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Fivebranes, Dualities and Non-Geometry[1]

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Abstract—In this contribution we would like to review and clarify the relation between a large class of extended objects in string theory and a set of generalized fluxes appearing in string compactifications. The main focus of this work is on the interplay between branes and dualities, which leads to novel states which are important for string physics. The main result that we would like to describe is the coupling of these new branes to the background fields of string theory and the exact correspondence between them and generalized stringy fluxes.

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The proposal of a U-duality symmetry underlying type II string theory by Hull and Townsend [1] and the description of Dp branes as R-R-charged objects by Polchinski [2] paved the road for a better understanding of non-perturbative string physics. Branes and dualities still remain important objects of research in string theory and useful tools in the efforts to link this fundamental theory to low energy physics.

Two very interesting aspects of branes and dualities play a key part in the present contribution. The first is that their interplay reveals a large class of new extended objects in string theory, which are obtained via repeated application of duality transformations on the well-known D-branes. These new states have a set of unconventional properties and they were recently classified and studied from rather different viewpoints in refs. [3–5].

On the other hand, at the flux compactification front, it was realized several years ago that applying duality transformations to known string backgrounds can lead to situations where the global or even local geometry appears to be ill-defined. Such cases were termed non-geometric backgrounds; the essence of the term "non-geometric" lies in the fact that the standard transformations of differential geometry, namely diffeomorphisms and gauge transformations, appear to be insufficient for gluing background fields along different patches of the compactification manifold. The simplest of such cases is often depicted with the following chain of generalized fluxes:

$$H_{abc} \overset{T_a}{\leftrightarrow} f^{a}{}_{bc} \overset{T_b}{\leftrightarrow} Q^{ab}{}_{c} \overset{T_c}{\leftrightarrow} R^{abc}.$$

Reading from left to right, this chain connects four different situations which are physically equivalent. First we encounter the H-background, which represents a three-dimensional torus penetrated by NS-NS flux H. A single T-duality along an isometry direction of the torus leads to the f-background. The latter is pure geometry, in the sense that the flux is not due to the expectation value of some p-form background field but rather it is a result of a non-trivial twist in the fibration of a two-dimensional torus over a circle. Such cases have a precise mathematical description via geometric spaces called nilmanifolds. A second and a third T-duality lead to the Q- and R-backgrounds respectively. These have the property that was mentioned above, namely that the background fields are patched along different neighborhoods by T-dualities rather than just with diffeomorphisms or gauge transformations. A lot of progress in understanding such cases has been recently achieved. Our aim here is to advocate for a correspondence between these fluxes and the exotic branes mentioned before [6]. Pictorially,

$$\begin{array}{ccccccccc}
F_{abc} & \overset{S}{\leftrightarrow} & H_{abc} & \overset{T_a}{\leftrightarrow} & f^{a}{}_{bc} & \overset{T_b}{\leftrightarrow} & Q^{ab}{}_{c} & \overset{S}{\leftrightarrow} & P^{ab}{}_{c} \\
\updownarrow & & \updownarrow & & \updownarrow & & \updownarrow & & \updownarrow \\
\mathrm{D5} & \overset{S}{\leftrightarrow} & \mathrm{NS5} & \overset{T}{\leftrightarrow} & \mathrm{KKM} & \overset{T}{\leftrightarrow} & 5^2_2 & \overset{S}{\leftrightarrow} & 5^2_3
\end{array}$$

and we would like to provide evidence for this diagram. Part of it is readily explicable. The upper horizontal chain represents an enhancement of the previous one. The lower horizontal chain depicts the duality relations among the set of IIB fivebranes. The vertical arrows treats branes as sources of some flux. Indeed the D5 brane is a source of F_3 flux, F_3 being the field strength of C_2. The NS5 brane sources NS-NS flux H, while the KKM is a source of geometric flux f. In the course of this contribution we will explain the rest of the quantities that appear in this diagram.

[1] The article is published in the original.

