ISSN 1547-4771, Physics of Particles and Nuclei Letters, 2023, Vol. 20, No. 3, pp. 425–428. © Pleiades Publishing, Ltd., 2023.

PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

Radiation of an Electron in a Lorentz-Violating Vacuum[1]

A. V. Borisov*

Faculty of Physics, Lomonosov Moscow State University, Moscow, 119991 Russia

**e-mail: borisov@phys.msu.ru*

Received October 27, 2022; revised November 25, 2022; accepted December 5, 2022

Abstract—The electromagnetic radiation of an electron in a constant background tensor field that violates Lorentz invariance is investigated in the framework of the Standard Model Extension. It is shown that the radiation effect can manifest itself under astrophysical conditions at ultrahigh electron energy.

DOI: 10.1134/S1547477123030160

INTRODUCTION

The Standard Model (SM) received its final experimental confirmation after the discovery of the Higgs boson in 2012 (for a recent review see [1]). However, it is not a complete theory, since a number of fundamental problems cannot be solved within its framework (see, e.g., [2]): electric charge quantization, a huge hierarchy of particle masses, dark matter and dark energy, baryon asymmetry in the Universe, etc. This circumstance causes the active development of theories generalizing the SM [2, 3].

Some such theories involve Lorentz violation. They include a theory called the Standard Model Extension (SME, see [4] and references therein), and we use it in the present paper. The SME Lagrangian is represented as the sum of the SM Lagrangian and additional terms—various combinations of SM fields with free tensor indices (it violates Lorentz invariance) convoluted with constant coefficients of the corresponding tensor ranks. These coefficients are considered as constant background fields that simulate the complex structure of the vacuum due to new physics not described by the SM.

Within the framework of the SME, various effects were studied. We indicate only a few works (see also references therein) that consider a number of processes in the axial-vector background field (AVBF): the electron-positron pair production by a photon and the photon emission by an electron and a positron [5, 6], synchrotron radiation of an electron taking into account its anomalous magnetic moment and interaction with the AVBF [7], its influence on the radiation of a hydrogen-like atom [8], generation of a vacuum current by the AVBF [9].

[1] Based on a talk given at the the International Conference on Quantum Field Theory, High-Energy Physics, and Cosmology (BLTP, JINR, Dubna, 18–21 July 2022).

In the present work, we consider the radiation of an electron in a tensor-type background field based on the extended Lagrangian of quantum electrodynamics as part of the SME Lagrangian[2]:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{T}}. \quad (1)$$

Here

$$\mathcal{L}_{\text{QED}} = \bar{\psi}\left(\gamma^{\mu}\left(i\partial_{\mu} + eA_{\mu}\right) - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\left(\partial_{\mu}A^{\mu}\right)^{2} \quad (2)$$

is the Lagrangian of the standard QED in the Lorentz gauge, ψ is the electron-positron field (m and $-e < 0$ are the electron mass and charge), A^{μ} and $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$ are the 4-potential and tensor of the electromagnetic field;

$$L_{\text{T}} = -\frac{1}{2}\bar{\psi}\sigma^{\mu\nu}H_{\mu\nu}\psi \quad (3)$$

is the Lagrangian of interaction with a tensor constant background field $H_{\mu\nu}$.

WAVE FUNCTIONS AND AN ELECTRON PROPAGATOR IN A TENSOR BACKGROUND FIELD

The wave function of an electron is a solution to the Dirac equation, which follows from (1)–(3):

$$\left(i\gamma^{\mu}\partial_{\mu} - m - \frac{1}{2}\sigma^{\mu\nu}H_{\mu\nu}\right)\psi = 0. \quad (4)$$

[2] A system of units is used in which $\hbar = c = 1$, $\alpha = e^{2}/4\pi \simeq 1/137$, and a pseudo-Euclidean metric with signature (+−−−); $\gamma^{5} = i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$, $\sigma^{\mu\nu} = i[\gamma^{\mu}, \gamma^{\nu}]/2$.

