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A New Method for Calculating the Plumb Line Deflection Based on *S*- and *R*-Approximations: Testing in the Atlantics

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Abstract—In many fields of geophysics and geodesy, it is required to know the deflection of the plumb line (PLD). With the airborne gravity measurements, by applying the gravimetric method, one can calculate the PLD in both the flat and mountainous terrain conditions. In the last case, the formulas for the calculations should include the correction for the effects of the topographic masses (the terrain correction). By using the method of *S*-approximations, which is based on representing the harmonic functions by the sum of the potentials of the simple and double layers on a certain support (e.g., on a horizontal plane), we have reconstructed the gravity field in each spatial point (at each measurement height), in particular, on the surface of the reference ellipsoid. We have developed the programs for computing the PLD by the Vening Meinesz formulas, which yield the zero approximation of PLD, and suggested the method for reconstructing the anomalous fields based on the *S*-approximations. The interpretation of the gravity data was also carried out by the method of *R*-approximations, which relies on the Radon transform. We present the results of the practical calculations for two regions of the Atlantic Ocean.

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INTRODUCTION

The deflection of the plumb line (the deviation of the direction of actual gravity from the vector of normal gravity) arises because the actual gravity field of the Earth, which reflects all the complexity of the Earth's interior and figure, differs from the normal field. In order to calculate the deflection of the plumb line (PLD) in the different points of the Earth, it is required to know the spatial distribution of the anomalous gravitational field, i.e., to have the gravimetry data.

In many regions of the world, gravity surveying directly on the Earth's surface is barely practical or even impossible. The detailed marine geophysical surveys are traditionally carried out from the ships; however, these projects are quite expensive. The airborne geophysical surveys are less expensive and suitable for assessing the mineral potential of the particular hardly accessible regions. At present, the inertial corrections are determined as accurately as within 1 mGal. For calculating the PLD, it is required to have the maps of gravity (mainly, of a scale of 1 : 200000) on the surface of the reference ellipsoid rather than at the altitude of the measurements (flight).

The ultimate objective of any airborne geophysical survey is to digitally describe the studied field on the Earth's surface (on the selected ellipsoid). For obtaining this digital description, certain approximations of the physical fields of the Earth are applied. Using the gravimetry data, one can then determine the quasigeoid height and calculate the PLD components.

Boyarsky et al. (2010) suggested an efficient algorithm for determining the PLD from the data on the anomalies of gravity. A remarkable feature of this approach is a simple procedure of allowing for the far zone effects in the calculations of PLD.

The purpose of the present paper is to refine and improve the technique for calculating the PLD, which was suggested in (Koneshov et al., 2007) by incorporating the results of the airborne gravity measurements over the hardly accessible regions. Besides the way of calculating the PLD with the use of *S*-approximations described in (Koneshov et al., 2007), in the present paper we also present the results of testing the method of *R*-approximations in two practical examples (two regions in the Atlantics).

The need for designing the refined (universal) method is dictated by two factors. First, the map of gravity anomalies based on the results of the airborne gravity survey is initially determined for the flight altitude, because this map is constructed in the coordinates of the GPS observations relative to the ellipsoid. Second, when calculating the PLD in the barely accessible regions with the rugged topography, one should allow for the terrain effects using the algo-

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rithms that are most adequate for the criteria of universality of the calculations.

THE APPROXIMATION OF THE ANOMALOUS POTENTIAL FIELDS

In a number of the methods that are currently used in the interpretation of the gravity and magnetic measurements, certain approximations of the anomalous geophysical fields are suggested.

The approximation of the Earth's gravity field by the segments of the solid spherical harmonic series has become very popular since the middle and end of the 20th century. Extensive research in this field was carried out by scientists in Russia and abroad (Naidu, 1968; Balmino et al., 1990). Since the 1950s, the interpretation of gravity and magnetic anomalies has come to widely use the methods of spectral analysis (Serkerov, 1991; etc.). The present-day theories of interpretation of the geophysical data have a common drawback: they use some idealizations, which do not comply with the regularities observed in the nature or with the experimental investigation of the studied fields. As the examples, we cite the idealizations of the two-dimensional field and a plane boundary between the solid Earth and the air, specifying a certain element of the field in the nodes of a regular grid.

The suggested methods of *S*- and *R*-approximations comply with the real geophysical practice and are free of the idealizations listed above. The methods of *S*- and *R*-approximations are the modifications of the method of linear integral representations suggested by V.N. Strakhov (1999).

