PHYSICS OF ELEMENTARY PARTICLES
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Relation between the Charged and Neutral Pion–Nucleon Coupling Constants in the Yukawa Model

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Abstract—In the Yukawa-model framework for NN forces, a simple relation between the charged and neutral pion–nucleon coupling constants is derived. The relation implies that the charged pion–nucleon constant is larger than the neutral one since the np interaction is stronger than the pp interaction. The derived value of the charged pion–nucleon constant shows a very good agreement with one of the recent measurements. In relative units, the splitting between the charged and neutral pion–nucleon constants is predicted to be practically the same as that between the charged and neutral pion masses. The charge dependence of the NN scattering length arising from the mass difference between the charged and neutral pions is also analyzed.

1. INTRODUCTION

The pion–nucleon coupling constants are fundamental characteristics of nuclear forces that play an important role in investigations of nucleon–nucleon and pion–nucleon interactions [1–5]. Exact knowledge of their values is instrumental for quntitatively describing and qualitatively understanding a broad variety of hadron- and nuclear-physics phenomena [3–8]. For this reason, the pion–nucleon constants have been investigated and their values have been refined throughout the whole period of nuclear-physics studies. Historically, the evolution of these investigations is detailed in [5, 7, 9].

That charge-independence of the pion–nucleon constant may be violated, or pion–nucleon constants may differ for neutral and charged π mesons, is a problem that has lately attracted much attention. That different analyses yield different values of the charged pion–nucleon constant $g_{\pi^\pm}^2$ renders this problem particularly important. Recent experimental estimates of $g_{\pi^\pm}^2$ vary between 13.54 [9, 10] and 14.74 [11]. As for the neutral pion–nucleon constant $g_{\pi^0}^2$, its value has been reliably and accurately measured as 13.5–13.6 [5, 9, 12–15].

Thus, the values of the charged-pion constant $g_{\pi^\pm}^2$ measured in [5, 7, 9, 10, 13, 16–19] are close to that of the neutral-pion constant $g_{\pi^0}^2$ in agreement with the charge independence of the pion–nucleon coupling. On the other hand, the measurements [5–7, 11, 14, 20–23] yielded significantly larger values of $g_{\pi^\pm}^2$ than that of $g_{\pi^0}^2$. Therefore, the charge-independence of the pion–nucleon coupling is still an open fundamental problem which calls for further experimental and theoretical investigation.

In this paper we investigate the pion–nucleon coupling constant relying on the standard classical Yukawa model [1–3, 24] for the nucleon–nucleon interaction and invoking contemporary experimental data on low-energy parameters of nucleon–nucleon scattering. It is well known [1–3] that, unfortunately, no accurate quantitative description of the nucleon–nucleon system can be obtained with the Yukawa potential as soon as its parameters are defined and selected using fundamental quantities of the field theory: the pion masses and coupling constants. Therefore, in selecting the parameter values of the Yukawa potential, we rather rely on measured low-energy parameters of nucleon–nucleon scattering in the effective range theory.

Analyzing the pion–nucleon coupling constants in the Yukawa-model framework is justified, since the one-pion exchange which corresponds to the Yukawa potential is the dominant mechanism of the nucleon–nucleon interaction at the lowest collision energies, which imply long-range interactions. The two-pion exchange and the heavier ρ- and ω-meson exchanges are dominant at medium and small distances, and quark–gluon degrees of freedom gain an important role at the smallest interaction ranges. Quite understandably, in determining the pion–nucleon coupling

constant as a characteristic of the pion–nucleon interaction, one should rely on the data for long-range (or peripheral) nucleon–nucleon interaction, which is dominated by one-pion exchange.

