &H\frac{dT(t,\omega)}{dt} + A\overline{G}(\overline{T})A^T\overline{T}(t) + A\overline{V}(\overline{T})A^T\overset{0}{T}(t,\omega) + A\overset{0}{Q}(\omega)A^T\overline{T}(t) \\ &= \Phi(\omega) + A\overline{G}(\overline{T})\overline{T_a}(t) + A\overset{0}{Q}(\omega)\overline{T_a}(t) + A\overline{V}(\overline{T})\,\overset{0}{T}_a(t,\omega), \end{aligned} \tag{2}$$

$$T(0,\omega) = T_a(0,\omega),$$

where $\overline{T}(t) = (\overline{T}_1(t), \overline{T}_2(t), \ldots, \overline{T}_{N+1}(t))^T$ is the $N+1$ vector of the mathematical expectations of the temperatures in the thermal scheme nodes; $\overset{0}{T}(t,\omega) = \left(\overset{0}{T}_1(t,\omega), \overset{0}{T}_2(t,\omega), \ldots, \overset{0}{T}_{N+1}(t,\omega)\right)^T$ is the stochastic $(N+1)$ vector of the centered stochastic temperatures in the thermal scheme nodes; $\overline{T}_a(t) = (0,0,\ldots,0,\overline{T}_e(t),\overline{T}_{a,in}(t))^T$ is the $(M+1)$ vector of the mathematical expectations of the stochastic temperatures $T_e(t,\omega)$ and $T_{a,in}(t,\omega)$; $\overset{0}{T}_a(t,\omega) = \left(0,0,\ldots,\overset{0}{T}_e(t,\omega),\overset{0}{T}_{a,in}(t,\omega)\right)^T$ is the stochastic $(M+1)$ vector of the centered stochastic temperatures $T_e(t,\omega)$ and $T_{a,in}(t,\omega)$; $\overline{G}(\overline{T}) = diag(\overline{g}_1,$

$\overline{g}_2, \ldots, \overline{g}_{M+1})$ is the diagonal $(M+1)\times(M+1)$ matrix of the mathematical expectations of the heat transfer rates in the lines $\overline{g}_k = \overline{g}_k(\overline{T}_i, \overline{T}_j)$, $k = 1, 2, \ldots, M+1$; $\overline{V}(\overline{T}) = diag(\overline{v}_1, \overline{v}_2, \ldots, \overline{v}_{M+1})$ is the diagonal $(M+1)\times(M+1)$ matrix with elements $\overline{v}_k$, $k = 1, 2, \ldots, M+1$, equal to the first derivatives of the heat transfer rates $g_k = g_k(T_i, T_j, \omega)$ with respect to the temperatures T_i and T_j taken at the mathematical expectations $\overline{T}_i$ and $\overline{T}_j$ in the expansion of the non-linear heat transfer rates $g_k = g_k(T_i, T_j, \omega)$, $k = 1, 2, \ldots, M+1$, in the Taylor series; $\overset{0}{Q}(\omega) = diag\left(\overset{0}{g}{}_1^{cond}(\omega), \overset{0}{g}{}_2^{cond}(\omega), \ldots, \overset{0}{g}{}_{M-1}^{cond}(\omega), 0, 0\right)$ is the stochastic diagonal $(M+1)\times(M+1)$ matrix of the centered stochastic conductive heat transfer rates.

Assessment of the error of the linearization developed in works [10, 11] shows that the Taylor series expansion linearization with only the linear terms kept makes it possible to model the thermal processes in ESs with an accuracy sufficient for engineering practice.

The stochastic thermal process in the ES described by Eq. (8) with the initial conditions is fully determined by the statistical measures of the following stochastic factors:

—mathematical expectation $\overline{T}_e(t)$ and variance $D_{Te}(t)$ of the interval stochastic temperature of the environment $T_e(t, \omega)$;

—mathematical expectation $\overline{T}_{a,in}(t)$ and variance $D_{Ta,in}(t)$ of the interval stochastic temperature of the fluid flow $T_{a,in}(t, \omega)$ at the ES inlet;

—mathematical expectations $\overline{\Phi}_i(\omega)$ and variances $D_{\Phi i}(t)$ of the interval stochastic powers of the internal sources of heat with a power $\Phi_i(\omega)$, $i = 1, 2, \ldots, N+1$;

—mathematical expectations $\overline{g}_{ij}^{cond}$ and variances D_{gij} of the interval stochastic conductive heat transfer rates $g_{ij}^{cond}(\omega)$, $i, j = 1, 2, \ldots, N$;

—mathematical expectation $\overline{T}_e(0)$ and variance $D_{Te}(0)$ of the component initial interval stochastic temperatures $T_e(0, \omega)$, $i = 1, 2, \ldots, N+1$.