Based on the method of the linear integral representations (Strakhov and Stepanova, 2002a; 2002b), one can construct the linear analytical approximations of the anomalous potential fields and the functions describing the topography of the terrain, calculate the linear transformations of the fields, and solve other problems of the interpretation type.

It is worth noting that most of the transformation procedures (analytical extension of the field towards the disturbing masses, recalculation of the field into the higher-order derivatives of the potential) that have the highest resolution are unstable.

To date, a wide range of the methods for transformation of the potential fields has been suggested. These methods are extensively published in the geophysical literature. Most of the existing methods have a common drawback: they are inadequate for the real geophysical practice because of the series of the embedded idealizations such as the disregard of the irregular pattern and different heights of the gravimetric networks and many others.

We applied computer-aided technologies for constructing the analytical approximations of the anomalous gravity and magnetic fields, as well as the terrain topography in the local version in the rectangular Cartesian coordinates. These technologies are based on the representation of the anomalous field in terms of the potentials of the simple and double layers distributed on certain surfaces. We considered the problems with relatively few points (at most 10000). Therefore, the arising systems of the linear equations (SLAEs) were solved by three methods: by the methods of M.M. Lavrent'ev, by the regularization of the Cholesky decomposition, and by the iterative method developed by Strakhov. In the examples with a larger number of points (~15000–30000), the SLAEs were solved by two modifications of the block-coordinate descent method (Strakhov, V. and Strakhov, A., 1999a; 1999b; Strakhov and Stepanova, 2001).

S-APPROXIMATION: LOCAL VERSION

Knowing the components of the magnetic or gravity field (e.g., the first derivative of the potential with respect to z on a certain surface topography above the physical surface of the Earth), we can represent the potential of the field as a sum of a simple and double layers, which are formed by a horizontal plane located below the given relief (Koshlyakov et al., 1962):

$$V(M) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\rho_1(\xi_1,\xi_2)d\xi_1 d\xi_2}{\sqrt{(x-\xi_1)^2+(y-\xi_2)^2+z^2}} + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\rho_2(\xi_1,\xi_2)z d\xi_1 d\xi_2}{[\sqrt{(x-\xi_1)^2+(y-\xi_2)^2+z^2}]^3}, \quad (1)$$

$$M = (x,y,z),\quad \hat{x} = (x,y),\quad \xi = (\xi_1,\xi_2,\xi_3),\quad \hat{\xi} = (\xi_1,\xi_2).$$

We select the coordinate system in such a way that the plane of the simple and double layers is described by the equation $z = 0$. Then, the derivative with respect to z of the potential V taken with the opposite sign has the following form:

$$-\frac{\partial V}{\partial z}(M) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\rho_1(\hat{\xi})z d\hat{\xi}}{[\sqrt{(x-\xi)^2+(y-\eta)^2+z^2}]^3} + \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{\rho_2(\hat{\xi})(2z^2-(x-\xi_1)^2-(y-\xi_2)^2)^2 d\hat{\xi}}{[\sqrt{(x-\xi_1)^2+(y-\xi_2)^2+z^2}]^{5/2}}, \quad (2)$$

$$M = (x,y,z).$$

The functions ρ_1 and ρ_2 are not known. Let the components of the field be specified in a finite set of the points M_i, $i = 1, 2, \ldots, N$; $M_i = (x_i, y_i, z_i)$. We denote the integrand function in the first term of (2) at point M_i by $Q_1^{(i)}$, and in the second term, by $Q_2^{(i)}$. Then, we obtain

$$-\frac{\partial V(M_i)}{\partial z} \equiv f_i = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\rho_1(\hat{\xi})Q_1^{(i)}(\hat{\xi}) + \rho_2(\hat{\xi})Q_2^{(i)}(\hat{\xi}))d\hat{\xi}, \quad i = 1, 2, \ldots, N. \tag{3}$$