Let us concentrate on the type IIB superstring, which in addition to being related by T-duality to the type IIA one it also possesses an S-duality as a self-duality. The type IIB superstring has a massless spectrum consisting of the common NS-NS sector, which includes the dilaton ϕ, the Kalb-Ramond 2-form B and the metric G, and its R-R sector, which includes the p-form potentials C_p with $p = 0, 2, 4, 6, 8$. These are the bosonic components of the ten-dimensional chiral IIB supergravity. Extended objects of the type IIB superstring couple to all these background fields, either electrically or magnetically. In particular, fundamental strings couple electrically to the Kalb-Ramond gauge potential. Their tension is independent of the string coupling and they are perturbative states of the theory. Dp branes couple to the R-R forms C_{p+1}; thus IIB superstring theory contains D-instantons, D-strings, D3, D5 and D7 branes. Their tension is proportional to g_s^{-1} and therefore they are non-perturbative states. NS5 branes couple magnetically to the Kalb-Ramond field. This can be equivalently stated as an electric coupling to the standard magnetic dual of B, which is a 6-form B_6 defined via the equation

$$\mathrm{d}B_6 = \mathrm{e}^{-2\phi} \star \mathrm{d}B,$$

at least when the R-R forms vanish. We emphasize that we speak about the standard magnetic dual because we shall shortly encounter a non-standard, inequivalent dual, which gives rise to a different brane. The NS5 brane has tension proportional to g_s^{-2}. Finally, the KKM couples magnetically to a KK gauge field arising from a dimensional reduction and its tension scales with g_s^{-2} too. NS5 and KKM are essentially the dual objects to the fundamental string and the graviton.

It is well-known that dualities map branes into branes. T-duality, which does not mix the NS-NS and R-R sectors or the perturbative and non-perturbative regimes, maps Dp branes to D(p ± 1) branes, depending on whether the duality is parallel (minus sign) or transverse (plus sign) to the brane world volume. Moreover, it maps NS5 to NS5 or KKM, again depending on the direction. T-duality relates IIA and IIB and therefore two T-dualities have to be performed in order to stay within the same theory. On the other hand IIB S-duality maps the D-string to the fundamental string, the D5 brane to the NS5 brane and the KKM to itself. All these relations are well-known but eventually they are just the tip of the iceberg. Let us explain this with an example inspired by ref. [3]. Consider a D5 brane in type IIB along the directions 056789. This is an object with mass

$$M_{\mathrm{D5}} = \frac{1}{g_s l_s^6}\prod_{i=5}^{9} R_i,$$

where R_i are the corresponding radii. Suppose that we perform the following chain of duality transformations on the D5 brane: ST_4T_3S. Using the duality rules

$$T_a : R_a \to \frac{l_s^2}{R_a}, \quad g_s \to \frac{l_s}{R_a} g_s \text{ and } S : g_s \to \frac{1}{g_s},$$
$$l_s \to \sqrt{g_s} l_s,$$

this yields

$$M_{\mathrm{D5}} \xrightarrow{S} \frac{1}{g_s^2 l_s^6}\prod_{i=5}^{9} R_i = M_{\mathrm{NS5}} \xrightarrow{T_4} \frac{R_4^2}{g_s^2 l_s^8}\prod_{i=5}^{9} R_i = M_{\mathrm{KKM}}$$
$$\xrightarrow{T_3} \frac{(R_3R_4)^2}{g_s^2 l_s^{10}}\prod_{i=5}^{9} R_i = M_{5_2^2} \xrightarrow{S} \frac{(R_3R_4)^2}{g_s^3 l_s^{10}}\prod_{i=5}^{9} R_i = M_{5_3^2},$$

where in the first line the standard branes appear and in the second line new exotic states. The notation that we use is taken from ref. [3]; b_α^c means b world volume directions, c special transverse directions (like the NUT of the KKM) and tension proportional to $g_s^{-\alpha}$. We observe that the fivebranes of type IIB consist of D5, NS5, KKM and two exotic states 5_2^2 and 5_3^2. In lower dimensions one encounters more and more exotic states. For example in three dimensions the total number of extended objects is 240, as the analysis of refs. [3–5] shows. The above example serves as an indication for the general fact that that exotic branes exhibit a diversity in their

—weight; in particular their tension scales as $g_s^{-\alpha}$ with $\alpha = 1, 2, 3, 4$.