4. OBTAINING EQUATIONS FOR STATISTICAL MEASURES OF ES COMPONENT STOCHASTIC TEMPERATURES

We will obtain equations that describe the following statistical measures of the stochastic temperatures $T_i(t, \omega)$ of components $i = 1, 2, \ldots, N+1$ of the ES thermal model:

—the temperature mathematical expectation vector $\overline{T}(t) = (\overline{T}_1(t), \overline{T}_2(t), \ldots, \overline{T}_{N+1}(t))^T$, where $\overline{T}_i(t) = E\{T_i(t, \omega)\}$;

—the temperature covariance matrix $K_{T,T}(t) = E\left\{\overset{0}{T}(t, \omega)\overset{0}{T}{}_{N+1}^{T}(t, \omega)\right\}$, the elements of which are equal to the covariances $K_{Ti,Tj}(t) = E\left\{\overset{0}{T}_i(t, \omega)\overset{0}{T}_j(t, \omega)\right\}$ between the temperatures $T_i = T_i(t, \omega)$ and $T_j = T_j(t, \omega)$ of components i and j $(i, j = 1, 2, \ldots, N+1)$;

—the temperature variance vector $D_T(t) = (D_{T1}(t), D_{T2}(t) \ldots D_{T,N=1}(t))^T$, where $D_{Ti}(t) = E\left\{(\overset{0}{T}_i(t, \omega))^2\right\}$ are the diagonal elements of the covariance matrix $K_{T,T}(t)$;

—the component temperature mean square deviation vector $\sigma_T(t) = (\sigma_{T1}(t), \sigma_{T2}(t), \ldots, \sigma_{T,N=1}(t))^T$, where $\sigma_{Ti}(t) = \sqrt{D_{Ti}(t)}$.

The found mathematical expectations $\overline{T}_i(t)$ and mean square deviations $\sigma_{Ti}(t)$ of the stochastic temperatures $T_j(t, \omega)$ of components $i = 1, 2, \ldots N+1$, permit determining the lower $T_{Bot,i}(t)$ and upper $T_{Up,i}(t)$ boundaries of the temperature intervals $[T_{Bot,i}(t), T_{Up,i}(t)]$ of the real values of the ES component temperatures, id est,

$$T_{Bot,i}(t) = \overline{T}_i(t) - \varepsilon \cdot \sigma_{Ti}(t), T_{Up,i}(t) = \overline{T}_i(t) + \varepsilon \cdot \sigma_{Ti}(t), \tag{3}$$

where ε is the coefficient that determines the width of the interval that covers the real values of the ES component temperatures with a specified probability value P. The coefficient ε value may be assessed using Chebyshev's inequality [6, 13] $P\left\{\overset{0}{T}_i(t,\omega) \le \varepsilon \cdot \sigma_{Ti}(t)\right\} \ge 1 - 1/\varepsilon^2$ with an accuracy sufficient for the engineering practice.

Without belittling the generality and simplification of the following results, let us accept that the stochastic temperatures $T_e(\omega)$ and $T_{a,in}(\omega)$ do not depend on the time and are random values with known mathematical expectations and variances.

