
**PHYSICS OF ELEMENTARY PARTICLES
AND ATOMIC NUCLEI. THEORY**

Relativistic Two-Body Interaction Current in the Elastic eD Scattering¹

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Abstract—The reaction of the elastic electron-deuteron scattering is considered within the Bethe–Salpeter approach with separable kernel of interaction. A consistent description of the Mandelstam current obtaining is given. The structure of matrix elements of the deuteron electromagnetic relativistic two-body interaction current is studied in details. All technicalities are thoroughly considered.

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1. INTRODUCTION

Nowadays the interest in some different old researches comes back to life. It happens due to increasing capacities of different old facilities and appearing of new ones. One of such cases is the elastic electron-deuteron scattering, that is of interest in the light of current upgrade of the JLab facilities up to 12 GeV. Indeed, the elastic electron-deuteron scattering is of the great interest because of deuteron being the simplest strongly interacting system of bound nucleons.

Especially we want to study the high energy behavior of the deuteron form factors and structure functions and its dependence on models of the nucleon form factors. In such a way we are interested in reducing or improving of other factors that have to be estimated, such as correct and complete representation for the deuteron electromagnetic current and constructing more realistic separable kernel of interaction. That finally would allow us to study electromagnetic structure of bound nucleons carefully.

The relativistic two-body interaction current was investigated for pion and other mesons form factors in Refs. [1–5] where different schemes of the two-body current construction are discussed. The importance of the relativistic two-body interaction current contribution at high energy transfer is shown.

Another investigations [6, 7] were done for the deuteron electromagnetic form factors taking into account two-body current within the one-boson exchange model.

This work is aimed directly to the analysis of the interacting part of the hadron electromagnetic current in the sense of taking it into calculations. In the paper we follow two works which are very close to the discussed problems, [8] and [9].

The paper is organized as: section 2 is dedicated to the derivation of Mandelstam current in the S-matrix formalism. In section 3 exact expressions for both one-particle and two-particle parts of such current are reviewed. Section 4 considers the problem of the gauge-invariance. In section 5 and appendices of the article you may see all technicalities that appear in calculations of such current in the Bethe–Salpeter approach. In section 6 some conclusions are given.

2. GENERAL EXPRESSION FOR THE AMPLITUDE OF THE ELASTIC eD SCATTERING

The S-matrix element of the elastic eD scattering in the one-photon approximation has the form:

$$S_{fi} = \langle D; \text{out} | \gamma D; \text{in} \rangle \\ = -i\epsilon_\mu \int d^4\xi e^{iq\xi} H^\mu(P' M'; \xi; PM), \quad (1)$$

where

$$H^\mu(P' M'; \xi; PM) = \langle P' M'; \text{out} | j^\mu(\xi) | PM; \text{in} \rangle \quad (2)$$

$\epsilon_\mu = \bar{u}(k')\gamma_\mu u(k)/q^2$ is the virtual photon polarization vector, $P, P'(M, M')$ are the initial-, final deuteron total momenta (the projection of the angular momenta); j^μ is the electromagnetic current density.

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Let us express $H^\mu(P'M'; \xi; PM)$ in terms of the 5-point Green function:

$$G_5^\mu(x_1, x_2; \xi; y_1, y_2) = \langle 0 | T \psi(x_1) \bar{\psi}(x_2) j^\mu(\xi) \bar{\psi}(y_1) \bar{\psi}(y_2) | 0 \rangle, \quad (3)$$

where T is the Wick operator.

Introducing the complete set of the eigenstates of the 4-momentum operator and separating the contribution of the two-nucleon bound state with mass M_d :

$$\begin{aligned} G_5^\mu(x_1, x_2; \xi; y_1, y_2) &= \theta(\min\{x_1^0, x_2^0\} - \xi^0) \\ &\times \theta(\xi^0 - \max\{y_1^0, y_2^0\}) \int \theta(P^0) \delta^{(4)}(P^2 - M_d^2) \\ &\times \frac{d^4 P'}{(2\pi)^3} \theta(P^0) \delta^{(4)}(P^2 - M_d^2) \frac{d^4 P}{(2\pi)^3} \Phi_M(P', x_1, x_2) \\ &\times H^\mu(P'M'; \xi; PM) \bar{\Phi}_M(P, y_1, y_2) \\ &+ R^\mu(x_1, x_2; \xi; y_1, y_2), \end{aligned} \quad (4)$$

where R is the regular function of P^2 and P^2 .

The Bethe–Salpeter amplitudes are introduced in a following way:

$$\Phi_M(P, y_1, y_2) \equiv \langle 0 | T \bar{\psi}(y_1) \bar{\psi}(y_2) | PM \rangle = e^{-iPY} \Phi_M(P, y), \quad (5)$$

$$\bar{\Phi}_M(P', x_1, x_2) \equiv \langle P'M' | T \psi(x_1) \psi(x_2) | 0 \rangle = e^{+iP'X} \bar{\Phi}_M(P, x), \quad (6)$$

and²

$$Y = \frac{1}{2}(y_1 + y_2), \quad y = y_1 - y_2, \quad (7)$$

$$X = \frac{1}{2}(x_1 + x_2), \quad x = x_1 - x_2. \quad (8)$$

Rewriting θ -functions as

$$\theta(\min\{x_1^0, x_2^0\} - \xi^0) = \theta\left(X^0 - \frac{|x^0|}{2} - \xi^0\right), \quad (9)$$

$$\theta(\xi^0 - \max\{y_1^0, y_2^0\}) = \theta\left(\xi^0 - Y^0 - \frac{|y^0|}{2}\right), \quad (10)$$

and using the integral representation for them as

$$\theta(z) = i \int \frac{d\lambda}{2\pi} \frac{e^{-i\lambda z}}{z + i0}, \quad (11)$$

we write

$$\begin{aligned} G_5^\mu(x_1, x_2; \xi; y_1, y_2) &= i^2 \int \frac{d\lambda_x}{2\pi} \frac{d^3 P'}{2E_{P'}(2\pi)^3} \frac{d\lambda_y}{2\pi} \frac{d^3 P}{2E_P(2\pi)^3} \\ &\times \frac{d^4 p'}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \Psi_M(P', p'; \lambda_x) H^\mu(P'M'; \xi; PM) \\ &\times \bar{\Psi}_M(P, p; \lambda_y) e^{-ip'x + ipy} \frac{e^{-i(E_P + \lambda_x)X^0 + iP'X + i\lambda_x \xi^0}}{\lambda_x + i0} \\ &\times \frac{e^{+i(E_P + \lambda_y)Y^0 - iPY - i\lambda_y \xi^0}}{\lambda_y + i0} + R^\mu(P'M'; \xi; PM), \end{aligned} \quad (12)$$

