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## PHYSICS OF ELEMENTARY PARTICLES AND ATOMIC NUCLEI. THEORY

# Comparative Analysis of Finite Field-dependent BRST Transformations[1]

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**Abstract**—We review our recent study [1–6], introducing the concept of finite field-dependent BRST and BRST-antiBRST transformations for gauge theories and investigating their properties. An algorithm of exact calculation for the Jacobian of a respective change of variables in the path integral is presented. Applications to the Yang–Mills theory, in view of infra-red (Gribov) peculiarities, are discussed.

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## 1. INTRODUCTION

BRST transformations [7, 8] for gauge theories in Lagrangian formalism were first examined in the capacity of *field-dependent (FD)* BRST transformations within the field-antifield approach [9] in order to prove the independence from small gauge variations (expressed through the gauge fermion $\psi$) of the path integral $Z_\psi$: $Z_\psi = Z_{\psi+\delta\psi}$, with the choice $\mu = -\frac{i}{\hbar}\delta\psi$ for the Grassmann-odd parameter of FD BRST transformations. Originally introduced as the case of a special $N = 1$ SUSY transformation, being a change of the field variables $\phi^A$,

$$\begin{gathered}\phi^A \to \phi^{A'} = \phi^A + \delta_\mu \phi^A,\\ \mathcal{I}^\psi_\phi = d\phi \exp\left\{\frac{\imath}{\hbar} S_\psi(\varphi)\right\},\ Z_\psi = \int \mathcal{I}^\psi_\varphi,\end{gathered} \tag{1}$$

in the integrand $\mathcal{I}^\psi_\phi$ with a quantum action $S_\psi(\phi)$, BRST transformations were extended, by means of antiBRST transformations [10, 11] in Yang–Mills theories, to $N = 2$ BRST-antiBRST transformations (in Yang–Mills [12] and general gauge theories [13]), which were associated with an Sp(2)-doublet of Grassmann-odd parameters, $\mu_a$, $a = 1, 2$.

The concept of *finite* FD BRST transformations was introduced [14] in Yang–Mills theories, as a sequence of infinitesimal FD BRST transformations, in order to prove the gauge-independence of the path integral within the family of $R_\xi$-gauges and their nonlinear deformations in the field variables. The authors of [15] suggested an analysis of so-called *soft BRST symmetry breaking* in Yang–Mills theories, with reference to the Gribov problem [16] in the long-wave spectra of field configurations, which also involves the Zwanziger proposal [17] for a horizon functional joined additively to a BRST invariant quantum action. The study of [18] investigated the scope of problems related to [15] in the field-antifield formalism and suggested an equation for the BRST non-invariant addition $M(\phi, \phi^*)$ to the quantum action $S_\psi(\phi, \phi^*)$ of a general gauge theory. The validity of this equation preserves the gauge-independence of the corresponding vacuum functional $Z_{\psi,M}(0)$, see (4) for a definition,

$$\begin{gathered}\left[M_A\left(\frac{\hbar}{\imath}\vec{\partial}_{(J)}, \phi^*\right)\left(\vec{\partial}^{*A} - \frac{\hbar}{\imath}M^{*A}\left(\frac{\hbar}{\imath}\vec{\partial}_J, \phi^*\right)\right)\right.\\ \left.\times\ \delta\psi\left(\frac{\hbar}{\imath}\vec{\partial}_{(J)}\right) + \delta M\left(\frac{\hbar}{\imath}\vec{\partial}_{(J)}, \phi^*\right)\right] Z_{\psi,M}(J, \phi^*) = 0,\end{gathered} \tag{2}$$

where it is assumed that $[M_A, M^{*A}] \equiv [M\overleftarrow{\partial}_A, \vec{\partial}^{*A}M]$. In terms of the vacuum expectation value, in the presence of external sources $J_A$, and with a given gauge $\psi$, relation (2) acquires the form,[2] $\frac{\vec{\delta} S_\psi}{\delta\phi^*_A} \equiv \vec{\partial}^{*A} S_\psi$,

$$\begin{gathered}\left\langle \delta M + M\overleftarrow{s}\frac{\imath}{\hbar}\delta\psi(\phi)\right\rangle = \langle \delta M - M\overleftarrow{s}\mu(\delta\psi)\rangle = 0,\\ \text{where } \overleftarrow{s} = \overleftarrow{\partial}_A S_\psi \vec{\partial}^{*A} S_\psi : \delta_\mu\phi^A \equiv \phi^A \overleftarrow{s}\mu,\end{gathered} \tag{3}$$

where $\overleftarrow{s}$ is the generator of BRST transformations. This fact was established in [1]. The authors of [19]

