ISSN 1547-4771, Physics of Particles and Nuclei Letters, 2018, Vol. 15, No. 4, pp. 431–433. © Pleiades Publishing, Ltd., 2018.

PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

Nucleons in Nuclear Matter and Properties of Nuclei[1]

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Received December 20, 2017

Abstract—In this contribution we discuss the properties of nucleons and atomic nuclei in the framework of in-medium modified chiral soliton model. The mesonic Lagrangian of the model takes into account an influence of surrounding nuclear environment to the nucleon properties. The model correctly describes the Equations of State of isospin-symmetric and isospin-asymmetric nuclear matter near the nuclear saturation density ρ_0. An extrapolation of the results to high density region allows to describe the properties of neutron stars. The model also can be applied to analysis of the properties of mirror nuclei.

Keywords: skyrmion, nucleon, nuclear matter, mirror nuclei
DOI: 10.1134/S1547477118040246

1. INTRODUCTION

The studies of nucleon properties and the isospin breaking effects in nuclear matter have fundamental interest in traditional nuclear physics and nuclear astrophysics [1, 2]. In particular, the possible modifications of neutron-proton mass difference in nuclear matter $\Delta m^*_{\rm np}$ may be relevant (i) to the physical phenomena in early formation stage of universe, (ii) to the Equations of State of asymmetric nuclear matter and, (iii) also for describing the properties of mirror nuclei. For example, the Nolen-Schiffer anomaly [3] observed in mirror nuclei may be explained in terms of the effective neutron-proton mass difference in nuclear medium. In this context, an in-medium modified chiral soliton approach to the properties of the mirror nuclei [4] dictated the stringent constraint for the nucleon mass changes in nuclear matter. The constraint was in agreement with the phenomenological observations, i.e. with the semi-empirical Bethe–Weizsäcker mass formula. Consequently, in this contribution we discuss a generalized form of chiral soliton approach which takes into account not only an outer-shell [4] but also an inner-core [5] modifications of nucleons in nuclear matter. We consider a broad range of nuclear densities starting from the ordinary nuclear densities up to extreme high densities which may exist in interiors of neutron stars.

2. THE MODEL LAGRANGIAN

We start from the generalized mesonic Lagrangian which describes the properties of nucleons in nuclear medium, the properties of infinite nuclear matter and finite nuclei at same footing in terms of the nucleons as chiral topological solitons. In general, the Lagrangian of model can be separated into two parts in the following form [5]

$$\mathcal{L}^* = \mathcal{L}^*_{\rm sym} + \mathcal{L}^*_{\rm asym}, \tag{1}$$

where $\mathcal{L}^*_{\rm sym}$ is isospoin-symmetric and $\mathcal{L}^*_{\rm asym}$ is isospin-assymmetric terms, respectively. The isospin-symmetric part contains three terms

$$\mathcal{L}^*_{\rm sym} = \mathcal{L}^*_2 + \mathcal{L}^*_4 + \mathcal{L}^*_{\chi SB}, \tag{2}$$

$$\mathcal{L}^*_2 = \frac{F_\pi^2}{16}\left\{\alpha_\tau {\rm Tr}\left(\partial_0 U \partial_0 U^\dagger\right) - \alpha_s {\rm Tr}(\partial_i U \partial_i U^\dagger)\right\}, \tag{3}$$

$$\mathcal{L}^*_4 = -\frac{1}{16e^2\zeta_\tau}{\rm Tr}[U^\dagger\partial_0 U, U^\dagger\partial_i U]^2 + \frac{1}{32e^2\zeta_s}{\rm Tr}[U^\dagger\partial_i U, U^\dagger\partial_j U]^2, \tag{4}$$

$$\mathcal{L}^*_{\chi SB} = \frac{F_\pi^2 m_\pi^2}{8}\alpha_m {\rm Tr}(U-1). \tag{5}$$

Here $\mathcal{L}^*_2$, $\mathcal{L}^*_4$ and $\mathcal{L}^*_{\chi\rm SB}$ are in medium modified large-N_c quadratic, quartic and chiral symmetry breaking terms, respectively. They describe the nucleon properties in isospin-symmetric nuclear matter and the properties of isospin-symmetric nuclear matter too. The isospin-asymmetric part is also separated into two parts

$$\mathcal{L}^*_{\rm asym} = \Delta\mathcal{L}_{\rm mes} + \Delta\mathcal{L}^*_{\rm env}. \tag{6}$$