As a result, according to the method developed in works [10, 11, 12], we will obtain the following equations relative to the statistical measures of the stochastic temperatures $T_i(t,\omega)$ of the ES thermal model components, $i = 1, 2, \ldots, (N+1)$:

—the Equation for the Temperature Mathematical Expectation Vector $\overline{T}(t)$

$$H\frac{d\overline{T}(t)}{dt} + A\overline{G}(\overline{T})A^T\overline{T}(t) = \overline{\Phi} + A\overline{G}(\overline{T})\overline{T}_a, \tag{4}$$

$$\overline{T}(0) = \overline{T}_a(0),$$

—the Equation for the Temperature Covariance Matrix $K_{TT}(t)$

$$\begin{gathered}\frac{dK_{TT}(t)}{dt} + H^{-1}A\overline{V}(\overline{T})A^TK_{TT}(t) + K_{TT}(t)A\overline{V}(\overline{T})A^TH^{-1} + H^{-1}AK_{QT}(t) + K_{QT}^T(t)A^TH^{-1} \\ = H^{-1}K_{\Phi T}(t) + K_{\Phi T}^T(t)H^{-1} + H^{-1}A\overline{V}(\overline{T})K_{TaT}(t) + K_{TaT}^T(t)\overline{V}(\overline{T})A^TH^{-1}, \\ K_{TT}(0) = D_{Te}I_1,\end{gathered} \tag{5}$$

where I_1 is the $(N+1) \times (N+1)$ matrix all the elements of which equal 1.

As it follows from Eq. (4), in order to obtain its solution relative to the covariance matrix $K_{TT}(t)$, it is also necessary to have some equations for determining the matrices $K_{\Phi T}(t)$, $K_{TaT}(t)$, and $K_{QT}(t)$ and included in it.

According to [10–12], we obtain

$$\frac{dK_{\Phi T}(t)}{dt} + K_{\Phi T}(t)A\overline{V}(\overline{T})A^TH^{-1} = K_{\Phi\Phi}H^{-1}, \tag{6}$$

$$K_{\Phi T}(0) = 0,$$

where $K_{\Phi\Phi} = E\left\{\overset{0}{\Phi}(\omega)\overset{0}{\Phi}{}^T(\omega)\right\}$ is the known covariance matrix of the stochastic powers of the independent sources of heat in the ES components,

$$\frac{dK_{TaT}(t)}{dt} + K_{TaT}(t)A\overline{V}(\overline{T})A^TH^{-1} = K_{TaTa}\overline{V}(\overline{T})A^TH^{-1}, \tag{7}$$

$$K_{TaT}(0) = K_{TaTa} \cdot I_2^T,$$

where $I_2^T = (11\ldots1)^T$ is the $N+1$ vector with elements equal to 1; $K_{TaTa} = E\left\{\overset{0}{T}_a(\omega)\overset{0}{T}{}_a^T(\omega)\right\}$ is the known covariance matrix of the environment's stochastic temperatures, and

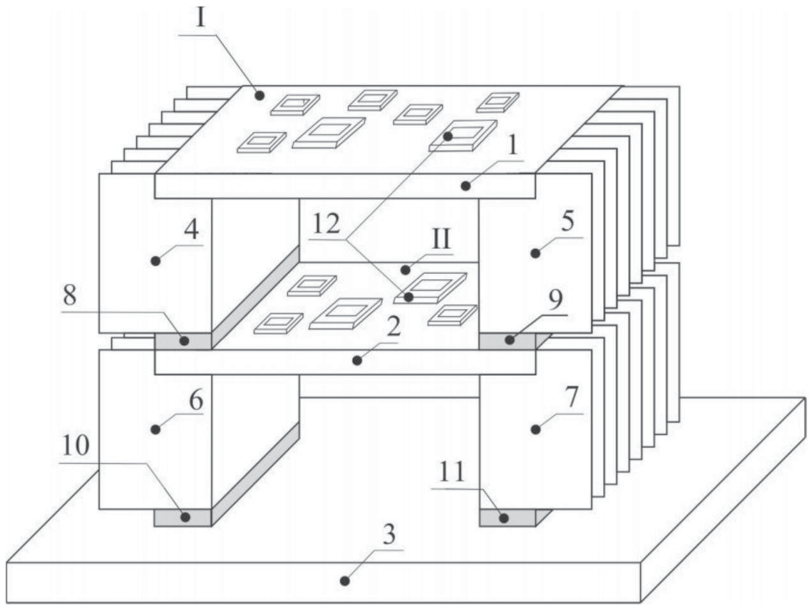

Fig. 2. ES structure with electronic devices I and II and heat-conducting plate 3. Electronic devices I and II consist of electronic modules EM1 and EM2, their edges attached to ribbed radiators 4, 5, 6, and 7 through heat-conducting elastic gaskets 8, 9, 10, and 11. Electronic modules EM1 and EM2 contain multilayer printed circuit boards 1 and 2, as well as active and passive components 12 (ICs, resistors, heat-releasing electronic components, radiators, electrical connectors, condensers, etc.) installed on them.