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[2] In fact, the horizon functional in the family of $R_\xi$-gauges for small $\xi$ was found explicitly in [18], see Eq. (5.20) therein, by using FD BRST transformations with a small odd-valued parameter.

attempted to use FD BRST transformations [14] for relating the vacuum functionals in YM and GZ (Gribov–Zwanziger) theories under different gauges. An explicit calculation of the functional Jacobian for a change of variables induced by FD BRST transformations in YM theories with a finite parameter $\mu$ was made in [20], to establish the gauge-independence of $Z_{\psi,M}\big|_{M=0}$ under a finite change of the gauge, $\psi \to \psi + \Delta\psi$, and afterwards in [21], to solve equation (3), with $M(\phi,\phi^*) = H(A)$ for GZ theory, in a way different from anticanonical transformations, as compared to [18].

The present article reviews the study of finite BRST and BRST-antiBRST (special $N = 1, 2$ SUSY) transformations (including the case of field-dependent parameters), and the way they influence the properties of the quantum action and path integral in conventional quantization. We use the DeWitt condensed notation and the conventions of [1, 2], e.g., we use $\epsilon(F)$ for the value of Grassmann parity of a quantity $F$. Derivatives with respect to (anti)field variables $\phi^A, \phi^*_A$ and sources $J_A$ are denoted by $\overleftarrow{\partial}^A, (\overrightarrow{\partial}^*_A)$ and $\overleftarrow{\partial}^A_{(J)}$. The raising and lowering of Sp(2) indices, $(\overleftarrow{s}^a, \overleftarrow{s}_a) = (\varepsilon^{ab}\overleftarrow{s}_b, \varepsilon_{ab}\overleftarrow{s}^b)$, are carried out by a constant antisymmetric tensor $\varepsilon^{ab}$, $\varepsilon^{ac}\varepsilon_{cb} = \delta^a_b$, $\varepsilon^{12} = 1$.

## 2. PROPOSALS FOR FINITE BRST TRANSFORMATIONS

The problem of softly broken BRST symmetry (SB BRST) in general gauge theories was solved in [1] on a basis of finite FD BRST transformations (invariance transformations for the integrand in (4) at $J = M = 0$) with finite odd-valued parameters $\mu(\phi,\phi^*)$ depending on external antifields $\phi^*_A$, $\epsilon(\phi^*_A) + 1 = \epsilon(\phi^A) = \epsilon_A$, and internal fields $\phi^A$ whose contents include the classical fields $A^i$, $i = 1,..,n$, with gauge transformations $\delta A^i = R^i_\alpha(A)\xi^\alpha$, $\alpha = 1,..,m < n$, the ghost, antighost, and Nakanishi–Lautrup fields $C^\alpha, \overline{C}^\alpha, B^\alpha$, $\epsilon(A^i, \xi^\alpha, C^\alpha, \overline{C}^\alpha, B^\alpha) = (\epsilon_i, \epsilon_\alpha, \epsilon_\alpha + 1, \epsilon_\alpha + 1, \epsilon_\alpha)$, as well as the additional towers of fields depending on the (ir)reducibility of the theory. The generating functional of Green's functions depending on external sources $J_A$, $\epsilon(J_A) = \epsilon_A$, with an SB BRST symmetry term $M$, $\epsilon(M) = 0$, is given by

$$Z_{\psi,M}(J,\phi^*) = \int d\varphi \exp\left\{\frac{\imath}{\hbar}S_\psi(\phi,\phi^*) + M(\phi,\phi^*) + J_A\phi^A\right\}, \quad (4)$$

$$\overleftarrow{s}_e = \overleftarrow{\partial}_A \overrightarrow{\partial}^{*A} S_\psi \equiv \overleftarrow{\partial}_A S^{*A}_\psi,$$