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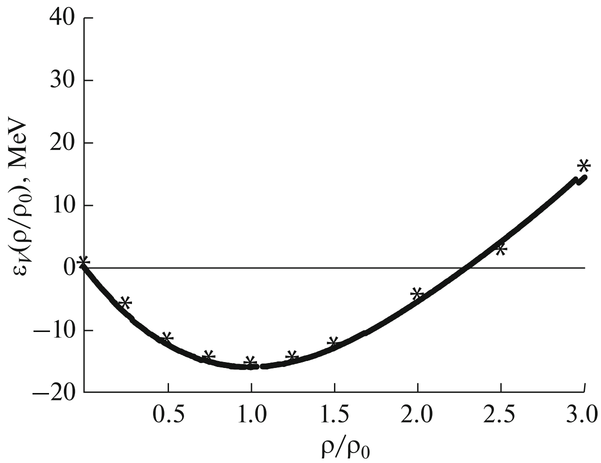

Fig. 1. The volume energy ε_V of symmetric nuclear matter (solid curve) as a function of normalized nuclear density ρ/ρ_0. The model parameters correspond to the Set I given in Table 1 of Ref. [5]. Akmal–Pandharipande–Ravenhall (APR) predictions [7] are marked by stars.

$$\Delta\mathcal{L}_{\rm mes} = -\frac{F_\pi^2}{32}\sum_{a=1}^{2}(m_{\pi^\pm}^2 - m_\pi^2){\rm Tr}(\tau_a U){\rm Tr}(\tau_a U^\dagger), \quad (7)$$

$$\Delta\mathcal{L}_{\rm env}^{*} = -\frac{F_\pi^2}{16} m_\pi \alpha_e \varepsilon_{ab3}{\rm Tr}(\tau_a U){\rm Tr}(\tau_b \partial_0 U^\dagger), \quad (8)$$

where $\Delta\mathcal{L}_{\rm mes}$ and $\Delta\mathcal{L}_{\rm env}^{*}$ are the explicit isospin-symmetry breaking term in mesonic sector and the symmetry breaking term in isospin asymmetric environment, respectively. They describe the isospin breaking effects in nucleon properties and an isospin-asymmetric nuclear matter properties.

Density functions α_τ, α_s, ζ_τ, ζ_s and α_e describe the influence of surrounding nuclear environment to the properties of nucleon under the consideration. At an infinite nuclear matter approximation they become the global parameters depending on the value of nuclear density. The explicit forms of density functions are given in Ref. [6].

The input parameters in the mesonic sector are the pion decay constant $F_\pi = 108.783$ MeV, the skyrme parameter $e = 4.854$ and the neutral pion mass $m_\pi = 134.977$ MeV. These parameters are chosen in such a way that the exact PDG-values of nucleons masses in the free space, $m_{\rm p} = 938.27$ MeV and $m_{\rm n} = 939.56$ MeV, are reproduced correctly. Consequently, ignoring the electromagnetic effects in pion masses, all of these choices of the parameters induce the following value of charged pion mass $m_{\pi^\pm} = 135.015$ MeV. The complete formalism and other technical details of the approach can be found in Refs. [5, 6].

3. NUCLEONS IN INFINITE NUCLEAR MATTER AND EQUATIONS OF STATE

As we mentioned above, in infinite nuclear matter approach the density functions become the external parameters and one can apply a spherically symmetric approximation to the solutions of field equations (see Ref. [5]). It can be achieved using the spherically symmetric hedgehog ansatz in the form of

$$U = \exp\{i\vec{\tau}\vec{n}F(r)\}, \quad \vec{n} = \vec{r}/r, \quad (9)$$

and formulating the corresponding boundary conditions: $F(0) = \pi$ and $(\infty) = 0$. In this scheme the baryons appear as rotational modes of the classical topological soliton. The topological winding number of the classical soliton ($B = 1$) is obtained from the integration of the zeroth component of topological current

$$B^\mu = \frac{\varepsilon^{\mu\nu\alpha\beta}}{24\pi^2}{\rm Tr}[(\partial_\nu U)U^\dagger(\partial_\alpha U)U^\dagger(\partial_\beta U)U^\dagger], \quad (10)$$

and corresponds to the baryon number of nucleon.

One can study the properties of nucleons in nuclear matter in terms of the external density parameters describing the influence of the surrounding nuclear environment to the mesonic fields of nucleons. In a self consistent approach, by considering the nucleon in a meanfield of other nucleons, one can relate the density functions to the Equations of State (EoS) of infinite nuclear matter. In such a way, the approach becomes the five parametric model of asymmetric nuclear matter which correctly describes EoS at densities around the saturation point of nuclear matter (see Ref. [5]). For example, in Fig. 1 we represent the volume energy $\varepsilon_V = m_N^{*S} - m_N^S$ of nuclear matter (in terms of the isoscalar masses of nucleons in nuclear matter and in free space, respectively) as a function of normalized nuclear matter density ρ/ρ_0. For comparison, we represent also the well-known Akmal–Pandharipande–Ravenhall predictions [7]. One can see, that the present model can easily reproduce APR predictions. Moreover, an extrapolation of results to a high extreme density region will reproduce the properties of two solar mass $2M_\odot$ neutron stars [8]. In such a way, the model correctly describes the isospin-asymmetric nuclear matter properties in the broad range, from the low up to very high extreme densities.