here the functions $\Psi(\bar{\Psi})$ are introduced as follows:

$$\Psi_M(P', x; \lambda_x) = e^{i\lambda_x \frac{|x^0|}{2}} \Phi_M(P', x), \quad (13)$$

$$\bar{\Psi}_M(P, y; \lambda_y) = e^{i\lambda_y \frac{|y^0|}{2}} \bar{\Phi}_M(P, y) \quad (14)$$

and their Fourier transformation reads:

$$\Psi_{M'}(P', x; \lambda_x) = \int \frac{d^4 p'}{(2\pi)^4} e^{-ip'x} \Psi_M(P', p'; \lambda_x), \quad (15)$$

$$\bar{\Psi}_M(P, y; \lambda_y) = \int \frac{d^4 p}{(2\pi)^4} e^{ipy} \bar{\Psi}_M(P, p; \lambda_y). \quad (16)$$

Now we can change the integration variables as

$$\lambda_x = P'_0 - E_{P'}, \quad \lambda_y = P_0 - E_P;$$

$$\begin{aligned} G_5^\mu(x_1, x_2; \xi; y_1, y_2) &= i^2 \int \frac{d^4 P'}{2E_{P'}(2\pi)^4} \frac{d^4 P}{2E_P(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} \\ &\times \frac{d^4 p}{(2\pi)^3} \Psi_M(P', p'; P'_0 - E_{P'}) H^\mu(P'M'; \xi; PM) \\ &\times \bar{\Psi}_M(P, p; P_0 - E_P) e^{-ip'x + ipy} \frac{e^{-iP'X + i(P'_0 - E_{P'})\xi^0}}{P'_0 - E_{P'} + i0} \\ &\times \frac{e^{+iPY - i(P_0 - E_P)\xi^0}}{P_0 - E_P + i0} + R^\mu(P'M'; \xi; PM). \end{aligned} \quad (17)$$

Defying the Fourier transformation for the Green-function G_5 as:

$$\begin{aligned} G_5^\mu(P'p'; \xi; Pp) &= \int \frac{d^4 P'}{(2\pi)^4} \frac{d^4 p'}{(2\pi)^4} \frac{d^4 P}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \\ &\times e^{-iP'X - ip'x + iPY + ipy} G_5^\mu(x_1, x_2; \xi; y_1, y_2), \end{aligned} \quad (18)$$

we have

$$\begin{aligned} G_5^\mu(P'p'; \xi; Pp) &= i^2 \Psi_M(P', p'; P'_0 - E_{P'}) \\ &\times H^\mu(P'M'; \xi; PM) \bar{\Psi}_M(P, p; P_0 - E_P) \\ &\times \frac{e^{+i(P'_0 - E_{P'})\xi^0}}{2E_{P'}(P'_0 - E_{P'} + i0)} \frac{e^{-i(P_0 - E_P)\xi^0}}{2E_P(P_0 - E_P + i0)} \\ &+ R^\mu(P'M'; \xi; PM), \end{aligned} \quad (19)$$

and, therefore,

$$\begin{aligned} i^2 \Phi_M(P', p') H^\mu(P'M'; \xi; PM) \bar{\Phi}_M(P, p) \\ = \lim_{P'^2, p'^2 \rightarrow M_d^2} \theta(P^0) \delta^{(4)}(P^2 - M_d^2) \theta(P^0) \\ \times \delta^{(4)}(P^2 - M_d^2) G_5^\mu(P'p'; \xi; Pp). \end{aligned} \quad (20)$$

In the last expression it was taken into account that

$$\lim_{P'^2 \rightarrow M_d^2} \Psi_M(P', p'; P'_0 - E_{P'}) = \Phi_M(P', p'), \quad (21)$$

$$\lim_{P^2 \rightarrow M_d^2} \bar{\Psi}_M(P, p; P_0 - E_P) = \bar{\Phi}_M(P, p). \quad (22)$$

² The corresponding momenta are in the Appendix A.

So, finally we have

$$\sum_{M,M'} \Phi_{M'}(P', p') S_{fi} \bar{\Phi}_M(P, p) = -i\epsilon_\mu \lim_{P^2, p^2 \rightarrow M_d^2} \theta(P^0) \quad (23)$$

$$\times \delta^{(4)}(P'^2 - M_d^2) \theta(P^0) \delta^{(4)}(P^2 - M_d^2) G_5^\mu(P', p'; q; Pp).$$

The full Fourier transformation for the Green-function G_5 in the following form

$$(2\pi)^4 \delta^{(4)}(p_1' + p_2' - p_1 - p_2 - q) G_5^\mu(p_1' p_2'; q; p_1 p_2)$$

$$= \int d^4 x_1 d^4 x_2 d^4 y_1 d^4 y_2 d^4 \xi e^{i(p_1' x_1 + p_2' x_2 - p_1 y_1 - p_2 y_2 - q \xi)} \quad (24)$$

$$\times G_5^\mu(x_1 x_2; \xi; y_1 y_2),$$

was used in the last expression.

To make the relation (20) more clear let us introduce the generalized EM current Λ_μ (the so called Mandelstam current):

$$G_5^\mu(p_1' p_2'; q; p_1 p_2) = \int d^4 k_1' d^4 k_2' d^4 k_1 d^4 k_2$$

$$\times \delta^{(4)}(p_1' + p_2' - k_1' - k_2') \delta^{(4)}(p_1 + p_2 - k_1 - k_2) \quad (25)$$

$$\times G_4(p_1' p_2'; k_1' k_2') \Lambda^\mu(k_1' k_2'; q; k_1 k_2) G_4(k_1 k_2; p_1 p_2).$$

As it is known the two-body Green-function G_4 has the following form near the pole $P^2 = M_d^2$:

$$G_4(p_1' p_2'; k_1' k_2') = \frac{i}{(2\pi)^4} \frac{\theta(P^0)}{P^2 - M_d^2}$$

$$\times \sum_M [\Phi_M(P', p') \bar{\Phi}_M(P, p)] + R(p_1' p_2'; k_1' k_2'), \quad (26)$$

with R being the regular part, and the expression takes the form:

$$S_{fi} = -i^3 \epsilon_\mu \delta^{(4)}(P' - P - q) \int \frac{d^4 p'}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \quad (27)$$

$$\times \bar{\Phi}_{M'}(P', p') \Lambda^\mu(p', p; P', P) \Phi_M(P, p).$$

So, the relations between the matrix element and the Bethe–Salpeter amplitudes and the Mandelstam current are obtained. The form and structure of the current will be discussed in the next section.