where the generator $\overleftarrow{s}_e$ reduces at $\phi^* = 0$ to the usual generator $\overleftarrow{s}$ of (FD) BRST transformations, $\delta_\mu\phi^A = S^{*A}_\psi(\phi,0)\mu$, and fails to be nilpotent, $(\overleftarrow{s}_e)^2 = \overleftarrow{\partial}_A(S^{*A}_\psi\overleftarrow{\partial}_B)S^{*B}_\psi \neq 0$, due to the quantum master equation for $S_\psi$, $\Delta\exp\left\{\frac{\imath}{\hbar}S_\psi\right\} = 0$, with $\Delta = (-1)^{\epsilon_A}\overrightarrow{\partial}_A\overrightarrow{\partial}^{*A}$.

The construction of finite BRST-antiBRST Lagrangian transformations solving the same problem within a suitable quantization scheme (starting from YM theories), is problematic in view of the BRST-antiBRST non-invariance of the gauge-fixed quantum action $S_F$, in a form more than linear in $\mu_a$, $S_F(g_l(\mu_a)\phi) = S_F(\phi) + O(\mu_1\mu_2)$, with the gauge condition encoded by a gauge boson $F(\phi)$. This problem was solved by finite BRST-antiBRST transformations in a group form, $\{g(\mu_a)\}$, using an appropriate set of variables $\Gamma^p$, according to [2]

$$\left\{G(g(\mu_a)\Gamma) = G(\Gamma) \text{ and } G\overleftarrow{s}^a = 0\right\} \Rightarrow g(\mu_a) = 1 + \overleftarrow{s}^a\mu_a + \frac{1}{4}\overleftarrow{s}^a\overleftarrow{s}_a\mu^2 = \exp\left\{\overleftarrow{s}^a\mu_a\right\}, \quad (5)$$

where $G$ is a certain functional with the indicated conditions, $\mu^2 \equiv \mu_a\mu^a$, and $\overleftarrow{s}^a$, $\overleftarrow{s}^2 \equiv \overleftarrow{s}^a\overleftarrow{s}_a$ are the generators of BRST-antiBRST and mixed BRST-antiBRST transformations in the space of $\Gamma^p$. These transformations, however, cannot be presented as group elements (in terms of an exp-like relation) for an $Sp(2)$ doublet $\mu_a$ which is not closed under BRST-antiBRST transformations: $\mu_a\overleftarrow{s}_b \neq 0$.

In YM theories, the construction of finite $N = 2$ BRST transformations (5) is straightforward [2] and uses the explicit form of BRST-antiBRST generators [13] in the space of fields $\phi^A = (A^i, C^\alpha, \overline{C}^\alpha, B^\alpha)$ arranged into $Sp(2)$-symmetric tensors, $(A^i, C^{\alpha a}, B^\alpha) = (A^{\mu m}, C^{ma}, B^m)$.

In general gauge theories, such as reducible ones or those with an open gauge algebra, the corresponding space of triplectic variables $\Gamma^p_{tr} = (\phi^A, \phi^*_{Aa}, \overline{\phi}_A, \pi^{Aa}, \lambda^A)$ in the $Sp(2)$-covariant Lagrangian quantization scheme [13] contains, in addition to $\phi^A$, 3 sets of antifields $\phi^*_{Aa}$, $\overline{\phi}_A$, $\epsilon(\phi^*_{Aa}, \overline{\phi}_A) = (\epsilon_A + 1, \epsilon_A)$, as sources to BRST, antiBRST and mixed BRST-antiBRST transformations, and 3 sets of Lagrangian multipliers $\pi^{Aa}, \lambda^A$, $\epsilon(\pi^{Aa}, \lambda^A) = (\epsilon_A + 1, \epsilon_A)$, introducing the gauge. The corresponding generating functional of Green's functions, $Z_F(J)$,

$$Z_F(J) = \int d\Gamma \exp\left\{(\imath/\hbar)\left[S + \phi^*_a\pi^a + \bar{\phi}\lambda - \frac{1}{2}F\bar{U}^2 + J\phi\right]\right\}, \quad (6)$$
$$\bar{U}^a = \bar{\partial}_A\pi^{Aa} + \varepsilon^{ab}\bar{\partial}^{(\pi)}_{Ab}\lambda^A$$