We restrict ourselves to the case of a background field of the quasimagnetic type:

$$H^{\mu\nu}H_{\mu\nu} > 0, \quad H^{\mu\nu}\tilde{H}_{\mu\nu} = 0,$$

where $\tilde{H}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}H^{\alpha\beta}/2$ ($\varepsilon_{0123} = -\varepsilon^{0123} = -1$). Then, in a special reference frame, the nonzero components of the tensors are as follows:

$$H_{21} = -H_{12} = h, \quad \tilde{H}_{03} = -\tilde{H}_{30} = h, \tag{5}$$

so that the tensor field is equivalent to the axial vector (we put $h > 0$)

$$\mathbf{h} = \mathbf{h}\mathbf{e}_z, \quad h = \left[H^{\mu\nu}H_{\mu\nu}/2\right]^{1/2}. \tag{6}$$

The wave function, which is a solution to Eq. (4), in the specified reference frame has the form:

$$\psi_{p\zeta}(t,r) = \frac{1}{\sqrt{V}}u(p,\zeta)\exp(-iEt + i\mathbf{p}\cdot\mathbf{r}),$$
$$u(\mathbf{p},\zeta) = 2^{-3/2}\begin{pmatrix} A_+(B_+ + \zeta B_-) \\ -\zeta A_-(B_+ - \zeta B_-)e^{i\phi} \\ A_+(B_+ - \zeta B_-) \\ \zeta A_-(B_+ + \zeta B_-)e^{i\phi} \end{pmatrix}; \tag{7}$$
$$A_\pm = \left(1 \pm \zeta\frac{m}{\varepsilon_\perp}\right)^{1/2}, \quad B_\pm = \left(1 \pm \frac{p_z}{E}\right)^{1/2},$$

where V is the normalization volume. This function describes the stationary state of an electron with the energy

$$E = \left[(\varepsilon_\perp - \zeta h)^2 + p_z^2\right]^{1/2} \tag{8}$$

that depends on the spin quantum number $\zeta = \pm 1$, the longitudinal p_z and transverse $p_\perp = \sqrt{p_x^2 + p_y^2}$ (via $\varepsilon_\perp = \sqrt{m^2 + p_\perp^2}$) components of the (canonical) momentum $\mathbf{p}$ with respect to the field direction (see (6)); the angle ϕ in (7) specifies $\mathbf{p}_\perp = (p_x, p_y, 0) = p_\perp(\cos\phi, \sin\phi, 0)$. The spin number ζ is related to the eigenvalue of the operator of the spin projection onto the direction $\mathbf{h}$:

$$\hat{\Pi} = \gamma^5\left(\gamma^0 p_z - \gamma^3 E\right), \quad \hat{\Pi}\psi_{p\zeta} = (\zeta\varepsilon_\perp - h)\psi_{p\zeta}.$$

The function (7) can be obtained from the wave function of a neutron moving in a constant magnetic field [10] by replacing $\mu_n F_{\mu\nu} \to H_{\mu\nu}$, where μ_n is the anomalous magnetic moment of the neutron.

The explicit form of the electron propagator (Green's function of Eq. (4)) in the momentum representation follows from the expression obtained in [11] for the propagator of a neutrino moving in a constant magnetic field by the obvious renaming:

$$\begin{aligned} G(q) &= \hat{Q}(q)R(q), \\ \hat{Q}(q) &= \{\left(q^2 - m^2\right)(\gamma\cdot q + m) \\ &- h^2(\gamma\cdot q - m) - 2H_{\mu\nu}(Hq)^\nu\gamma^\mu + 2m(\widetilde{H}q)_\mu\gamma^\mu\gamma^5 \\ &+ .\sigma^{\mu\nu}[\tfrac{1}{2}\left(q^2 + m^2 - h^2\right)H_{\mu\nu} - 2(Hq)_\mu q_\nu]\}, \\ R(q) &= \Big[\left(q^2 - m^2\right)\left(q^2 - m^2 + i0\right) \\ &- 2h^2\left(q^2 + m^2\right) + 4(Hq)^2 + \frac{1}{2}h^2\Big]^{-1}. \end{aligned} \tag{9}$$

Here $(Hq)^\mu = H^{\mu\nu}q_\nu$, $(\tilde{H}q)^\mu = \tilde{H}^{\mu\nu}q_\nu$.