In practice, the components of the field are typically known with a certain error, therefore the input data in our problem are composed of the values of $f_{i,\delta}$. Using the solution of the variational problem

$$\Omega(\rho) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\rho_1^2(\hat{\xi}) + \rho_2^2(\hat{\xi}))d\hat{\xi} = \min_{\rho}, \tag{4}$$

$$f_{i,\delta} - \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(\rho_1(\hat{\xi})Q_1^{(i)}(\hat{\xi}) + \rho_2(\hat{\xi})Q_2^{(i)}(\hat{\xi}))d\hat{\xi} = 0, \quad i = 1, 2, \ldots, N, \tag{5}$$

we find that the sought functions should have the form

$$\rho_1^{(a)}(\hat{\xi}) = \tilde{\rho}_1(\hat{\xi}, \lambda), \quad \rho_2^{(a)}(\hat{\xi}) = \tilde{\rho}_2(\hat{\xi}, \lambda),$$
$$\tilde{\rho}_1(\hat{\xi}, \lambda) = \sum_{i=1}^{N}\lambda_i Q_1^{(i)}(\hat{\xi}), \quad \tilde{\rho}_2(\hat{\xi}, \lambda) = \sum_{i=1}^{N}\lambda_i Q_2^{(i)}(\hat{\xi}). \tag{6}$$

Thus, we come to the following system of the linear equations, whose matrix in our case has the elements

$$a_{ij} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(Q_1^{(i)}(\hat{\xi})Q_1^{(j)}(\hat{\xi}) + Q_2^{(i)}(\hat{\xi})Q_2^{(j)}(\hat{\xi}))d\hat{\xi}, \quad 1 \le i \le N, \quad 1 \le j \le N. \tag{7}$$

In our case, coefficients a_{ij} can be calculated in the explicit form with the use of the Poisson integral:

$$a_{ij} = 2\pi\left\{\frac{z_i + z_j}{\left(\sqrt{(z_i + z_j)^2 + (x_i - x_j)^2 + (y_i - y_j)^2}\right)^3} - \frac{(z_i + z_j)(9[(x_i - x_j)^2 + (y_i - y_j)^2] - 6(z_i + z_j)^2)}{\left(\sqrt{(z_i + z_j)^2 + (x_i - x_j)^2 + (y_i - y_j)^2}\right)^7}\right\}, \quad 1 \le i \le N, \quad 1 \le j \le N. \tag{8}$$

Based on the values of λ_i, $i = 1, 2, \ldots, N$, which are found by solving the system (6)–(8), we can determine the values of the functionals p_s, $s = 1, 2, \ldots, S$ (see (Strakhov and Stepanova, 2002a)). In our study, we sought for the spatial distribution of the gravity field.

Below we describe the main principles of constructing the R-approximations according to (Helgason, 1983; Gel'fand et al., 2000).

The application of the Radon transform to both sides of the expression gives

$$\hat{V}_{x_3}(\omega, p) = \int_{-\infty}^{+\infty}[\hat{\rho}_1(\omega, q)\hat{Q}_1^{(i)}(\omega, p - q) + \hat{\rho}_2(\omega, q)\hat{Q}_2^{(i)}(\omega, p - q)]dq, \tag{9}$$

since the Radon transform of the convolution of the functions is equal to the convolution of the product of the Radon transforms of the corresponding functions. We can find the quantities (or, to be more exact, the functions) $\hat{Q}_1^{(i)}, \hat{Q}_2^{(i)}$:

$$\begin{gathered}\int_{-\infty}^{\infty}\frac{x_3}{\left[\sqrt{(x_1 + t\sin\varphi - p\cos\varphi)^2 + (x_2 - t\cos\varphi - p\sin\varphi)^2 + x_3^2}\right]^3}dt \\ = \frac{2x_3}{x_3^2 + p^2 - 2px_1\cos\varphi - 2px_2\sin\varphi + (x_1\cos\varphi + x_2\sin\varphi)^2}, \\ \omega = (\cos\varphi, \sin\varphi). \\ \int_{-\infty}^{\infty}\frac{(2x_3^2 - ((x_1 + t\sin\varphi - p\cos\varphi)^2 + (x_2 - t\cos\varphi - p\sin\varphi)^2)}{\left[\sqrt{(x_1 + t\sin\varphi - p\cos\varphi)^2 + (x_2 - t\cos\varphi - p\sin\varphi)^2 + x_3^2}\right]^5}dt \\ = \frac{\partial}{\partial x_3}\left(\frac{2x_3}{x_3^2 + p^2 - 2px_1\cos\varphi - 2px_2\sin\varphi + (x_1\cos\varphi + x_2\sin\varphi)^2}\right), \\ \omega = (\cos\varphi, \sin\varphi).\end{gathered} \tag{10}$$