3. MANDELSTAM CURRENT

The procedure of obtaining the electromagnetic current of the two-nucleon bound system is a great problem. It is possible to implement this easily only for the kernel that includes exchange particle explicitly. However, there are some obstacles when we are going to use kernels that describe effective nucleon-nucleon interaction like a separable one. In Ref. [9] the problem was solved and the Mandelstam current that is

suitable for any kernel of interaction had been proposed. Such the current Λ_μ consists of several parts:

$$\Lambda_\mu(p, p'; P, P') = \Lambda_\mu^{[1]} + \Lambda_\mu^{[2]}, \quad (28)$$

$$\Lambda_\mu^{[2]} = \Lambda_\mu^{[2,dir]} + \Lambda_\mu^{[2,exc]}, \quad (29)$$

where $\Lambda_\mu^{[1]}$ is the one-particle part, that corresponds to the relativistic impulse approximation (RIA) (if nucleon form factors are supposed to be the on-mass-shell), which was calculated in the [10]. Values $\Lambda_\mu^{[2,dir]}$, $\Lambda_\mu^{[2,exc]}$ are the direct and exchange parts of the two-particle part $\Lambda_\mu^{[2]}$, so called the interaction current (IC), of the Mandelstam current.

The currents are equal to

$$\Lambda_\mu^{[1]}(p, p'; P, P') = i(2\pi)^4 \left\{ \delta^{(4)} \left(p' - p - \frac{q}{2} \right) \right.$$

$$\left. \times \Gamma_\mu^{(1)} \left(\frac{P'}{2} + p', \frac{P}{2} + p \right) S^{(2)} \left(\frac{P}{2} - p \right)^{-1} + (1 \longleftrightarrow 2) \right\}, \quad (30)$$

$$\Gamma_\mu(p + q, p) = e \left\{ \left[\gamma_\mu - \frac{\partial}{\partial p_\mu} \int_0^1 \Sigma(p + qt) dt \right] \right.$$

$$\left. \times f(q^2) + i \frac{\sigma_{\mu\nu} q^\nu}{2m} g(q^2) \right\}, \quad (31)$$

$$\Lambda_\mu^{[2,dir]}(p', p; P', P) = i^2 e (2\pi)^4 \sum_{l=1,2} f^{(l)}(q^2)$$

$$\times \int_0^1 \left[\frac{\partial}{\partial P} - \frac{1}{2} (-1)^l \left(\frac{\partial}{\partial p'} + \frac{\partial}{\partial p} \right) \right]_\mu$$

$$\times V_{dir}(k^{(l)}, k^{(l)}; K) dt, \quad (32)$$

$$\Lambda_\mu^{[2,exc]}(p, p'; P, P') = i^2 e (2\pi)^4 \sum_{l=1,2} f^{(l)}(q^2)$$

$$\times P_\tau \int_0^1 \left[\frac{\partial}{\partial P} - \frac{1}{2} (-1)^l \left(\frac{\partial}{\partial p'} - \frac{\partial}{\partial p} \right) \right]_\mu V_{exc}(k^{(l)}, k^{(l)}; K) dt, \quad (33)$$

where Γ_μ is the vertex of γNN -interaction and $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$. All the momenta are clarified in the Appendix A.

Kernels V_{dir} and V_{exc} are defined as:

$$V_{dir} = V_0 - V_1, \quad V_{exc} = 2V_1, \quad (34)$$

where V_0, V_1 describe the most general form of the interaction kernel:

$$V(p', p; P) = \sum_{T=0,1} \Pi_T V_T(p', p; P), \quad (35)$$

Π_T is the projector on a state with a total isospin momentum T .

In the case of elastic eD scattering only the V_0 (further V_0 equals just to V) part contributes to the interaction current. So, finally, we get the expression for the interaction current:

$$\begin{aligned} \Lambda_\mu^{[2]}(p', p; P', P) &= i^2 e (2\pi)^4 \sum_{l=1,2} f^{(l)}(q^2) \\ &\times \int_0^1 \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'}, \frac{\partial}{\partial p}; \frac{\partial}{\partial P} \right) V(k^{(l)}, k^{(l)}; K) dt \\ &\times \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'}, \frac{\partial}{\partial p}; \frac{\partial}{\partial P} \right) = \left[\frac{\partial}{\partial P} - \frac{1}{2} (-1)^l \left(\frac{\partial}{\partial p'} + \frac{\partial}{\partial p} \right) \right]_\mu. \end{aligned} \quad (36)$$

We neglect the mass operator Σ and put nucleons on-mass-shell, in this case the functions $f^{(l)}(q^2)$ and $g^{(l)}(q^2)$ are exactly $F_1^{(l)}(q^2)$ and $F_2^{(l)}(q^2)$ – Dirac and Pauli form factors.

4. GAUGE-INVARIANCE OF THE CURRENT

The gauge-invariance (GI) condition leads to the zero value for the divergence of the total current:

$$q^\mu (\Lambda_\mu^{[1]} + \Lambda_\mu^{[2]}) = 0. \quad (37)$$

It is shown in [9] that in order to satisfy the GI condition the divergence of $\Lambda_\mu^{[2]}$ should be equal to

$$\begin{aligned} iq\Lambda^{[2]}(p', p; P', P) &= e(2\pi)^4 \sum_{l=1,2} f^{(l)}(q^2) \\ &\times \left[V(p' + (-1)^l \frac{q}{2}, p; P) - V(p', p - (-1)^l \frac{q}{2}; P + qt) \right]. \end{aligned} \quad (38)$$

Let us check that the current (36) indeed is in compliance with this requirement.