In quantum field theory, the pion–nucleon interaction may be described by either a pseudoscalar (PS) or a pseudovector (PV) Lagrangian formulated as [3, 9, 20, 25, 26]

$$\mathscr{L}_{\pi N}^{PS} = g_\pi \sqrt{4\pi}(i\bar{\psi}\gamma_5\psi)\phi, \qquad (1)$$

$$\mathscr{L}_{\pi N}^{PV} = \frac{f_\pi}{m_s} \sqrt{4\pi}(\bar{\psi}\gamma_\mu\gamma_5\psi)\partial^\mu\phi, \qquad (2)$$

where in the latter formula the scaling mass m_s is introduced so as to render the pseudovector pion–nucleon coupling constant f_π dimensionless. In Eqs. (1) and (2), the nucleon and pion fields are denoted as $\psi(x)$ and $\phi(x)$, respectively. As was demonstrated in [9, 27], the Lagrangian forms (1) and (2) are interrelated by a gauge transformation and therefore are equivalent. Also note that our Lagrangian definitions (1) and (2) feature explicit $\sqrt{4\pi}$ factors so that the 4π factor no longer enters the coupling constants themselves. In other words, we follow the convention adopted in the review paper [9] and write simply g_π^2 instead of the often-used denotation $g_\pi^2/4\pi$.

According to the convention adopted in [9, 25], the scaling mass m_s is usually assumed to be equal to the charged-pion mass: $m_s = m_{\pi^\pm}$. Then, the pseudoscalar pion–nucleon coupling constant g_π and the pseudovector coupling constant f_π are interrelated by the equivalence equation [9, 27] implied by the equivalence of Lagrangian forms (1) and (2),

$$\frac{g_\pi}{M_1 + M_2} = \frac{f_\pi}{m_{\pi^\pm}}, \qquad (3)$$

where M_1 and M_2 are the masses of interacting nucleons. Therefore, the π^0 and $\pi^\pm$ pseudoscalar coupling constants, g_{π^0} and $g_{\pi^\pm}$, and the corresponding pseudovector constants f_{π^0} and $f_{\pi^\pm}$ may be interrelated through [5, 9, 14]

$$g_{\pi^0} = \frac{2M_p}{m_{\pi^\pm}} f_{\pi^0}, \qquad (4)$$

$$g_{\pi^\pm} = \frac{M_p + M_n}{m_{\pi^\pm}} f_{\pi^\pm}, \qquad (5)$$

where M_p and M_n denote the proton and neutron masses, respectively.

2. DERIVATION AND DISCUSSION OF MAJOR EQUATIONS BETWEEN THE CHARGED AND NEUTRAL PION–NUCLEON COUPLING CONSTANTS IN THE YUKAWA MODEL

According to the meson-field theory, low-energy strong interaction between two nucleons is dominated by the exchange of a virtual π meson, which determines the form of the long-range nucleon–nucleon interaction. The classical one-pion-exchange potential of the nucleon–nucleon interaction in the meson-field theory, referred to as the Yukawa potential, for the pure singlet $^1S_0$ state has a simple and well-known form of [1−3, 24, 26]

$$V_Y(r) = -V_0 \frac{e^{-\mu r}}{\mu r}. \qquad (6)$$

Here, r is the distance between the nucleons and μ is expressed through the π-meson mass m_π as

$$\mu = \frac{m_\pi c}{\hbar}, \qquad (7)$$

where c and $\hbar$ are the speed of light and the reduced Planck constant, respectively. According to (7), the nuclear-force radius $R \equiv 1/\mu$ is inversely proportional to the pion mass m_π and has a small value of $R \sim 1.4$ fm. For this case, the depth of the Yukawa potential, V_0, is expressed through the pseudovector pion–nucleon coupling constant via a simple relation [1−3, 5, 26, 28]

$$V_0 = m_\pi c^2 f_\pi^2, \qquad (8)$$

which, like the Yukawa-potential form (6), is a direct consequence of the quantum meson-field theory represented by the Lagrangians (1) and (2).

For the interaction between two protons mediated by the exchange of a neutral π^0 meson, the μ_{pp} and V_0^{pp} parameters of the Yukawa potential (6) are expressed through the π^0 mass m_{π^0} and the coupling constant f_{π^0} according to equations (7) and (8). The neutron–proton interaction involves the exchanges of both the neutral π^0 mesons and charged $\pi^\pm$ mesons. In estimating the μ_{np} and V_0^{np} parameters of the potential (6) for the latter case, one should [8, 29] substitute the averaged π-meson mass

$$\bar{m}_\pi = \frac{1}{3}(m_{\pi^0} + 2m_{\pi^\pm}) \qquad (9)$$

and the averaged pion–nucleon coupling constant

$$\bar{f}_\pi^2 = \frac{1}{3}(f_{\pi^0}^2 + 2f_{\pi^\pm}^2). \qquad (10)$$

We know [1−3] that, unfortunately, no accurate quantitative description of the nucleon–nucleon system can be obtained with the Yukawa potential as soon as its parameters are defined and selected using funda-

mental quantities of the field theory: the pion masses and coupling constants. Therefore, in this paper the parameters of the Yukawa potential are assigned values consistent with the measured parameters of low-energy nucleon−nucleon scattering in the effective range theory [1, 4, 30−36].