$$\frac{dK_{gT}(t)}{dt} + K_{gT}(t)A\overline{V}(\overline{T})A^T H^{-1} + K_{gg}\overline{R}(\overline{T})A^T H^{-1} = 0, \tag{8}$$

$$K_{gT}(t) = 0,$$

where $K_{gg} = E\left\{\overset{0}{g}{}^{cond}(\omega)\left(\overset{0}{g}{}^{cond}(\omega)\right)^T\right\}$ is the known covariance $(M+1)\times(M+1)$ matrix of the conductive heat transfer rates. Taking into account the independence of the random values $\overset{0}{g}{}^{cond}(\omega)$ for all the $k = 1, 2, \dots, M+1$, we will obtain that the correlation matrix K_{gg} is diagonal with its elements equal to the variances of the conductive heat transfer rates, id est, $K_{gg} = diag(D_{gcond,1} D_{gcond,2}, \dots, D_{gcond,M-1}, 0, 0)$.

Thus, the statistical measures of the stochastic temperatures $T_i(t,\omega)$, $i = 1, 2, \dots, N+1$, of the ES thermal model components are determined from the solution of non-linear matrix differential equations (4) and (5). In so doing, in order to obtain the solution of equations (5) relative to the covariance matrix $K_{TT}(t)$, it is necessary to additionally solve non-linear matrix differential equations (6), (7), and (8).

Equations (4)–(8) are a system of matrix non-stationary non-linear differential equations of the first order in ordinary derivatives. However, despite the substantially non-linear character of these equations, they are easily computed with personal computers and do not require much machine time.

5. APPLICATION IN PRACTICE

Let us exemplify the application of the method elaborated in the article by an ES (Fig. 3) that includes electronic devices I and II, as well as heat-conducting plate 3, on which electronic device II is installed. The electronic devices consist of electronic modules EM1 and EM2 (1 and 2, Fig. 2), their edges attached to ribbed radiators 4, 5, 6, and 7 through heat-conducting elastic gaskets 8, 9, 10, and 11. Each of the electronic modules EM1 and EM2 contains a multilayered printed circuit board with active

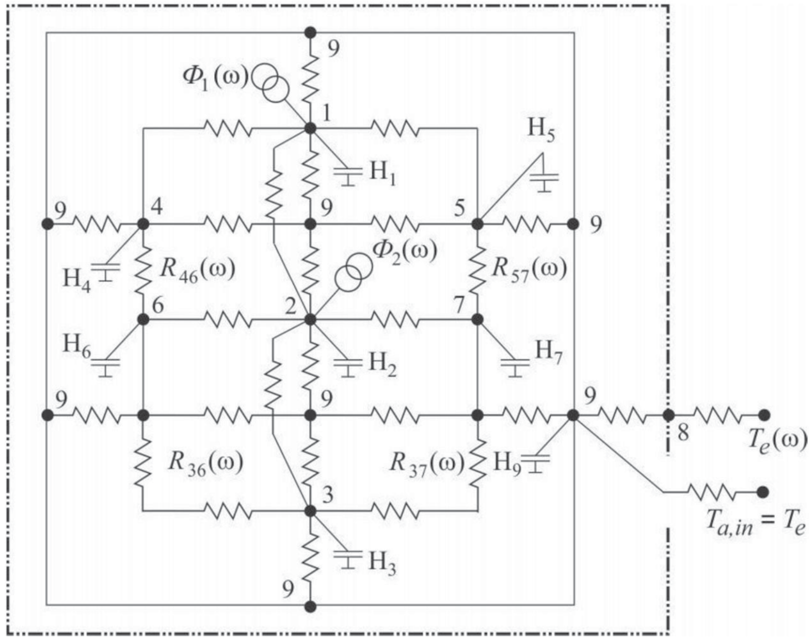

Fig. 3. The thermal scheme that corresponds to the ES structure in Fig. 2.

and passive electronic components 12 (ICs, resistors, heat-releasing electronic components, radiators, electrical connectors, condensers, etc.) installed on it. When the ES works, the powers consumed by the active electronic components transform into heat, which warms up the electronic modules and the whole ES; the heat exchange among all the ES components and the environment takes place simultaneously through the natural convection, radiation, and conduction.