Consider $l = 1$ and $\{0, z\}$ components for simplicity.

$$\Lambda_\mu^{[2]} \sim \left(\frac{\partial}{\partial P} + \frac{1}{2} \frac{\partial}{\partial p'} + \frac{1}{2} \frac{\partial}{\partial p} \right) V(k', k; K), \quad (39)$$

where $k' = p' - \frac{q}{2}(1-t)$, $k = p + \frac{q}{2}t$, $K = P + qt$.

Then the divergence of the current can be written as:

$$\begin{aligned} q\Lambda^{[2]} &= q_0 \Lambda^{[2]0} - q_z \Lambda^{[2]z} \sim \left(q_0 \frac{\partial}{\partial K_0} + \frac{q_0}{2} \frac{\partial}{\partial k'_0} \right. \\ &+ \left. \frac{q_0}{2} \frac{\partial}{\partial k_0} + q_z \frac{\partial}{\partial K_z} + \frac{q_z}{2} \frac{\partial}{\partial k'_z} + \frac{q_z}{2} \frac{\partial}{\partial k_z} \right) V(k', k; K), \end{aligned} \quad (40)$$

where the following property was used $\frac{\partial}{\partial P} = \frac{\partial K}{\partial P} \frac{\partial}{\partial K} = \frac{\partial}{\partial K}$, $\frac{\partial}{\partial p'} = \frac{\partial k'}{\partial p'} \frac{\partial}{\partial k'} = \frac{\partial}{\partial k'}$ and $\frac{\partial}{\partial p} = \frac{\partial k}{\partial p} \frac{\partial}{\partial k} = \frac{\partial}{\partial k}$.

Let us write the total derivative of V over t :

$$\begin{aligned} \frac{dV}{dt} &= \left(\frac{\partial V}{\partial K_0} \frac{\partial K_0}{\partial t} + \frac{\partial V}{\partial k'_0} \frac{\partial k'_0}{\partial t} + \frac{\partial V}{\partial k_0} \frac{\partial k_0}{\partial t} \right. \\ &+ \left. \frac{\partial V}{\partial K_z} \frac{\partial K_z}{\partial t} + \frac{\partial V}{\partial k'_z} \frac{\partial k'_z}{\partial t} + \frac{\partial V}{\partial k_z} \frac{\partial k_z}{\partial t} \right) \\ &= \left(q_0 \frac{\partial}{\partial K_0} + \frac{q_0}{2} \frac{\partial}{\partial k'_0} + \frac{q_0}{2} \frac{\partial}{\partial k_0} + q_z \frac{\partial}{\partial K_z} \right. \\ &+ \left. \frac{q_z}{2} \frac{\partial}{\partial k'_z} + \frac{q_z}{2} \frac{\partial}{\partial k_z} \right) V. \end{aligned} \quad (41)$$

It is seen that (41) coincides with (40). So the divergence can be rewritten as:

$$\begin{aligned} iq\Lambda^{[2]} &= i^2 e (2\pi)^4 \int_0^1 \frac{\partial V}{\partial t} = e(2\pi)^4 [V(0) - V(1)] \\ &= e(2\pi)^4 \left[V(p' - \frac{q}{2}, p; P) - V(p', p + \frac{q}{2}; P + q) \right]. \end{aligned} \quad (42)$$

The same logic is for the $l = 2$ case. So it is seen that the current (36) satisfies the equality (38), hence the condition of GI is fulfilled.

In the case when interaction kernel depends only on the k'^2 and k^2 we can write:

$$\Lambda_\mu^{[2]} \sim \frac{1}{2} \left(\frac{\partial}{\partial p'} + \frac{\partial}{\partial p} \right)_\mu V(k'^2, k^2) = \left(\frac{k'_\mu}{k'q} + \frac{k_\mu}{kq} \right) \frac{d}{dt} V(t),$$

where we used the following expressions:

$$\begin{aligned} \frac{\partial}{\partial p'_\mu} &= \frac{\partial}{\partial k'_\mu} = 2k'_\mu \frac{\partial}{\partial k'^2} = 2 \frac{k'_\mu}{k'q} \frac{d}{dt}, \\ \frac{\partial}{\partial p_\mu} &= \frac{\partial}{\partial k_\mu} = 2k_\mu \frac{\partial}{\partial k^2} = 2 \frac{k_\mu}{kq} \frac{d}{dt}. \end{aligned}$$

This again leads to the equation (42).

5. MATRIX ELEMENTS OF THE INTERACTION CURRENT

Now we are able to write matrix elements in form which is suitable for the direct calculations:

$$M_{M', M; \mu}^{[2]}(P', P) = \int \frac{d^4 p'}{(2\pi)^4} \frac{d^4 p}{(2\pi)^4} \bar{\Phi}_{M'}(P', p')_{\alpha_1, \alpha_2} \quad (43)$$

$$\times \Lambda_\mu^{[2]}(p', p; P', P)_{\alpha_1, \alpha_2; \beta_1, \beta_2} \Phi_M(P, p)_{\beta_1, \beta_2},$$

where

$$\begin{aligned} \Lambda_\mu^{[2]}(p', p; P', P)_{\alpha_1, \alpha_2; \beta_1, \beta_2} &= i^2 e (2\pi)^4 \sum_{l=1,2} f^{(l)}(q^2) \\ &\times \int_0^1 \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'}, \frac{\partial}{\partial p}; \frac{\partial}{\partial P} \right) V(k^{(l)}, k^{(l)}; K)_{\alpha_1, \alpha_2; \beta_1, \beta_2} dt. \end{aligned} \quad (44)$$