PROBABILITY AND POWER OF RADIATION

According to the optical theorem [12], the imaginary part of the one-loop radiative shift of the electron energy ΔE in the initial state determines the radiation probability w (see (7) and (9)):

$$\begin{aligned} w &= -2\,\mathrm{Im}\,\Delta E, \\ \Delta E &= -\frac{ie^2}{(2\pi)^4}\int d^4q D(p - q) \\ &\times R(q)\bar{u}(\mathbf{p},\zeta)\gamma^\mu\hat{Q}(q)\gamma_\mu u(\mathbf{p},\zeta), \end{aligned} \tag{10}$$

where the photon propagator $D(k) = (k^2 + i0)^{-1}$. According to the Cutkosky rules, $\mathrm{Im}\,\Delta E$ is determined as follows:

$$\begin{aligned} 2i\,\mathrm{Im}\,\Delta E &= \Delta E(D(p - q) \\ &\to -2\pi i\delta(D^{-1}(p - q)), \\ R(q) &\to +2\pi i\delta(R^{-1}(q))). \end{aligned} \tag{11}$$

From (9)–(11), and (5), we obtain the following representation of the total radiation probability:

$$\begin{aligned} w &= \frac{2\alpha}{\pi}\int d^4q\delta(X_\gamma)\delta(X_e)F(q), \\ X_\gamma &= (p - q)^2, \\ X_e &= (q^2 - m^2 - h^2)^2 - 4h^2(q_\perp^2 + m^2). \end{aligned} \tag{12}$$

Here $F(q) = \langle\cdots\rangle = \bar{u}(\mathbf{p},\zeta)(\cdots)u(\mathbf{p},\zeta)$ (see (10)), and explicitly (for $\zeta = -1$, see a comment below), taking into account (7) and setting without loss of generality $p_y = 0$ and the angle $\phi = 0$ in view of the axial symmetry of the background field (see (5) and (6)):

$$\begin{aligned} F(q) &= \frac{1}{E}\left(1 + \frac{h}{\varepsilon_\perp}\right)\left(q^2 - m^2 + h^2\right)\left(2m^2 + q_x p_x\right) \\ &- \left(q^2 - m^2 - h^2 - 2h\frac{m^2}{\varepsilon_\perp}\right)\left(q_0 - \frac{q_z p_z}{E}\right). \end{aligned} \tag{13}$$

We obtain the angular distribution of the radiation probability after the change of integration variables, $k = p - q$ (it is the photon 4-momentum),

$$d^4q = dk_0 d^3k, \quad \delta(X_\gamma) \to \frac{1}{2\omega}\delta(k_0 - \omega),$$
$$\omega = |\mathbf{k}|, \quad \mathbf{k} = \omega\mathbf{n}, \quad |\mathbf{n}| = 1,$$

and integration over k_0:

$$\frac{dw}{d\Omega} = \frac{\alpha}{\pi}\int d\omega\omega\delta(X_e)F(q), \quad q = (E - \omega, \mathbf{p} - \omega\mathbf{n}), \tag{14}$$

where $d\Omega$ is the solid angle element, $\mathbf{n}$ is the direction of radiation. The radiation frequency ω, i.e. the photon energy, found from the equation $X_e = 0$ (see (12)), is determined by the direction $\mathbf{n}$, and the radiative transition is due to the electron spin flip, and it also immediately follows from the law of energy conservation (see (8)):

$$\omega_n = E(\mathbf{p}, \zeta = -1) - E(\mathbf{p} - \mathbf{k}, \zeta' = +1) = \frac{2h(\varepsilon_\perp E_n - hn_x p_x)}{E_n^2 - h^2 n_\perp^2}, \quad E_\mathbf{n} = E - \mathbf{n}\cdot\mathbf{p}, \quad n_\perp^2 = 1 - n_z^2. \tag{15}$$

Using the relation (see (12))

$$\delta(X_e) = \frac{\delta(\omega - \omega_n)}{4\omega_\mathbf{n}\left(E_\mathbf{n}^2 - h^2 n_\perp^2\right)},$$

we obtain the angular distribution of the radiation probability after integration over ω in (14):

$$\frac{dw}{d\Omega} = \frac{\alpha F(q)}{4\pi(E_n^2 - h^2 n_\perp^2)}, \quad q = (E - \omega_\mathbf{n}, \mathbf{p} - \omega_\mathbf{n}\mathbf{n}). \tag{16}$$

Equation (16) is exact in terms of the background field strength h, for which there is a strict limitation [4]:

$$h \lesssim 10^{-17}\ \text{eV}. \tag{17}$$

Therefore, below we take into account only the leading terms of the expansion with respect to the parameter h/m.