Because of the interval stochastic technological spread in making, installing, and mounting the electronic modules in the ES, the thermal processes in the ES components, electronic modules, and the whole ES will have an interval stochastic character. In the example under consideration, we accept that the following ES factors and parameters are subject to a statistical spread: the powers consumed by the electronic modules EM1 and EM2 (the independent sources $\Phi_1(\omega)$ and $\Phi_2(\omega)$ on the thermal scheme, Fig. 4); the thicknesses of gaps 8 and 9 (the thermal resistances $R_{46}(\omega)$ and $R_{57}(\omega)$, Fig. 4) between electronic devices I and II; the thicknesses of gaps 10 and 11 (the thermal resistances $R_{36}(\omega)$ and $R_{37}(\omega)$, Fig. 3) between electronic device II and heat-conducting plate 3; the environment temperature $T_e(\omega)$ (the fluid temperature $T_{a,in}(\omega)$ at the ES inlet is equal to the environment temperature $T_e(\omega)$).

Non-stationary non-linear equations (4)–(8) relative to the statistical measures are solved by means of the Runge–Kutta numerical method using the STP-ES (Simulation of Thermal Processes in Electronic Systems) software complex specially developed by the authors, which permits computer modeling of interval stochastic thermal processes in electronic systems of any complexity. The computations are made at the following interval spreads of the initial data:

- the interval stochastic temperature of the environment $T_e(\omega) \in [19.5; 26.5]$, °C, and its mathematical expectation $\overline{T}_e = 23$°C;
- the interval stochastic powers consumed by the electronic modules EM1 $\Phi_1(\omega) \in [12; 18]$ and EM2 $\Phi_2(\omega) \in [16.5; 19.5]$ as well as their mathematical expectations $\overline{\Phi}_1 = 15\ W$ and $\overline{\Phi}_2 = 18\ W$;
- the interval stochastic rates of the conductive heat transfer through the gaps $g_{46}(\omega), g_{57}(\omega), g_{36}(\omega), g_{39}(\omega) \in [0, 12; 1, 32]$, W/K, and their mathematical expectations $\overline{g}_{46}, \overline{g}_{57}, \overline{g}_{36}, \overline{g}_{37} = 0.72$, W/K.

The non-stationary values of the lower $T_{Bot,i}(t)$ and upper $T_{Up,i}(t)$ boundaries of the intervals $[T_{Bot,i}(t), T_{Up,i}(t)]$, $i = 1, 2$, within which the real temperatures of the electronic modules EM1 and EM2 will vary (thermal scheme nodes $i = 1, 2$, Fig. 3) are calculated according to formulas (3) at $\varepsilon = 3$ and a probability $P = 0.89$.