—special transverse directions; these can be anything from 0 to 7.

—monodromy properties; exotic branes are generically U-folds.

The last statement can be supported by presenting the supergravity solutions associated to the 5_2^2 and 5_3^2 branes [7]. Let us concentrate on the first one due to lack of space. The non-vanishing background fields are

$$\mathrm{d}s^2 = H(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2) + HK^{-1}(\mathrm{d}x^{89})^2 + (\mathrm{d}x^{034567})^2,$$
$$e^{2\phi} = HK^{-1}, \quad B = -\theta K^{-1}\mathrm{d}x^{89}, \quad K = H^2 + \theta^2,$$

where H is the brane harmonic function. It is immediately observed that traversing the θ direction leads to global problems of the solution, since $B(\theta + 2\pi) \neq B(\theta) + \delta B$. This is reminiscent of the same situation encountered in T-dual situations of known flux vacua [8]. In particular it corresponds directly to the T-dual of the Heisenberg nilmanifold which is associated with a Q flux. The latter is often discussed in the context of T-folds [9]. T-folds are generalized manifolds where transition functions among different neighborhoods are allowed to be T-duality transformations. They find a mathematically precise incarnation in the context of generalized complex geometry [10, 11]. There one defines a generalized metric, whose general form is

$$\mathcal{H} = \begin{pmatrix} g - Bg^{-1}B & Bg^{-1} + g\beta \\ -g^{-1}B - \beta g & g^{-1} - \beta g\beta \end{pmatrix}, \text{ where } \beta = 1/2\beta^{ij}\partial_i \wedge \partial_j$$

is an antisymmetric 2-vector and we assumed that the (1,1) tensor due to the contraction $B\beta = 0$ vanishes. In the case at hand, although both the metric and B are globally ill-defined when $\beta = 0$, one can instead transform the fields into a different parametrization where they become well-defined. This is achieved when $B = 0$ and

$$ds_{89}^2 = H^{-1}((dx^8)^2 + (dx^9)^2), \quad \beta = -\theta\partial_8 \wedge \partial_9.$$

The existence of a non-vanishing 2-vector is related to the presence of Q flux and in the particular simple example it holds that $Q_\theta{}^{89} = \partial_\theta\beta^{89}$. This is a first piece of evidence that a 5_2^2 brane sources Q flux. The second piece of evidence comes from the Wess-Zumino term describing the coupling of the said brane to the background fields. Following a series of solid argument, we showed in ref. [6] that the top coupling of this brane is to an exotic magnetic dual of B that is defined through the 2-vector β as $\star e^{-2\phi} d\beta = d\beta_8^2$. This β_8^2 field is a mixed symmetry tensor of degree (2,8) and it was expected to be associated to the 5_2^2 brane via different arguments presented in ref. [4].

Similar results hold for the 5_3^2 brane. This is a U-fold associated to non-geometric R-R flux P, see ref. [12]. Its top coupling in the Wess–Zumino action is to a degree (8,2) exotic dual C_8^2 of C_2 that is defined through a 2-vector γ. The latter is to the 2-form C_2 what β is to the 2-form B. This provides an interesting example of R-R non-geometry that has been understudied in the literature so far.

According to the above, the main messages of this work can be summarized in the following points:

—There exists a plethora of non-perturbative objects in string theory due to U-duality, the exotic branes.

—Exotic branes couple to exotic duals of the standard gauge potentials.

—They are strongly related to non-geometric backgrounds and in particular they are sources of non-geometric fluxes.

Further study of the properties of exotic branes is expected to be very useful in order to gain a more complete understanding of string vacua both at a conceptual level as well as for phenomenological applications.

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