Consider the matrix element in the the laboratory system (which we define as a rest system of the initial deu-

teron). The boosting of the BS amplitudes and kernel to their rest systems should be done. Expressing boost operators as they described in the Appendix B we have:

$$M_{M',M;\mu}^{[2]}(P',P) = \frac{i^2 e}{(2\pi)^4} \sum_{l=1,2} f^{(l)}(q^2) \int d^4 p' d^4 p \int_0^1 dt \bar{\Phi}_{M'}(P'_{(0)}, p'_{(0)})_{\alpha_1, \alpha_2} \Lambda^{(1)-1}(P')_{\alpha_1 \gamma_1} \Lambda^{(2)-1}(P')_{\alpha_2 \gamma_2} \times \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'_{(0)}}, \frac{\partial}{\partial p}, \frac{\partial}{\partial P} \right) \left[\Lambda_{\gamma_1 \sigma_1}^{(1)}(K) \Lambda_{\gamma_2 \sigma_2}^{(2)}(K) \times V(k_{(0)}^{(l)}, k_{(0)}^{(l)}; P)_{\sigma_1 \sigma_2; \rho_1 \rho_2} \Lambda_{\rho_1 \beta_1}^{(1)-1}(K) \Lambda_{\rho_2 \beta_2}^{(2)-1}(K) \right] \times \Phi_M(P, p)_{\beta_1, \beta_2}. \quad (45)$$

We consider only positive-energy states (ρ -spins of both nucleons equal to $+\frac{1}{2}$), in this case there are only

two partial-waves $^3S_1^{++}$ and $^3D_1^{++}$, which differ from each other only by angular momentum $L = 0, 2$.

Now the partial-wave decomposition (see Appendix C) can be done. The obtained expressions are:

$$M_{M',M;\mu}^{[2]}(P',P) = \frac{i^2 e}{(2\pi)^4} \sum_{L_1 L_2 L_3 L_4} \sum_{M'} \sum_{l=1,2} f^{(l)}(q^2) \times \int d^4 p' d^4 p \int_0^1 dt \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'_{(0)}}, \frac{\partial}{\partial p}, \frac{\partial}{\partial P} \right) \Phi_{L_1}(p'_{(0)0}, |\mathbf{p}'_{(0)}|) \times \phi_{L_4}(p_0, |\mathbf{p}|) \check{V}_{L_2 L_3}(k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; s) \times \bar{\mathcal{Y}}_{M'}^{L_1}(p'_{(0)})_{\alpha_1, \alpha_2} \Lambda^{(1)-1}(P')_{\alpha_1 \gamma_1} \Lambda^{(2)-1}(P')_{\alpha_2 \gamma_2} \check{\Lambda}_{\gamma_1 \sigma_1}^{(1)}(K) \times \check{\Lambda}_{\gamma_2 \sigma_2}^{(2)}(K) \check{\mathcal{Y}}_{M'}^{L_2}(-\mathbf{k}_{(0)}^{(l)})_{\sigma_1 \sigma_2} \check{\mathcal{Y}}_{M'}^{L_3+}(\mathbf{k}_{(0)}^{(l)})_{\rho_1 \rho_2} \check{\Lambda}_{\rho_1 \beta_1}^{(1)-1}(K) \times \check{\Lambda}_{\rho_2 \beta_2}^{(2)-1}(K) \check{\mathcal{Y}}_{M'}^{L_4}(\mathbf{p})_{\beta_1, \beta_2}, \quad (46)$$

where the values marked as \check{a} are affected by the operator \hat{D}_μ .

Using the definition of the spin-angular functions given in the Appendix B the final expression can be written as

$$M_{M',M;\mu}^{[2]}(P',P) = \frac{i^2 e}{(2\pi)^4} \sum_{L_1 L_2 L_3 L_4} \sum_{l=1,2} i^{L_1+L_2+L_3+L_4} \times f^{(l)}(q^2) \int d^4 p' d^4 p \int_0^1 dt \hat{D}_\mu^{(l)} \left(\frac{\partial}{\partial p'_{(0)}}, \frac{\partial}{\partial p}, \frac{\partial}{\partial P} \right) \times \check{R}(p'_{(0)0}, |\mathbf{p}'_{(0)}|; k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; s; p_0, |\mathbf{p}|)_{L_1 L_2 L_3 L_4} \times \sum_{M''} \sum_{a_1 a_2 a_3 a_4} \check{C}(\hat{\mathbf{p}}'_{(0)}, -\hat{\mathbf{k}}_{(0)}^{(l)}, \hat{\mathbf{k}}_{(0)}^{(l)}, \hat{\mathbf{p}}_{(0)})_{a_1 a_2 a_3 a_4}^{M'' M'' M''} \times \check{A}^{(1)}(\mathbf{p}'_{(0)}, \mathbf{k}_{(0)}^{(l)})_{\mu, \mu_2} \check{A}^{(2)}(\mathbf{p}_{(0)}, \mathbf{k}_{(0)}^{(l)})_{\nu, \nu_2} \times \check{B}^{(1)}(\mathbf{k}_{(0)}^{(l)}, \mathbf{p})_{\mu_3 \mu_4} \check{B}^{(2)}(\mathbf{k}_{(0)}^{(l)}, \mathbf{p})_{\nu_3 \nu_4}, \quad (47)$$

where $a_i = \{L_i, m_{L_i}, m_{S_i}, \mu_i, \nu_i\}$ and

$$\check{R}(p'_{(0)0}, |\mathbf{p}'_{(0)}|, p_0, |\mathbf{p}|; s)_{L_1 L_2 L_3 L_4} = \phi_{L_1}(p'_{(0)0}, |\mathbf{p}'_{(0)}|) \times \phi_{L_4}(p_0, |\mathbf{p}|) \check{V}_{L_2 L_3}(k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; k_{(0)0}^{(l)}, |\mathbf{k}_{(0)}^{(l)}|; s), \quad (48)$$

$$\check{C}(\hat{\mathbf{p}}'_{(0)}, -\hat{\mathbf{k}}_{(0)}^{(l)}, \hat{\mathbf{k}}_{(0)}^{(l)}, \hat{\mathbf{p}}_{(0)})_{a_1 a_2 a_3 a_4}^{M'' M'' M''} = C_{L_1 m_{L_1} 1 m_{S_1}}^{1 M''} C_{\frac{1}{2} \mu_1 \frac{1}{2} \nu_1}^{1 m_{S_1}} \times C_{L_2 m_{L_2} 1 m_{S_2}}^{1 M''} C_{\frac{1}{2} \mu_2 \frac{1}{2} \nu_2}^{1 m_{S_2}} C_{L_3 m_{L_3} 1 m_{S_3}}^{1 M''} C_{\frac{1}{2} \mu_3 \frac{1}{2} \nu_3}^{1 m_{S_3}} C_{L_4 m_{L_4} 1 m_{S_4}}^{1 M''} C_{\frac{1}{2} \mu_4 \frac{1}{2} \nu_4}^{1 m_{S_4}} \times Y_{L_1 m_{L_1}}(\hat{\mathbf{p}}'_{(0)}) \check{Y}_{L_2 m_{L_2}}(-\hat{\mathbf{k}}_{(0)}^{(l)}) \check{Y}_{L_3 m_{L_3}}(\hat{\mathbf{k}}_{(0)}^{(l)}) Y_{L_4 m_{L_4}}(\hat{\mathbf{p}}_{(0)}), \quad (49)$$