4. NUCLEON IN FINITE NUCLEI AND FINITE NUCLEI PROPERTIES

When the nucleon is located inside the finite nuclei the density functions no more global parameters and become the local functions of coordinates. Therefore, the different parts of nucleon may be affected by a surrounding environment in the different ways depending on the nucleon's location inside nuclei (e.g. see Fig. 2. in Ref. [4]). Consequently, the local density functions enter into the equations of motion in a nontrivial way. In this case, a spherically symmetric approximation to

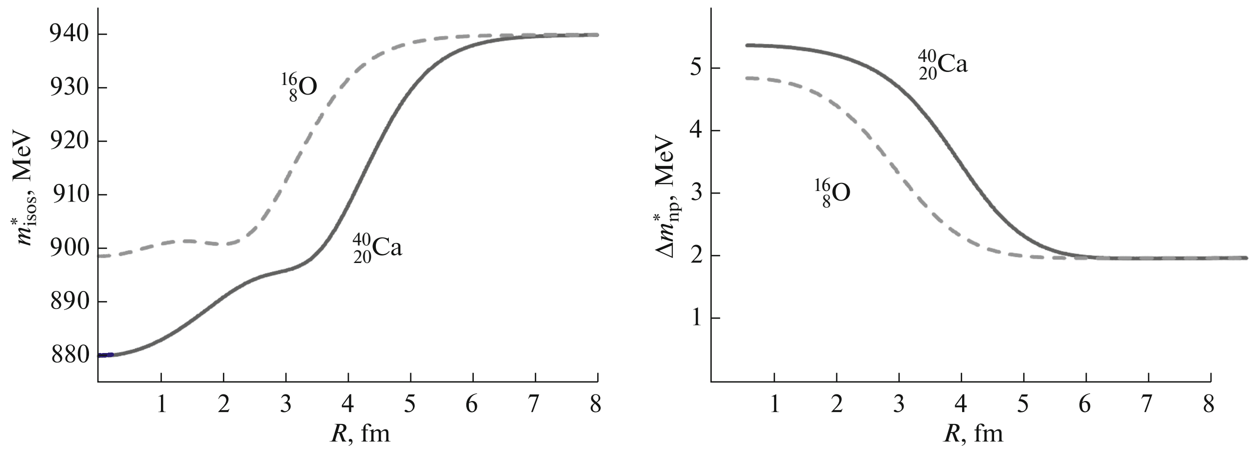

Fig. 2. Isoscalar mass of the nucleon (left panel) and strong part of the effective neutron-proton mass difference (right panel) in finite nuclei.

the solutions of field equations does not work and one have to apply *deformed* hedgehog ansatz

$$U = \exp\{i\vec{\tau}\vec{N}(\vec{r} - \vec{R})F(\vec{r} - \vec{R})\}. \quad (11)$$

Although, an axial symmetry approach to the problem will survive in the case of spherical nuclei.[2] The problem becomes more involved and one should solve the coupled partial differential equations instead of an ordinary differential equation corresponding to the spherically symmetric case. In order to simplify the calculations one can use a variational approach (for the details, see Ref. [4]). In such a way, the present approach is generalized to the case of finite nucleus which serves as a density environment and the properties of nucleons in the spherically symmetric finite nucleus can be studied.

As an example, in Fig. 2 we present isoscalar mass of the nucleon and an effective neutron-proton mass difference in spherical nuclei. Here we considered only the strong part of neutron-proton mass difference, while the changes in the electromagnetic part found to be small [4]. One can see that[3], relatively to the results of model in Ref. [4] where the nucleons core modifications in nuclear medium has not been taken into account, the present model takes into account the core modifications in nuclear matter and much better satisfies the stringent restriction to the effective nucleon mass in nuclear medium. Moreover, the changes in neutron-proton mass difference in finite nuclei in the present work is more pronounced relatively to the results in Ref. [4]. Consequently, one has an opportunity not only for the qualitative but also for the quantitative description of the Nolen–Schiffer anomaly in framework of the present approach.

[2] The nuclei with the closed shells or the nuclei near the shell closure.

[3] Compare the left panel of Fig. 2 with Fig. 3 of Ref. [4].

5. SUMMARY

In summary, we discussed the in-medium chiral soliton model which can be applied to the studies of the nucleon properties in nuclei as well as the properties of neutron stars and the properties of finite nuclei, at same footing. The approach presented in the present work can be extended to study the properties of SU(3) baryons in nuclear matter. The corresponding studies are under the way.

ACKNOWLEDGMENTS

This work is supported by the Basic Science Research Program through the National Research Foundation (NRF) of Korea funded by the Korean government (Ministry of Education, Science and Technology, MEST), grant no. 2016R1D1A1B03935053.

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