In this approximation, the function F in (16) has the form (see (13)):

$$F = 4h^3\sqrt{1 - v_z^2}\frac{f(\mathbf{v}, \mathbf{n})}{(1 - \mathbf{v}\cdot\mathbf{n})^2},$$
$$f(\mathbf{v}, \mathbf{n}) = (1 - n_z v_z)^2\left(1 + \frac{1 - v^2}{1 - v_z^2}\right) - (1 - v^2)(1 - n_z^2) - v_x^2 n_x^2, \tag{18}$$

where $\mathbf{v} = \mathbf{p}/\varepsilon = (v_x, 0, v_z)$ is the velocity of a free electron ($\varepsilon = E(h = 0) = \sqrt{m^2 + \mathbf{p}^2}$, electron motion is semiclassical). From (18) and (16), we obtain in the leading approximation for the angular distributions of the probability and power of radiation:

$$\frac{dw}{d\Omega} = \frac{\alpha h^3}{\pi m^2}\sqrt{1 - v_z^2}\frac{1 - v^2}{(1 - \mathbf{v}\cdot\mathbf{n})^4}f(\mathbf{v}, \mathbf{n}),$$
$$\frac{dW}{d\Omega} = \omega_n\frac{dw}{d\Omega} = \frac{2\alpha h^4}{\pi m^2}(1 - v_z^2)\frac{1 - v^2}{(1 - \mathbf{v}\cdot\mathbf{n})^5}f(\mathbf{v}, \mathbf{n}), \tag{19}$$

where the photon energy (see (15))

$$\omega_\mathbf{n} = \frac{2h\sqrt{1 - v_z^2}}{1 - \mathbf{v}\cdot\mathbf{n}}. \tag{20}$$

The angular distribution of the radiation of a relativistic electron ($\gamma = \varepsilon/m \gg 1$) demonstrates the well-known "projector" effect (due to the presence in the denominator of the fifth degree of the expression $1 - \mathbf{v}\cdot\mathbf{n}$): the radiation is concentrated in a narrow cone with an opening angle of about $1/\gamma$ and an axis along the vector $\mathbf{v}$.

Integration over angles in (19) gives the total probability and radiation power:

$$w = \sqrt{1 - v_z^2}w^{(0)} = \frac{8\alpha h^3}{3m^2}\sqrt{1 - v_z^2}\frac{2 + v_0^2}{1 - v_0^2},$$
$$W = W^{(0)} = \frac{32\alpha h^4}{3m^2}\frac{1 + v_0^2}{(1 - v_0^2)^2} \tag{21}$$

with $v_0 = p_\perp/\varepsilon_\perp = v_\perp/\sqrt{1 - v_z^2}$, $v_\perp = \sqrt{v^2 - v_z^2}$. Here the index 0 marks the quantities related to the reference frame moving with the velocity v_z along the axis Oz (the corresponding boost does not change the configuration of the quasimagnetic background field (6), compare with the theory of synchrotron radiation [13]), and Eqs. (21) are consistent with the special relativity. For an unpolarized electron, one should introduce an additional factor $1/2$ into the right-hand sides of (21).

RESULTS AND DISCUSSION

Equations (19) and (21) are valid for an arbitrary angle between the electron momentum $\mathbf{p}$ and the direction of the background field $\mathbf{h}$.

For special cases of transverse ($v_z = 0, v_0 = v$) and longitudinal ($v_z = v, v_0 = 0$) motions, we obtain from (21), respectively:

$$w_\perp = \frac{8\alpha h^3}{3m^2}(2 + v^2)\gamma^2,$$
$$W_\perp = \frac{32\alpha h^4}{3m^2}(1 + v^2)\gamma^4; \tag{22}$$
$$w_\parallel = \frac{16\alpha h^3}{3m^2}\gamma^{-1}, \quad W_\parallel = \frac{32\alpha h^4}{3m^2},$$

where $\gamma = \varepsilon/m = (1-v^2)^{-1/2}$ is the Lorentz factor. Note that for $v = 0$, both cases give the same result, corresponding to the radiation of an electron at rest due to spin flip (see (15) and (20)).

The average photon energy,

$$\langle\omega\rangle = \frac{\int \omega dw}{\int dw} = \frac{W}{w}, \tag{23}$$

is emitted during the time interval

$$\tau_R = 1/w. \tag{24}$$

As a result of the spin flip, the electron transits into a radiation-stable state (see (15)). Thus, the initially unpolarized electron beam becomes completely polarized along the background field direction during the characteristic time (24). A similar effect of radiative polarization was indicated for neutrons moving in a magnetic field in [10].