is invariant, at $J = 0$, with respect to finite $N = 2$ BRST transformations (for constant $\mu_a$) in the space of $\Gamma^p_{tr}$, which are given by (5) with a functional $G_{tr} = G(\Gamma^p_{tr})$:

$$\Gamma^p_{tr} \to \Gamma'^p_{tr} = \Gamma^p_{tr}\left(1 + \bar{s}^a\mu_a + \frac{1}{4}\bar{s}^2\mu^2\right) \equiv \Gamma^p_{tr}g(\mu_a) \Rightarrow \mathcal{I}^{(F)}_{\Gamma_{tr}g(\mu_a)} = \mathcal{I}^{(F)}_{\Gamma_{tr}} \quad (7)$$
$$\text{for } Z_F = \int \mathcal{I}^{(F)}_{\Gamma_{tr}},$$

where $\bar{s}^a = (\bar{\partial}_A, \bar{\partial}^{Aa}_{(\phi^*)}, \bar{\partial}^A_{(\bar{\phi})}, \bar{\partial}^{(\pi)}_{Ab})$ $(\pi^{Aa}, S_{,A}(-1)^{\epsilon_A}, \varepsilon^{ab}\phi^*_{Ab}(-1)^{\epsilon_A+1}, \varepsilon^{ab}\lambda^A)^T$, $\{\bar{s}^a, \bar{s}^b\} \neq 0$, provided that

$$\left(\Delta^a + (\imath/\hbar)\varepsilon^{ab}\phi^*_{Ab}\bar{\partial}^A_{(\bar{\phi})}\right) \times \exp\left\{\frac{\imath}{\hbar}S\right\} = 0, \text{ for } \Delta^a = (-1)^{\epsilon_A}\bar{\partial}_A\bar{\partial}^{*Aa}. \quad (8)$$

## 3. JACOBIANS OF FINITE $N = 1, 2$ BRST TRANSFORMATIONS

The Jacobian induced by a change of variables[3] $\phi^A \to \phi'^A = \phi^A(1 + \bar{s}_e\mu)$ is given by [1]

$$\mathrm{Sdet}\left\|\phi'^A\bar{\partial}_B\right\| = \exp\left\{\mathrm{Str}\ln\left(\delta^A_B + (S^{*A}_\psi\mu)\bar{\partial}_B\right)\right\} = \exp\left\{\mathrm{Str}\sum_{n=1}\frac{(-1)^{n+1}}{n}((S^{*A}_\psi\mu)\bar{\partial}_B)^n\right\} \quad (9)$$
$$= (1 + \mu\bar{s}_e)^{-1}\{1 + \bar{s}_e\mu\}\{1 + (\Delta S_\psi)\mu\}$$

and reduces, in a rank-1 theory with a closed gauge algebra, $[\Delta S_\psi, \bar{s}^2] = [0,0]$, where $\bar{s}_e = \bar{s}$, to the form $\mathrm{Sdet}\left\|\Phi'^A\bar{\partial}_B\right\| = (1 + \mu\bar{s})^{-1}$, which is the same as in YM theories. The Jacobian (9) allows one to solve the problem of SB BRST symmetry in general gauge theories [1] and was examined in detail [5] for an equivalent representation of $Z_{\psi,M}(J,\phi^*)$ with BRST transformations $\Gamma^p \to \Gamma^{p'} = \Gamma^p(1 + \bar{s}\mu) = \Gamma^p(1 + \bar{s}\mu)$, for $\mu(\Gamma)$ and $\Gamma^p\bar{s} = (\phi^A, \tilde{\phi}^*_A, \lambda^A)\bar{s} = (\lambda^A, S\bar{\partial}_A, 0)$, in an extended space $\Gamma^p$ of fields $\phi^A$, internal antifields $\tilde{\phi}^*_A$, and Lagrangian multipliers $\lambda^A$ to Abelian hypergauge conditions, $G_A(\phi,\phi^*) = \phi^*_A - \psi(\phi)\bar{\partial}_A$, with the result given by

$$\mathrm{Sdet}\left\|\Gamma^{p'}\bar{\partial}^\Gamma_q\right\| = (1 + \mu\bar{s})^{-1}\{1 + (\Delta S_\psi)\mu\} + O(\mu\bar{s}\mu). \quad (10)$$