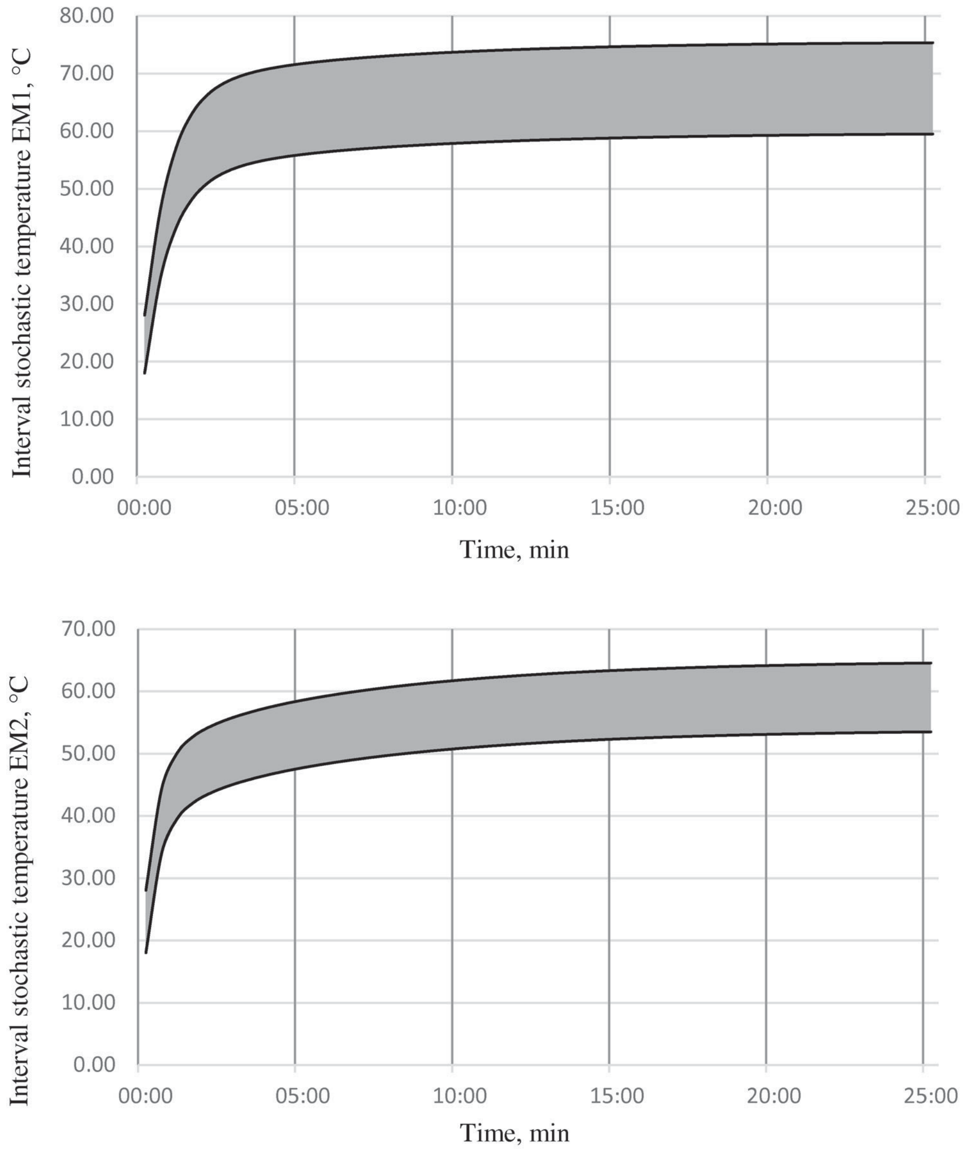

Fig. 4. Dynamics of variation of interval stochastic temperatures (°C) of electronic modules EM1 and EM2 with time (minutes).

6. RESULTS AND ANALYSIS

The obtained results (Fig. 4) show that in the electronic modules EM1 and EM2, the interval stochastic temperatures set in starting from the moment of time $t \geq 25$ min. In the thermal process that has set in, the temperature of the electronic module EM1 varies within the interval [59.7; 75.5], °C, at a mathematical expectation $\overline{T}_1 = 67.6$°C, and the temperature of the electronic module EM2 lies within the interval [53.8; 64.9], °C, at a mathematical expectation $\overline{T}_1 = 59.4$°C (Fig. 4). Thus, the interval stochastic temperatures $T_1(t, \omega)$ and $T_2(t, \omega)$ that settled in the real electronic modules EM1 and EM2 can have any values within the intervals $T_1(\omega) \in [59.7; 75.5]$, °C, and $T_2(\omega) \in [53.8; 64.9]$, °C, with a probability of not less than 0.89. The spread of the EM1 and EM2 temperature variation intervals—$T_1(\omega)$ and $T_2(\omega)$—amounts to 15.8°C and 11.1°C respectively.

The found interval values of the temperatures are of a great practical importance since they make it possible to prognosticate —as early as at the stage of thermally designing the ESs —the real biggest and smallest temperatures of components that will take place in practice during the operation of real ESs and to assess the electric and reliability parameters of the designed ESs more accurately.