Indeed, one may estimate the "effective" mass $m_{\pi^0}^Y$ and pion−nucleon coupling constant $(f_{\pi^0}^Y)^2$ for the neutral π^0 meson from the measured nuclear proton−proton scattering length and effective range, assuming the Yukawa form for the proton−proton potential. Thus estimated $m_{\pi^0}^Y$ and $(f_{\pi^0}^Y)^2$ values prove to significantly exceed the directly measured values [37], so that we have

$$m_{\pi^0}^Y = C_1 m_{\pi^0}, \quad (f_{\pi^0}^Y)^2 = C_2 f_{\pi^0}^2, \qquad (11)$$

where the factors C_1 and C_2 can be computed numerically [37]. Their exact values are not needed for our purposes. It is quite natural to assume that equations analogous to (11) also hold for the masses and pion−nucleon coupling constants of charged $\pi^{\pm}$ mesons, and therefore also for the averaged pion mass (9) and pion−nucleon coupling constant (10).

Under the latter assumption based on Eqs. (11), from (8)−(10) we obtain the following relations between the parameters of the neutron−proton Yukawa potential, μ_{np} and V_0^{np}, and the analogous parameters of the proton−proton potential, μ_{pp} and V_0^{pp}:

$$\mu_{np} = \frac{\bar{m}_{\pi}}{m_{\pi^0}} \mu_{pp}, \qquad (12)$$

$$V_0^{np} = \frac{\bar{m}_{\pi}}{m_{\pi^0}} \frac{\bar{f}_{\pi}^2}{f_{\pi^0}^2} V_0^{pp}. \qquad (13)$$

Equations (10) and (13) directly imply a relation between the pseudovector pion−nucleon coupling constants for the charged and neutral π mesons,

$$f_{\pi^{\pm}}^2 = C_f^2 f_{\pi^0}^2, \qquad (14)$$

where the factor C_f^2 is expressed as

$$C_f^2 = \frac{1}{2}\left(3\frac{m_{\pi^0}}{\bar{m}_{\pi}}\frac{V_0^{np}}{V_0^{pp}} - 1\right). \qquad (15)$$

From Eqs. (4), (5), and (14) we obtain that the pseudoscalar charged and neutral pion−nucleon coupling constants are interrelated as

$$g_{\pi^{\pm}}^2 = C_g^2 g_{\pi^0}^2, \qquad (16)$$

where

$$C_g^2 = \left(\frac{M_p + M_n}{2M_p}\right)^2 C_f^2. \qquad (17)$$

Equation (15) features directly measured pion masses, so in the considered model the proportion between the charged and neutral pion−nucleon coupling constants is fully determined by that between the depths of the neutron−proton and proton−proton Yukawa potentials, V_0^{np}/V_0^{pp}. Further we demonstrate that the neutron−proton potential is appreciably deeper than the proton−proton one: $V_0^{np} > \frac{\bar{m}_{\pi}}{m_{\pi^0}} V_0^{pp}$. As a consequence, the factor in parentheses in Eq. (15) is in excess of two. Therefore, in the considered formalism, the charged pion−nucleon coupling constant proves to be larger than the neutral one:

$$f_{\pi^{\pm}}^2 > f_{\pi^0}^2, \qquad g_{\pi^{\pm}}^2 > g_{\pi^0}^2. \qquad (18)$$

That the charged pion−nucleon constant is greater than the neutral one is, within the considered scheme, a consequence of stronger np than pp interactions in the spin-singlet $^1S_0$ state, which is a reliably established phenomenon. One of its manifestations is that the absolute singlet length of np scattering is larger than the purely nuclear pp scattering length: $|a_{np}| > |a_{pp}|$.