For BRST-antiBRST transformations in YM theories, the technique of calculating the Jacobian was first examined for functionally-dependent parameters $\mu_a = \Lambda(\phi)\bar{s}_a$ with an even-valued functional $\Lambda$ and was developed in [2]. The result is given by, $\phi'^A \equiv \phi^A g(\Lambda(\phi)\bar{s}_a)$,

$$J_{\Lambda(\phi)\bar{s}_a} = \mathrm{Sdet}\left\|\phi'^A\bar{\partial}_B\right\| = \exp\left\{\mathrm{Str}\ln\left(\delta^A_B + M^A_B\right)\right\},$$
$$\text{for } M^A_B = P^A_B + Q^A_B + R^A_B = \phi^A\bar{s}^a(\mu_a\bar{\partial}_B) + \mu_a[(\phi^A\bar{s}^a)\bar{s}_B - \frac{1}{2}(\phi^A\bar{s}^2)(\mu^a\bar{\partial}_B)](-1)^{\epsilon_A+1} + \frac{1}{4}\mu^2(\phi^A\bar{s}^2\bar{s}_B), \quad (11)$$

$$\mathrm{Str}(P + Q + R)^n = \mathrm{Str}(P + Q)^n + C^1_n\mathrm{Str}P^{n-1}R, \text{ for } C^k_n = n!/k!(n-k)!, \quad (12)$$

$$\mathrm{Str}(P + Q)^n = \begin{cases} \mathrm{Str}P^n + n\mathrm{Str}P^{n-1}Q + C^2_n\mathrm{Str}P^{n-2}Q^2, & n = 2, 3, \\ \mathrm{Str}P^n + n\sum_{k=0}^{2}\mathrm{Str}P^{n-k}Q^k + K_n\mathrm{Str}P^{n-3}QPQ, & n > 3 \end{cases} \quad (13)$$

$$\Rightarrow J_{\Lambda(\phi)\bar{s}_a} = \exp\left\{\sum_{n=1}(-1)^{n-1}n^{-1}\mathrm{Str}(P^A_B)^n\right\} = \left(1 - \frac{1}{2}\Lambda\bar{s}^2\right)^{-2}, \quad (14)$$

where $K_n = \left[\frac{n+1}{2} - 2\right]C^1_n + ((n+1)\mathrm{mod}2)C^1_{\left[\frac{n}{2}\right]}$, with $[x]$ being the integer part of $x \in R$. For functionally-independent FD parameters $\mu_a(\phi) \neq \Lambda\bar{s}_a$, the above algorithm (11)–(14) involves a generalization of (14), examined separately for odd and even $n$, which leads to [6]

$$J_{\mu_a} = \exp\left\{\mathrm{tr}\sum_{n=1}(-1)^{n-1}n^{-1}\mathrm{Str}(P^A_B)^n\right\} = \exp\{-\mathrm{tr}\ln(e + m)\}, \; m^a_b = \mu_b\bar{s}^a, \quad (15)$$

[3] In the case $\mu\bar{s}_e \neq 0$, the set $\{g(\mu)\}$, for $\phi' = \phi g(\mu)$, cannot be presented as Lie group elements: $g(\mu) \neq \exp(\bar{s}_e\mu)$.

where $(e)^a_b$ and tr denote $\delta^a_b$ and trace over Sp(2) indices. The Jacobian (15) is generally not BRST-antiBRST exact; however, it reduces at $\mu_a = \Lambda\bar{s}_a$ to the Jacobian (14), due to

$$\mathrm{tr}m^a_b = \mathrm{tr}\Lambda\bar{s}_b\bar{s}^a = -(1/2)\mathrm{tr}\delta^a_b\Lambda\bar{s}^2 \\ \Rightarrow \mathrm{tr}m^n = 2[-(1/2)\Lambda\bar{s}^2]^n \Rightarrow J_{\mu_a} = J_{\Lambda\bar{s}_a}. \tag{16}$$