As follows from (22), the effect of Lorentz violation is most significant for a high-energy electron moving across the direction of the background field. For $\gamma \gg 1$ and $\gamma h/m \ll 1$, the average photon energy (see (23) and (22)) and the radiative polarization length $L_R = v\tau_R$ (see (24)) are:

$$\langle\omega\rangle_\perp \simeq \frac{8}{3}\gamma^2 h, \quad L_R \simeq \frac{c}{w_\perp} \simeq \frac{\bar{\lambda}_e}{8\alpha}\left(\frac{m}{h}\right)^3 \gamma^{-2}, \tag{25}$$

where $\bar{\lambda}_e$ is the Compton wavelength of the electron.

Let us estimate these quantities numerically by setting $h = 10^{-17}$ eV (see (17)) and $\varepsilon = 10^{16}$ GeV (the energy scale of the Grand Unification of the three fundamental interactions [2, 3]). Then we get from (25): $\langle\omega\rangle_\perp \simeq 10^{13}$ GeV that two orders of magnitude greater than the maximum registered energy of particles in cosmic rays $\simeq 10^{11}$ GeV (see the review [14]) and $L_R \simeq 2.3\times10^{20}$ cm (for comparison, the distance from the Sun to the nearest star $\simeq 4\times10^{18}$ cm, and from the Sun to the center of the Galaxy $\simeq 2.5\times10^{22}$ cm).

CONCLUSIONS

Within the framework of the Standard Model Extension, we calculated the probability and power of electromagnetic radiation of an electron due to a spin flip in a constant tensor background field of a quasimagnetic type, which simulates the violation of Lorentz invariance. The calculation is performed using the optical theorem establishing the relationship between the imaginary part of the radiative shift of the electron energy in the initial state and the probability of photon emission. It is shown that, as a result of a radiative transition with spin flip, an initially unpolarized electron beam becomes completely polarized. The considered radiative effect can become noticeable under astrophysical conditions for high-energy electrons.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

REFERENCES

1. R. L. Workman et al. (Particle Data Group), "Review of particle physics," Prog. Theor. Exp. Phys. **2022**, 1–2270 (2022).
2. P. Langacker, *The Standard Model and Beyond*, 2nd ed. (CRC Press, 2017).
3. Y. Nagashima, *Beyond the Standard Model of Elementary Particle Physics* (Wiley-VCH, 2014).
4. V. A. Kostelecký and N. Russel, "Data tables for Lorentz and CPT violation," Rev. Mod. Phys. **83**, 11–31 (2011). arXiv:0801.0287v15 [hep-ph].
5. V. Ch. Zhukovsky, A. E. Lobanov, and E. M. Murchikova, "Radiative effects in the standard model extension," Phys. Rev. D **73**, 065016-1–06516-8 (2006).
6. V. Ch. Zhukovsky, A. E. Lobanov, and E. M. Murchikova, "Production of electron-positron pairs and photon emission by an electron in an axial-vector background field," Phys. Atom. Nucl. **70**, 1248–1252 (2007).
7. I. E. Frolov and V. Ch. Zhukovsky, "Synchrotron radiation in the standard model extension," J. Phys. A **40**, 10625-1–10640 (2007).
8. O. G. Kharlanov and V. Ch. Zhukovsky, "CPT and Lorentz violation effects in hydrogenlike atoms," J. Math. Phys. **9**, 092302-1–092302-16 (2007).
9. A. F. Bubnov, N. V. Gubina, and V. Ch. Zhukovsky, "Vacuum current induced by an axial-vector condensate and electron anomalous magnetic moment in a magnetic field," Phys. Rev. D **96**, 016011-1–016011-10 (2017).
10. I. M. Ternov, V. G. Bagrov, and A. M. Khapaev, "Electromagnetic radiation from a neutron in an external magnetic field," Sov. Phys. JETP **21**, 613–616 (1965).
11. V. Egorov and I. Volobuev, "Quantum field-theoretical description of neutrino oscillations in a magnetic field and the solar neutrino problem," J. Exp. Theor. Phys. **135**, 197–208 (2022). arXiv:2107.11570v2 [hep-ph]
12. V. B. Berestetskii, E. M. Lifshits, and L. P. Pitaevskii, *Course of Theoretical Physics Vol. 4: Quantum Electrodynamics* (Fizmatlit, Moscow, 2002; Pergamon Press, 1982).
13. A. A. Sokolov and I. M. Ternov, *Radiation from Relativistic Electrons* (Am. Inst. Phys., 1986).
14. L. A. Anchordoqui, "Ultra-high-energy cosmic rays," Phys. Rep. **801**, 1–93 (2019). arXiv:1807.09645v3 [astro-ph.HE].