In a number of experiments [6, 11, 20−23], measured values of pion−nucleon coupling constants obey the inequalities (18). On the other hand, the data of other experiments [9, 10, 16−19] are consistent with charge-independence of the pion−nucleon constant; i.e., the (approximate) equalities $f_{\pi^{\pm}}^2 \cong f_{\pi^0}^2$ and $g_{\pi^{\pm}}^2 \cong g_{\pi^0}^2$ hold within the experimental uncertainties. In the proposed model, charge-dependence of nuclear forces reveals itself as a violation of charge-independence of the pion−nucleon constant.

3. NUMERICAL RESULTS AND DISCUSSION OF THE VIOLATION OF CHARGE-INDEPENDENCE OF THE PION−NUCLEON COUPLING CONSTANT

The depth V_0^{NN} and radius $R_{NN} = 1/\mu_{NN}$ of the Yukawa nucleon−nucleon potential can be derived from the measured low-energy parameters of the effective range expansion. Substituting the known values of purely nuclear low-energy parameters of nucleon−nucleon scattering [4, 5, 28, 38−41]

$$a_{pp} = -17.3(4) \text{ fm}, \qquad r_{pp} = 2.85(4) \text{ fm}, \qquad (19)$$

$$a_{np} = -23.715(8) \text{ fm} \qquad (20)$$

and using the variable phase approach [42], for the Yukawa-potential parameters of pp and np interactions we obtain

$$V_0^{pp} = 44.8259 \text{ MeV}, \quad \mu_{pp} = 0.839241 \text{ fm}^{-1}, \quad (21)$$

$$V_0^{np} = 48.0706 \text{ MeV}, \quad \mu_{np} = 0.858282 \text{ fm}^{-1}. \quad (22)$$

Note that the neutron–proton interaction parameters have been obtained using Eq. (12) and substituting the neutron–proton scattering length as (20).

The Yukawa potential with parameters (22) results in an np-scattering effective radius of

$$r_{np} = 2.696 \text{ fm} \quad (23)$$

which fully agrees with the measured value [34, 40, 43, 44]

$$r_{np} = 2.70(9) \text{ fm}. \quad (24)$$

As expected, we have

$$V_0^{np} > \frac{\bar{m}_\pi}{m_{\pi^0}} V_0^{pp} = 45.8429 \text{ MeV}, \quad (25)$$

so that the aforementioned condition leading to inequalities (18) is satisfied.

Substituting the derived values (21) and (22) of the depths of pp and np potentials and the measured pion and nucleon masses [45] in (15) and (17), for the factors relating the charged and neutral pion–nucleon coupling constants we obtain

$$C_f^2 = 1.0729, \quad C_g^2 = 1.0744. \quad (26)$$

The C_f^2 and C_g^2 values are seen to be very similar, as expected from the proximity of the proton and neutron masses.

In contrast with the charged pion–nucleon coupling constant $g_{\pi^\pm}^2$, the value of the neutral constant $g_{\pi^0}^2$ has been reliably measured and is not subject to controversy. One of the latest measurements, $g_{\pi^0}^2 = 13.52(23)$ [15], fully agrees with the earlier experimental values of $g_{\pi^0}^2 = 13.55(13)$ [12] and $g_{\pi^0}^2 = 13.61(9)$ [13] and the mean value $g_{\pi^0}^2 = 13.6(3)$ quoted in [5, 14]. Substituting in (16) the latest experimental value of the pseudoscalar neutral constant

$$g_{\pi^0}^2 = 13.52(23) \quad [15] \quad (27)$$

and the C_g^2 value (26), for the pseudoscalar charged pion–nucleon coupling constant we find

$$g_{\pi^0}^2 = 14.53(25) \quad (28)$$

Using Eqs. (4), (7), (27), and (28), the pseudovector pion–nucleon coupling constants are determined as

$$f_{\pi^0}^2 = 0.07479(127), \quad (29)$$

$$f_{\pi^\pm}^2 = 0.08027(138). \quad (30)$$

The $g_{\pi^\pm}^2$ value (28) derived by us in the Yukawa-model framework practically coincides with one of the most recent measurements:

$$g_{\pi^\pm}^2 = 14.52(26) \quad [6] \quad (31)$$

The measurement (31), reported in [6] by the Uppsala group for neutron studies, is close to previous measurements of the same group, $g_{\pi^\pm}^2 = 14.62(35)$ [23] and $g_{\pi^\pm}^2 = 14.74(33)$ [11], and to the value $g_{\pi^\pm}^2 = 14.28(18)$ earlier obtained in [20−22]. On the other hand, the charged pion–nucleon coupling constant was measured by the Nijmegen group as $g_{\pi^\pm}^2 = 13.51(5)$ [9, 10], which practically coincides with the π^0 constant $g_{\pi^0}^2$. Other recent measurements [16−19] have yielded values of $g_{\pi^\pm}^2 \sim 13.7-13.8$ which are close to the neutral-pion constant $g_{\pi^0}^2$. Thus, possible charge-dependence of the pion–nucleon coupling constant, or possible difference between those for charged and neutral pions, is still an open problem which is of paramount and fundamental importance. The proposed model involves an explicit violation of charge-independence of the pion–nucleon constant; see Eqs. (27) and (28).

A measure of the violation of charge-invariance of pion–nucleon couplings is the difference between the charged and neutral pion–nucleon coupling constants:

$$\Delta f_{\text{CIB}}^2 \equiv f_{\pi^\pm}^2 - f_{\pi^0}^2, \quad \Delta g_{\text{CIB}}^2 \equiv g_{\pi^\pm}^2 - g_{\pi^0}^2. \quad (32)$$

In the considered model, Eqs. (14) and (16) imply the following explicit expressions for these quantities:

$$\Delta f_{\text{CIB}}^2 = \left(C_f^2 - 1 \right) f_{\pi^0}^2, \quad (33)$$

$$\Delta g_{\text{CIB}}^2 = \left(C_g^2 - 1 \right) g_{\pi^0}^2. \quad (34)$$

Substituting the numerical value (26) for the factor C_g^2, which relates the pseuscalar charged and neutral pion–nucleon coupling constants, as well as the reliably measured value (27) for the neutral constant $g_{\pi^0}^2$, for the absolute violation of charge-independence of pion–nucleon coupling constants in the Yukawa model we obtain

$$\Delta g_{\text{CIB}}^2 = 1.0055. \quad (35)$$

In relative units, the violation of charge-invariance in pion–nucleon coupling constants is formulated as

$$\frac{\Delta f_{\text{CIB}}^2}{f_{\pi^0}^2} = C_f^2 - 1 = 0.0729, \quad (36)$$

$$\frac{\Delta g_{\text{CIB}}^2}{g_{\pi^0}^2} = C_g^2 - 1 = 0.0744. \quad (37)$$

Thus, the relative violation of charge-independence of pion−nucleon coupling constants is as high as 7.4% in the considered scheme.

Equations (36) and (37) suggest that the relative violation of charge-invariance is larger by 0.15% for the pseudoscalar pion−nucleon coupling constant g_π^2 than for the pseudovector constant f_π^2. According to Eqs. (15) and (17), this effect arises from the neutron−proton mass difference ($M_n > M_p$). Therefore, even as soon as the pseudovector coupling constant is strictly charge-independent $(f_{\pi^\pm}^2 = f_{\pi^0}^2)$, charge-invariance should be violated for the pseudoscalar coupling constant g_π^2 [9].

It should be noted that the values (36) and (37) derived for relative violation of charge-invariance of coupling constants are actually independent of particular values of the charged and neutral pion−nucleon constants, but are rather determined by the pion and nucleon masses according to (15) and (17) and by the experimental input parameters of the model quoted in (19) and (20).

4. CONNECTION BETWEEN CHARGE SPLITTINGS OF THE PION−NUCLEON COUPLING CONSTANT AND OF THE PION MASS

Equations (14) and (26) imply that the charged and neutral pseudovector pion−nucleon coupling constants are in a ratio of

$$\frac{f_{\pi^\pm}}{f_{\pi^0}} = C_f = 1.0358, \tag{38}$$

which closely agrees with the ratio between measured masses of the charged and neutral π mesons [45]

$$\frac{m_{\pi^\pm}}{m_{\pi^0}} = 1.0340. \tag{39}$$

Therefore, to a high precision we have

$$\frac{f_{\pi^\pm}}{f_{\pi^0}} \cong \frac{m_{\pi^\pm}}{m_{\pi^0}}. \tag{40}$$

Thus, charge splitting of the pion−nucleon coupling constant practically coincides with that of the π-meson mass.