$$\check{A}^{(1)}(\mathbf{p}'_{(0)}, \mathbf{k}_{(0)}^{(l)})_{\mu, \mu_2} = \bar{u}_{\mu_1}^{(1)}(\mathbf{p}'_{(0)}) \Lambda^{(1)-1}(P') \check{\Lambda}^{(1)}(K) \check{u}_{\mu_2}^{(1)}(-\mathbf{k}_{(0)}^{(l)}), \quad (50)$$

$$\check{A}^{(2)}(\mathbf{p}'_{(0)}, \mathbf{k}_{(0)}^{(l)})_{\nu, \nu_2} = \bar{u}_{\nu_1}^{(2)}(-\mathbf{p}'_{(0)}) \Lambda^{(2)-1}(P') \check{\Lambda}^{(2)}(K) \check{u}_{\nu_2}^{(2)}(\mathbf{k}_{(0)}^{(l)}), \quad (51)$$

$$\check{B}^{(1)}(\mathbf{k}_{(0)}^{(l)}, \mathbf{p})_{\mu_3 \mu_4} = \check{u}_{\mu_3}^{(1)+}(\mathbf{k}_{(0)}^{(l)}) \check{\Lambda}^{(1)-1}(K) u_{\mu_4}^{(1)}(\mathbf{p}), \quad (52)$$

$$\check{B}^{(2)}(\mathbf{k}_{(0)}^{(l)}, \mathbf{p})_{\nu_3 \nu_4} = \check{u}_{\nu_3}^{(2)+}(-\mathbf{k}_{(0)}^{(l)}) \check{\Lambda}^{(2)-1}(K) u_{\nu_4}^{(2)}(-\mathbf{p}). \quad (53)$$

Actually we have to analyze the analytical structure of the matrix elements with certain kernel of interaction, because the function R has singularities. That will be considered elsewhere in further works.

6. NON-MINIMAL CONTRIBUTION

It is important to admit that the current investigated above is based on the minimal substitution method of introduction of the EM interaction. However, it does not allow us to represent the magnetic part of the current. In order to fix this problem let us consider the non-minimal substitution with derivative of the photonic field:

$$\frac{\partial}{\partial x_\alpha} \rightarrow \frac{\partial}{\partial x_\alpha} + ie A_\alpha(x) - \frac{ie}{8m} \gamma_\alpha \sigma_{\beta\delta} \frac{\partial A_\beta(x)}{\partial x_\delta}. \quad (54)$$

This substitution leads to the following modification of the expression for the two-body interaction current:

$$\Lambda_\mu^{[2]}(p', p; P', P) = i^2 e (2\pi)^4 \times \sum_{l=1,2} \left(F_1^{(l)}(q^2) \delta_{\mu\nu} + F_2^{(l)}(q^2) \frac{i}{4m} \sigma'_{\mu\beta} q^\beta \gamma_\nu \right) \times \int_0^1 \hat{D}_\nu^{(l)} \left(\frac{\partial}{\partial p'}, \frac{\partial}{\partial p}, \frac{\partial}{\partial P} \right) V(k^{(l)}, k^{(l)}; K) dt. \quad (55)$$

In this case, the functions A_1 and A_2 are modified as (consider only F_2 – term):

$$\begin{aligned} \check{A}^{(1)}(\mathbf{p}'_{(0)}, \mathbf{k}'_{(0)})_{\mu_1\mu_2} &= \bar{u}_{\mu_1}^{(1)}(\mathbf{p}'_{(0)})\Lambda^{(1)-1}(P')\left[\delta_{1,l}\sigma_{\mu\beta}q^\beta\gamma_\nu\right] \\ &\times \check{\Lambda}^{(1)}(K)\check{u}_{\mu_2}^{(1)}(-\mathbf{k}'_{(0)}), \\ \check{A}^{(2)}(\mathbf{p}'_{(0)}, \mathbf{k}'_{(0)})_{\nu_1\nu_2} &= \bar{u}_{\nu_1}^{(2)}(-\mathbf{p}'_{(0)})\Lambda^{(2)-1}(P')\left[\delta_{2,l}\sigma_{\mu\beta}q^\beta\gamma_\nu\right] \\ &\times \check{\Lambda}^{(2)}(K)\check{u}_{\nu_2}^{(2)}(\mathbf{k}'_{(0)}). \end{aligned} \quad (56)$$

In (55) we introduced F_1 and F_2 form factors, which are exactly Dirac and Pauli form factors for nucleons.

7. CONCLUSIONS

We have the consistent description of the Mandelstam current obtaining. The relativistic two-body interaction current component of the deuteron EM current is investigated. The procedure of the calculation of such a current in the reaction of elastic electron-deuteron scattering has been described. The implementation of such a procedure was obtained in the Bethe-Salpeter approach and suits for any type of kernel of interaction. Also the non-minimal contribution to the two-body interaction current is discussed. The calculation was done in the laboratory system. The complete description of all technicalities was given.