7. CONCLUSIONS

This article presents a method elaborated for mathematical modeling of non-linear non-stationary interval stochastic thermal processes in electronic systems (ESs); the method permits calculating the non-stationary temperatures of electronic and other components of ESs, as well as the intervals of the real temperature values. The method elaborated in the article makes it possible to obtain equations that describe the non-stationary statistical measures of the interval stochastic temperature distributions in an ES: mathematical expectations, variances, mean square deviations, and covariances between the temperatures of all the components. Equations (4)–(8) for calculating the statistical measures of the thermal processes in an ES are a system of non-stationary non-linear differential equations of the first order in ordinary derivatives. They are obtained under the most common conditions that do not use unrealistic assumptions about representing the stochastic factors in the form of white noises or Wiener processes such as Brownian motion.

The method elaborated in the article takes into account principal peculiarities of the structure of and heat exchange in an ES such as the stochastic character of heat exchange in the ES, conditioned by the statistical spread of the thermal, electrical, and structural factors and parameters at manufacturing, assembling, and mounting the ES, as well as by the interval stochastic character of the environment conditions;

—the non-linearity of the heat exchange processes, conditioned by the temperature dependence of the parameters and factors that determine the thermal processes in the ES;

—the non-stationary character of the thermal processes.

In order to model and calculate the statistical measures of the interval stochastic thermal processes in ESs, the authors developed a specialized software complex STP-ES (Simulation of Thermal Processes in Electronic Systems) for thermal design of ESs of any complexity; the software has shown its adequacy and effectiveness in designing and creating modern competitive ESs.

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REFERENCES

1. Adomian, G., *Stochastic Systems*, New York: Academic Press, 1983.
2. Wang, C., Qiu, Z., and Chen, X., Uncertainty Analysis for Heat Convection-Diffusion Problem with Large Uncertain-but-Bounded Parameters, *Acta Mech.*, 2015, vol. 226, no. 11, pp. 3831–3844.
3. Wang, C. and Qiu, Z., Hybrid Uncertain Analysis for Steady-State Heat Conduction with Random and Interval Parameters, *Int. J. Heat Mass Transfer*, 2015, vol. 80, pp. 319–328.
4. Claudio, R., Avila, da S.Jr., Azikri de Deus, H.P., Kozlik, A.Jr., and Garcia, O.S., Application of the Method of Galerkin to Non Linear Problem Stochastic Heat Conduction One-Dimensional, *Appl. Mech. Mater.*, 2015, vol. 751, pp. 325–330.
5. Ellison, G.N., *Thermal Computations for Electronics. Conductive, Radiative, and Convective Air Cooling*, New York: CRC Press, 2011.
6. Feller, W., *An Introduction to Probability Theory and Its Applications*, vol. 1, 3rd ed., New York: Wiley, 1970.
7. Keller, C.J. and Antonetti, V.W., Statistical Thermal Design for Computer Electronics, *Electronic Pack. Prod.*, 1979, vol. 19, no. 3, pp. 55–62.
8. Madera, A.G., Simulation of Stochastic Heat Conduction Processes, *Int. J. Heat Mass Transfer*, 1994, vol. 37, no. 16, pp. 2671–2677.
9. Madera, A.G., *Modelirovanie teploobmena v tekhnicheskikh sistemakh* (Modeling of Heat Transfer in Technical Systems), Moscow: Akad. V.A. Mel'nikov Sci. Found., 2005.
10. Madera, A.G., Hierarchical Method for Mathematical Modeling of Stochastic Thermal Processes in Complex Electronic Systems, *Comput. Res. Model.*, 2019, vol. 11, no. 4, pp. 613–630.
11. Madera, A.G., Interval-Stochastic Thermal Processes in Electronic Systems: Analysis and Modeling, *J. Eng. Thermophys.*, 2017, vol. 26, no. 1, pp. 17–28.
12. Madera, A.G., Interval-Stochastic Thermal Processes in Electronic Systems: Modeling in Practice, *J. Eng. Thermophys.*, 2017, vol. 26, no. 1, pp. 29–38.
13. Pugachev, V.S., *Probability Theory and Mathematical Statistics for Engineers*, Oxford: Pergamon Press, 1984.