Equation (40) has a simple physical interpretation. Since the pion−nucleon coupling constant f_π is a measure of the strength of π-meson-field action on the nucleon, this action is stronger the greater the π-meson mass m_π is. Since we have $m_{\pi^\pm} > m_{\pi^0}$, the nucleon is more strongly affected by the ambient field of charged $\pi^\pm$ mesons than by that of neutral π^0 mesons. Equation (40) directly implies that, to a high

precision, the ratio f_π/m_π is a charge-invariant quantity in contrast with the coupling constant f_π.

As soon as the equality in (40) is exact, for the charged pseudovector coupling constant one obtains

$$f_{\pi^\pm}^2 = 0.07997(136). \tag{41}$$

Under the same assumption, for the pseudoscalar coupling constant, Eqs. (5) and (41) imply the value

$$g_{\pi^\pm}^2 = 14.48(25), \tag{42}$$

which practically coincides with the value (28) computed with formula (16). Then, from (27) and (42), the charge-invariance violation in the pion−nucleon coupling constant arising from the mass difference between the $\pi^\pm$ and π^0 mesons ($\Delta m_\pi = 4.59$ MeV) is estimated as

$$\Delta g_{CIB}^2 = 0.96, \tag{43}$$

which amounts to 7% in relative units.

Pseudoscalar coupling constants obey the approximate equation

$$\frac{g_{\pi^\pm}}{g_{\pi^0}} \cong \frac{m_{\pi^\pm}}{m_{\pi^0}}, \tag{44}$$

which is analogous to Eq. (40) for pseudovector coupling constants. Formula (44) first attracted attention in [5, 14], but in these analyses it was treated as an accidental coincidence which has no physical meaning. In our analysis, Eqs. (40) and (44) result from implementing the traditional classical Yukawa model and relying on measured values of low-energy pp- and np-scattering parameters quoted in (19) and (20).

Equation (40) may be rewritten as

$$f_{\pi^\pm} R_{\pi^\pm} \cong f_{\pi^0} R_{\pi^0}, \tag{45}$$

where the radius of the meson-exchange potential for the π^0-exchange

$$R_{\pi^0} \equiv \frac{\hbar}{m_{\pi^0} c} = 1.4619 \text{ fm} \tag{46}$$

is larger than the radius $R_{\pi^\pm}$ corresponding to the $\pi^\pm$-exchange:

$$R_{\pi^\pm} \equiv \frac{\hbar}{m_{\pi^\pm} c} = 1.4138 \text{ fm}. \tag{47}$$

Thus, the pion−nucleon constant f_π and the radius of the meson-exchange potential R_π prove to be charge-dependent quantities due to the mass splitting between the $\pi^\pm$ and π^0 mesons. That $\pi^\pm$ mesons are heavier than the π^0 meson effectively increases the charged-pion constant $f_{\pi^\pm}$ with respect to the neutral-pion constant f_{π^0} and effectively reduces the

$\pi^\pm$-exchange radius $R_{\pi^\pm}$ with respect to the π^0-exchange radius R_{π^0}. As a result, the product of the pion–nucleon constant f_π and the π-exchange radius R_π is a charge-invariant quantity:

$$f_\pi R_\pi = B. \tag{48}$$

Substituting the well-measured value of the neutral pion–nucleon constant

$$f_{\pi^0} = 0.2735 \tag{49}$$

and the value (46) for the π^0-exchange radius, the constant B is numerically estimated as

$$B = 0.3998 \text{ fm.} \tag{50}$$

Thus, the pion–nucleon coupling constant f_π and the π-exchange radius R_π are correlated through the relation

$$f_\pi \cong \frac{B}{R_\pi}, \tag{51}$$

which is valid to high precision.