In general gauge theories (6)–(8), the calculation of Jacobians induced by FD BRST-antiBRST transformations was first carried out in [3, 5] with functionally-dependent parameters $\mu_a = \Lambda(\phi,\pi,\lambda)\overleftarrow{U}_a$, the restricted generators $\overleftarrow{U}^a = \bar{s}^a|_{\phi,\pi,\lambda}$ satisfying the algebra $\{\overleftarrow{U}^a, \overleftarrow{U}^b\} = 0$, and afterwards in [6] with arbitrary parameters $\mu_a(\Gamma_{\mathrm{tr}})$, including functionally-independent $\mu_a(\phi,\pi,\lambda)$. The result is given by

$$J_{\Lambda\overleftarrow{U}_a} = \mathrm{Sdet}\left\|\left[\Gamma^p_{tr} g(\Lambda\overleftarrow{U}_a)\right]\overleftarrow{\partial}^{\Gamma}_q\right\| \\ = \exp\left[-\left(\Delta^a S\right)\mu_a - \frac{1}{4}\left(\Delta^a S\right)\bar{s}_a\mu^2\right]\left(1-\frac{1}{2}\Lambda\bar{s}^2\right)^{-2}, \tag{17}$$

$$J_{\mu_a(\phi,\pi,\lambda)} = \exp\left\{-\left(\Delta^a S\right)\mu_a \right. \\ \left. - \frac{1}{4}\left(\Delta^a S\right)\bar{s}_a\mu^2 - \mathrm{tr}\ln(e+m)\right\}, \tag{18}$$

$$J_{\mu_a(\Gamma_{tr})} = J|_{\mu_a(\phi,\pi,\lambda)\to\mu_a(\Gamma_{tr})} \\ \times \exp\left\{\frac{1}{4}(\mu_a\overleftarrow{\partial}^{\Gamma}_p)\left[(e+m)^{-1}\right]^a_b\left(\Gamma^p_{tr}\bar{s}^2\bar{s}^b\right)\mu^2\right\}. \tag{19}$$

The second multiplier in (19) draws a difference between the Jacobians $J_{\mu_a(\phi,\pi,\lambda)}$ and $J_{\mu_a(\Gamma_{tr})}$, because $\bar{s}_a$ are not reduced to the nilpotent $\overleftarrow{U}_a$ as they act on $\Gamma^p_{tr}$. In generalized Hamiltonian formalism, the Jacobians of corresponding FD BRST-antiBRST transformations were calculated from first principles by the rules (11)–(15) in [4, 6].

## 4. IMPLICATIONS OF FINITE BRST TRANSFORMATIONS

For FD parameters, finite BRST transformations allow one to obtain a new form of the Ward identity and to establish the gauge-independence of the path integral under a finite change of the gauge, $\psi \to \psi + \psi'$, provided that the SB BRST symmetry term $M = M_\psi$ transforms to $M_{\psi+\psi'} = M_\psi(1+\bar{s}\mu(\psi'))$, with $\mu(\psi')$ being a solution of a so-called compensation equation:

$$Z_{\psi,M_\psi}(0,\phi^*) = Z_{\psi+\psi',M_{\psi+\psi'}}(0,\phi^*) \\ \Rightarrow \psi'(\phi,\lambda|\mu) = \frac{\hbar}{i}\left[\sum_{n=1}\tfrac{(-1)^{n-1}}{n}(\mu\bar{s})^{n-1}\right]\mu. \tag{20}$$

The Ward identity, depending on the FD parameter $\mu(\psi') = -\frac{i}{\hbar}g(y)\psi'$, for $g(y) = 1-\exp\{y\}/y$, $y \equiv (i/\hbar)\psi'\bar{s}$, and the gauge-dependence problem are described by the respective expressions [5]

$$\left\langle\left\{1+\frac{i}{\hbar}\left[J_A\phi^A + M_\psi\right]\bar{s}\mu(\psi')\right\} \right. \\ \left. -(1+\mu(\psi')\bar{s}s)^{-1}\right\rangle_{\psi,M,J} = 1, \tag{21}$$
$$\text{and } \left\langle(J_A\phi^A + M_\psi)\bar{s}\right\rangle_{\psi,M,J} = 0,$$

as one makes averaging with respect to $Z_{\psi,M_\psi}(J,\phi^*)$. The above equations are equivalent to those of [1, 18].