In the laboratory system:

$$\begin{aligned} P &= (M_d, 0), \quad q = (w, \mathbf{q}_z), \quad w = 2M_d\eta, \quad q_z = 2M_d\sqrt{\eta}\sqrt{1+\eta}, \\ P' &= (M_d + w, \mathbf{q}_z) = (M_d(1+2\eta), 0, 0, 2M_d\sqrt{\eta}\sqrt{1+\eta}), \\ K &= (M_d + wt, \mathbf{q}_zt) = (M_d(1+2\eta t), 0, 0, 2M_d\sqrt{\eta}\sqrt{1+\eta t}), \\ s(\eta, t) &= M_d^2(1+4\eta t(1-t)), \quad p = (p_0, p_x, p_y, p_z), \quad p' = (p'_0, p'_x, p'_y, p'_z), \\ k^{(1)} &= p + \frac{q}{2}t = \left(p_0 + \frac{w}{2}t, p_x, p_y, p_z + \frac{q_z}{2}t\right) \\ k'^{(1)} &= p' - \frac{q}{2}(1-t) = \left(p'_0 - \frac{w}{2}(1-t), p'_x, p'_y, p'_z - \frac{q_z}{2}(1-t)\right) \\ k^{(2)} &= p - \frac{q}{2}t = \left(p_0 - \frac{w}{2}t, p_x, p_y, p_z - \frac{q_z}{2}t\right) \\ k'^{(2)} &= p' + \frac{q}{2}(1-t) = \left(p'_0 + \frac{w}{2}(1-t), p'_x, p'_y, p'_z + \frac{q_z}{2}(1-t)\right) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \begin{array}{l} \text{1st term,} \\ \\ \text{2nd term,} \end{array}$$

Lorentz transformation to the rest frame:

since the matrix element is considered in the rest frame of the initial deuteron, then $P_{(0)} \equiv P$ and $p_{(0)} \equiv p$, and the P, p notations are used below.

KINEMATICS

Momenta:

$P = p_1 + p_2$ —total momentum of the initial deuteron, $P^2 = M_d^2$

$p = \frac{p_1 - p_2}{2}$ —relative momentum of the initial deuteron

$q = (w, \mathbf{q})$ —momentum transfer, $q^2 = -4M_d^2\eta$ — convenient substitution

$P' = p'_1 + p'_2 = P + q$ —total momentum of the final deuteron, $P'^2 = M_d^2$

$p' = \frac{p'_1 - p'_2}{2}$ —relative momentum of the final deuteron

$K = P + qt$ —total momentum for the kernel of interaction in IC, $K^2 = s$

$k^{(l)} = p - (-1)^l \frac{q}{2}t$, $l = 1, 2$ —initial relative momentum of the kernel of interaction in IC

$k'^{(l)} = p' + (-1)^l \frac{q}{2}(1-t)$, $l = 1, 2$ —final relative momentum for the kernel of interaction in IC

The Lorentz transformation operators:

$$L = \begin{pmatrix} 1 + \frac{w}{M_d} & 0 & 0 & \frac{q_z}{M_d} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{q_z}{M_d} & 0 & 0 & 1 + \frac{w}{M_d} \end{pmatrix} = \begin{pmatrix} 1 + 2\eta & 0 & 0 & 2\sqrt{\eta}\sqrt{1+\eta} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2\sqrt{\eta}\sqrt{1+\eta} & 0 & 0 & 1 + 2\eta \end{pmatrix},$$

$$L^{-1} = \begin{pmatrix} 1 + \frac{w}{M_d} & 0 & 0 & -\frac{q_z}{M_d} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{q_z}{M_d} & 0 & 0 & 1 + \frac{w}{M_d} \end{pmatrix} = \begin{pmatrix} 1 + 2\eta & 0 & 0 & -2\sqrt{\eta}\sqrt{1+\eta} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2\sqrt{\eta}\sqrt{1+\eta} & 0 & 0 & 1 + 2\eta \end{pmatrix},$$

$$L_t = \begin{pmatrix} 1 + \frac{wt}{M_d} & 0 & 0 & \frac{q_z t}{M_d} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{q_z t}{M_d} & 0 & 0 & 1 + \frac{wt}{M_d} \end{pmatrix} = \begin{pmatrix} 1 + 2\eta t & 0 & 0 & 2\sqrt{\eta}\sqrt{1+\eta}t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2\sqrt{\eta}\sqrt{1+\eta}t & 0 & 0 & 1 + 2\eta t \end{pmatrix},$$

$$L_t^{-1} = \begin{pmatrix} 1 + \frac{wt}{M_d} & 0 & 0 & -\frac{q_z t}{M_d} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{q_z t}{M_d} & 0 & 0 & 1 + \frac{wt}{M_d} \end{pmatrix} = \begin{pmatrix} 1 + 2\eta t & 0 & 0 & -2\sqrt{\eta}\sqrt{1+\eta}t \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -2\sqrt{\eta}\sqrt{1+\eta}t & 0 & 0 & 1 + 2\eta t \end{pmatrix}.$$

The Lorentz transformation of the momenta:

total momentum $P' = LP$, $P = L^{-1}P'$

total momentum $K = L_t K_{(0)}$, $K_{(0)} = P = L_t^{-1}K$

relative momentum $p' = Lp'_{(0)}$, $p'_{(0)} = L^{-1}p'$

$$\begin{cases} p'_{(0)0} = (1 + 2\eta)p'_0 - 2\sqrt{\eta}\sqrt{1+\eta}p'_z \\ p'_{(0)x} = p'_x \\ p'_{(0)y} = p'_y \\ p'_{(0)z} = -2\sqrt{\eta}\sqrt{1+\eta}p'_0 + (1 + 2\eta)p'_z \end{cases}$$

relative momentum $k^{(l)} = L_t k^{(l)}_{(0)}$, $k^{(l)}_{(0)} = L_t^{-1}k^{(l)}$:

1st term

$$\begin{cases} k^{(1)}_{(0)0} = (1 + 2\eta t)p'_0 - 2\sqrt{\eta}\sqrt{1+\eta}t p'_z + M_d \eta t(1 - 2t) \\ k^{(1)}_{(0)x} = p'_x \\ k^{(1)}_{(0)y} = p'_y \\ k^{(1)}_{(0)z} = -2\sqrt{\eta}\sqrt{1+\eta}t p'_0 + (1 + 2\eta t)p'_z + M_d \sqrt{\eta}\sqrt{1+\eta}t \end{cases}$$

2nd term

$$\begin{cases} k^{(2)}_{(0)0} = (1 + 2\eta t)p'_0 - 2\sqrt{\eta}\sqrt{1+\eta}t p'_z - M_d \eta t(1 - 2t) \\ k^{(2)}_{(0)x} = p'_x \\ k^{(2)}_{(0)y} = p'_y \\ k^{(2)}_{(0)z} = -2\sqrt{\eta}\sqrt{1+\eta}t p'_0 + (1 + 2\eta t)p'_z - M_d \sqrt{\eta}\sqrt{1+\eta}t \end{cases}$$

relative momentum $k^{(l)} = L_t k^{(l)}_{(0)}$, $k^{(l)}_{(0)} = L_t^{-1}k^{(l)}$:

1st term

$$\begin{cases} k^{(1)}_{(0)0} = (1 + 2\eta t)p'_0 - 2\sqrt{\eta}\sqrt{1+\eta}t p'_z \\ \quad - M_d \eta(2t^2 - 3t + 1) \\ k^{(1)}_{(0)x} = p'_x \\ k^{(1)}_{(0)y} = p'_y \\ k^{(1)}_{(0)z} = -2\sqrt{\eta}\sqrt{1+\eta}t p'_0 + (1 + 2\eta t)p'_z \\ \quad + M_d \sqrt{\eta}\sqrt{1+\eta}(t - 1) \end{cases}$$

2nd term

$$\left\{ \begin{array}{l} k'_{(0)0} = (1 + 2\eta t)p'_0 - 2\sqrt{\eta}\sqrt{1 + \eta t}p'_z \\ + M_d\eta(2t^2 - 3t + 1) \\ k'_{(0)x} = p'_x \\ k'_{(0)y} = p'_y \\ k'_{(0)z} = -2\sqrt{\eta}\sqrt{1 + \eta t}p'_0 + (1 + 2\eta t)p'_z \\ - M_d\sqrt{\eta}\sqrt{1 + \eta t}(t - 1) \end{array} \right.$$

APPENDIX B

LORENTZ BOOSTING

Boosting of different values:

$$\begin{aligned} & \Phi_{JM}(P, p)_{\alpha_1\alpha_2} \\ &= \Lambda^{(1)}(P)_{\alpha_1\beta_1}\Lambda^{(2)}(P)_{\alpha_2\beta_2}\Phi_{JM}(P_{(0)}, p_{(0)})_{\beta_1\beta_2}, \\ & \Gamma_{JM}(P, p)_{\alpha_1\alpha_2} \\ &= \Lambda^{(1)}(P)_{\alpha_1\beta_1}\Lambda^{(2)}(P)_{\alpha_2\beta_2}\Gamma_{JM}(P_{(0)}, p_{(0)})_{\beta_1\beta_2}, \\ & S^{(l)}\left(\frac{P}{2} + (-1)^{l+1}p\right)_{\alpha\beta_l} \\ &= \Lambda^{(l)}(P)_{\alpha\gamma_l}S^{(l)}\left(\frac{P_{(0)}}{2} + (-1)^{l+1}p_{(0)}\right)_{\gamma\delta_l}\Lambda^{(l-1)}(P)_{\delta\beta_l}, \\ & V(p', p; P)_{\alpha_1\alpha_2;\beta_1\beta_2} = \Lambda^{(1)}(P)_{\alpha_1\gamma_1}\Lambda^{(2)}(P)_{\alpha_2\gamma_2} \\ & \times V(p'_{(0)}, p_{(0)}; P_{(0)})_{\gamma_1\gamma_2;\delta_1\delta_2}\Lambda^{(1-1)}(P)_{\delta_1\beta_1}\Lambda^{(2-1)}(P)_{\delta_2\beta_2}. \end{aligned}$$

Boost operators:

$$\begin{aligned} \Lambda(P') &= \frac{M_d + P'\gamma\gamma_0}{\sqrt{2M_d(M_d + E_{P'})}} \\ &\text{in l.s.} \left[\frac{\sqrt{1+\eta}}{\sqrt{1+2\eta}} \left[1 - \sqrt{\frac{\eta}{1+\eta}}\gamma_3\gamma_0 \right] \right], \end{aligned}$$

$$\text{with } E_{P'} = \sqrt{M_d^2 + \mathbf{P}'^2}$$

$$\begin{aligned} \Lambda(K) &= \frac{\sqrt{s} + K\gamma\gamma_0}{\sqrt{2\sqrt{s}(\sqrt{s} + E_K)}} \\ &\text{in l.s.} \left[\frac{\sqrt{u'}}{\sqrt{2(1+2\eta)}} \left[1 - \frac{\sqrt{\eta}\sqrt{1+\eta t}}{u'}\gamma_3\gamma_0 \right] \right], \end{aligned}$$

$$\text{with } E_K = \sqrt{s + \mathbf{K}^2} \text{ and } M_d u' = \sqrt{s} + M_d(1 + 2\eta t).$$

APPENDIX C

Partial-wave decomposition (in the rest frame):

BS amplitude:

$$\Phi_{JM}(P, p)_{\gamma_1\gamma_2} = \sum_a \phi_a(p_0, |\mathbf{p}|) \mathcal{Y}_M^a(\mathbf{p})_{\gamma_1\gamma_2},$$

where $\phi_a(p_0, |\mathbf{p}|) = \sum_{ab} S_{ab}(p_0, |\mathbf{p}|; s) g_b(p_0, |\mathbf{p}|)$, and

$$\mathcal{Y}_M^a(\mathbf{p})_{\gamma_1\gamma_2} = i^L \sum_{m_l m_s m_\nu} C_{L m_l S m_s}^{1M} C_{\frac{1}{2} m_l \frac{1}{2} m_\nu}^{S m_s} Y_{L m_l}(\hat{\mathbf{p}}) \times u_\mu^{(1)}(\mathbf{p})_{\gamma_1} u_\nu^{(2)}(-\mathbf{p})_{\gamma_2}.$$

BS vertex function:

$$\Gamma_{JM}(P, p)_{\gamma_1\gamma_2} = \sum_a g_a(p_0, |\mathbf{p}|) \mathcal{Y}_M^a(-p)_{\gamma_1\gamma_2}.$$

Kernel of interaction:

$$\begin{aligned} V(p', p; P)_{\gamma_1\gamma_2;\sigma_1\sigma_2} &= \sum_{abM} v_{\alpha\beta}(p'_0, |\mathbf{p}'|; p_0, |\mathbf{p}|; s) \\ &\times \mathcal{Y}_M^a(-\mathbf{p}')_{\gamma_1\gamma_2} \otimes \mathcal{Y}_M^{b+}(\mathbf{p})_{\sigma_1\sigma_2}. \end{aligned}$$

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