5. CHARGE DEPENDENCE OF THE NUCLEON–NUCLEON SCATTERING LENGTH

Since the $^1S_0$ state of the two-nucleon system features a virtual level with a nearly zero energy, scattering length is the parameter which is most sensitive to small variations of the nucleon–nucleon potential. For this reason, the violation of charge-independence of nuclear forces is often quantitatively estimated using the difference between the proton–proton and neutron–proton scattering lengths,

$$\Delta a_{\text{CIB}} \equiv a_{pp} - a_{np}. \tag{52}$$

According to (19) and (20), the experimental value of this difference is

$$\Delta a_{\text{CIB}}^{\text{expt}} = 6.42(41) \text{ fm,} \tag{53}$$

which amounts to 30% in relative units. That this difference is nonzero significantly beyond the experimental uncertainty indicates that the hypothesis of charge-independence of nuclear forces is violated at low energies [28, 46–49]. The charge-dependence of nuclear forces is often attributed to the mass difference between charged and neutral π mesons [14, 28, 46, 50–53]. However, only a half of the difference $\Delta a_{\text{CIB}}^{\text{expt}}$ has been shown to be due to the mass difference between the $\pi^\pm$ and π^0 mesons [14, 46, 52, 53].

The value of the singlet np-scattering length computed by us assuming that equality (40) is exact,

$$a_{np} = -22.89(40) \text{ fm,} \tag{54}$$

differs from the pp-scattering length of $a_{pp} = -17.3(4)$ fm. The difference between the computed pp- and np-scattering lengths,

$$\Delta a_{\text{CIB}}^\pi = 5.59 \text{ fm,} \tag{55}$$

is consistent with experimental value (53).

Thus, within the discussed model framework, the violation of charge-independence of nuclear forces is fully explained by the mass difference between charged and neutral π mesons. The predicted difference between the pp- and np-scattering lengths, $\Delta a_{\text{CIB}}^\pi$, amounts to 90% of the corresponding experimental value $\Delta a_{\text{CIB}}^{\text{expt}}$. In contrast with this, the $\Delta a_{\text{CIB}}^\pi$ values derived in previous analyses reached only 50% of $\Delta a_{\text{CIB}}^{\text{expt}}$ [14, 52, 53].

6. MAJOR CONCLUSIONS AND SUMMARY

In this paper, which is based on the Yukawa meson-field model, we develop a physically consistent formalism of the nucleon–nucleon interaction in which the parameters of the np and pp systems in the spin-singlet state $^1S_0$ are related to major characteristics of the pion–nucleon interaction: the π-meson masses m_π and the pion–nucleon coupling constants f_π^2. Within this model framework, a simple relation between the charged and neutral pion–nucleon coupling constants is formulated by Eqs. (14)–(17).

The results of our analysis suggest that the charged pion–nucleon coupling constant $f_{\pi^\pm}^2$ is larger than the corresponding neutral constant $f_{\pi^0}^2$, so that charge-independence of nuclear forces is violated for the pseudovector and pseudoscalar pion–nucleon coupling constants, f_π^2 and g_π^2.

Using the derived formulae, for the pseudoscalar charged constant we obtain the value $g_{\pi^\pm}^2 = 14.53(25)$, which virtually coincides with the recent experimental value reported by the Uppsala group for neutron studies [6]: $g_{\pi^\pm}^2 = 14.52(26)$.

The absolute value of the difference between the charged and neutral pseudoscalar pion–nucleon coupling constants, $\Delta f_{\text{CIB}} \equiv f_{\pi^\pm} - f_{\pi^0}$, is derived by us as $\Delta f_{\text{CIB}} = 0.0093$. In relative units, the ratio $\Delta f_{\text{CIB}}/f_{\pi^0} = 3.58\%$ proves to be very close to $\Delta m_\pi/m_{\pi^0} = 3.40\%$. Therefore, in relative units the charge splitting of the pion–nucleon coupling constants is practically the same as that of the π-meson mass.

Our analysis demonstrates that, while both the pion–nucleon coupling constant f_π and the

π-meson-exchange radius R_π are charge-dependent, their product $f_\pi R_\pi$ is, to high precision, a charge-independent quantity. In relative units, the difference between such products for charged $\pi^\pm$ mesons and neutral π^0 mesons does not exceed 0.2%.

In our model approach, 90% of the difference between experimental values of the pp- and np-scattering lengths is accounted for by the mass difference between the $\pi^\pm$ and π^0 mesons, $\Delta m_\pi = 4.59$ MeV.

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