FD BRST-antiBRST transformations solve the same problem under a finite change of the gauge, $F \to F + F'$, provided that the SB BRST-antiBRST symmetry term $M_F$ transforms to $M_{F+F'} = M_F(1+\bar{s}^a\mu_a(F') + \frac{1}{4}\bar{s}^2\mu^2(F'))$, with $\mu_a(F';\phi,\pi,\lambda) = \Lambda\overleftarrow{U}_a$ being a solution to the corresponding compensation equation based on (6):

$$Z_F(0) = Z_{F+F'}(0) \Rightarrow F'(\phi,\pi,\lambda|\mu_a) \\ = 4\imath\hbar\left[\sum_{n=1}\tfrac{(-1)^{n-1}}{2^n n}\left(\Lambda\overleftarrow{U}^2\right)^{n-1}\Lambda\right]. \tag{22}$$

As a result, the corresponding Ward identity, with the FD parameters $\mu_a(F') = \frac{i}{2\hbar}g(y)F'\overleftarrow{U}_a$, $\Lambda(\Gamma|F') = \frac{i}{2\hbar}g(y)F'$, for $y \equiv (i/4\hbar)F'\overleftarrow{U}^2$, and the gauge-dependence problem acquire the form [5]

$$\left\langle\left\{1+\frac{i}{\hbar}J_A\phi^A\left[\overleftarrow{U}^a\mu_a(\Lambda) + \frac{1}{4}\overleftarrow{U}^2\mu^2(\Lambda)\right] \right.\right. \\ \left. - \frac{1}{4}\left(\frac{i}{\hbar}\right)^2 J_A\phi^A\overleftarrow{U}^a J_B(\phi^B)\overleftarrow{U}_a\mu^2(\Lambda)\right\} \\ \left. \times \left(1-\tfrac{1}{2}\Lambda\overleftarrow{U}^2\right)^{-2}\right\rangle_{F,J} = 1, \tag{23}$$

$$Z_{F+F'}(J) = Z_F(J)\left\{1+\left\langle\frac{i}{\hbar}J_A\phi^A\left[\overleftarrow{U}^a\mu_a(\Gamma|-F')\right] \right.\right. \\ + \frac{1}{4}\overleftarrow{U}^2\mu^2(\Gamma|-F') - (-1)^{\varepsilon_B}\left(\frac{i}{2\hbar}\right)^2 \\ \left.\left. \times J_B J_A\left(\phi^A\overleftarrow{U}^a\right)\left(\phi^B\overleftarrow{U}_a\right)\mu^2(\Gamma|-F')\right\rangle_{F,J}\right\}, \tag{24}$$

with a source-dependent average expectation value with respect to $Z_F(J)$ corresponding to a gauge-fixing $F(\phi)$.

By choosing the $N = 1$ or $N = 2$ SB BRST symmetry term $M(\phi)$ as the horizon functional $H(A)$ in Landau gauge and assuming the gauge-independence of

$Z_{H,\psi}$, $Z_{H,F}$ under a finite change of the gauge condition, $\psi \to \psi + \psi'$ or $F \to F + F'$, one can determine the functional $H(A)$ in a new reference frame, $\psi + \psi'$ or $F + F'$, of the respective $N = 1, 2$ BRST symmetry setting, with account taken of (20), (22):

$$H_{\psi'}(\phi) = H(A)\{1 + \bar{s}\mu(\psi')\} \text{ or}$$
$$H_{F'}(\phi) = H(A)\left\{1 + \bar{s}^a\mu_a(F') + \frac{1}{4}\bar{s}^2\mu^2(F')\right\}. \quad (25)$$

Notice in conclusion that the above $N = 1, 2$ FD BRST transformations make it possible to study their influence on the Yang–Mills, Gribov–Zwanziger, Freedman–Townsend models, and the Standard Model, as well as on the concept of average effective action [1–3, 5, 6].

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