Here, $\omega_1\xi + \omega_2\eta = p$ is the line along which the integration is conducted. If we write out the expression for $-\frac{\partial V}{\partial x_3}(M_i)$ using the formula of inversion of the Radon transform, we obtain

$$-\frac{\partial V}{\partial x_3}(M_i) = -\frac{1}{\pi}\int_0^{\infty} dp \int_0^{2\pi} d\varphi \frac{1}{p}\left[\int_{-\infty}^{+\infty}[\hat{\rho}_1(\omega,q)(\hat{Q}_1^{(i)})'_p(\omega,p-q) + \hat{\rho}_2(\omega,q)(\hat{Q}_2^{(i)})'_p(\omega,p-q)]dq\right]$$

$$= -\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_{-\infty}^{\infty}\left\{\left[\int_{-\infty}^{\infty} dp \frac{(\hat{Q}_1^{(i)})'_p(\omega,p-q)}{p}\right]\rho_1(\omega,q) + \left[\int_{-\infty}^{\infty} dp \frac{(\hat{Q}_2^{(i)})'_p(\omega,p-q)}{p}\right]\rho_2(\omega,q)\right\}dq.$$

Here, the integral with respect to the variable p is understood in the sense of its principal value. It can be calculated in the explicit form:

$$I_1(\omega,q) = \int_{-\infty}^{\infty}\frac{(Q_1)'_p(\omega,p-q)dp}{p} = 2\frac{x_3^2-(q-r\cos\varphi)^2}{\left(x_3^2+(q-r\cos\varphi)^2\right)^2}.$$

$$I_2(\omega,q) = \int_{-\infty}^{\infty}\frac{(Q_2)'_p(\omega,p-q)dp}{p} = 8\frac{x_3(q-r\cos\varphi)^2}{\left(x_3^2+(q-r\cos\varphi)^2\right)^2}.$$

In practice, the components of the field are typically specified with a certain error, therefore the input information is composed of the values of $f_{i,\delta}$. By solving the variational problem (in the general form, the variational statement of the problem is described in (Strakhov and Stepanova, 2002a; 2002b))

$$\Omega(\rho) = \int_0^{+\infty}\int_{-\infty}^{+\infty} dq \int_0^{2\pi}(\hat{\rho}_1^2(\omega,q)+\hat{\rho}_2^2(\omega,q))dpd\varphi = \min_{\rho}, \quad (11)$$

$$f_{i,\delta} = -\frac{1}{2\pi}\int_0^{2\pi} d\varphi \int_{-\infty}^{\infty}\left\{\left[\int_{-\infty}^{\infty} dp \frac{(\hat{Q}_1^{(i)})'_p(\omega,p-q)}{p}\right]\rho_1(\omega,q) + \left[\int_{-\infty}^{\infty} dp \frac{(\hat{Q}_2^{(i)})'_p(\omega,p-q)}{p}\right]\rho_2(\omega,q)dq,\right. \quad (12)$$

we find that the sought functions should have the form

$$\hat{\rho}_1^{(a)}(\omega,q) = \tilde{\hat{\rho}}_1(\omega,q,\lambda), \quad \hat{\rho}_2^{(a)}(\omega,q,\lambda) = \tilde{\hat{\rho}}_2(\omega,q,\lambda),$$
$$\tilde{\hat{\rho}}_1(\omega,q,\lambda) = -\frac{1}{2\pi}\sum_{i=1}^{N}\lambda_i\int_{-\infty}^{\infty}\frac{(\hat{Q}_1^{(i)})'_p(\omega,p-q)}{p}dp, \quad (13)$$
$$\tilde{\hat{\rho}}_2(\omega,q,\lambda) = -\frac{1}{2\pi}\sum_{i=1}^{N}\lambda_i\int_{-\infty}^{\infty}\frac{(\hat{Q}_2^{(i)})'_p(\omega,p-q)}{p}dp.$$

Thus, we obtain the following system of linear equations

$$A\lambda = f_\delta, \quad \lambda = (\lambda_1,\ldots,\lambda_N), \quad f_\delta = (f_{1,\delta},\ldots,f_{N,\delta}), \quad (14)$$

whose matrix in our case has the following elements:

$$a_{ij} = \frac{1}{4\pi^2}\int_0^{2\pi}\int_{-\infty}^{\infty}\{I_{1,i}(\omega,q)I_{1,j}(\omega,q) + I_{2,i}(\omega,q)I_{2,j}(\omega,q)\}d\varphi dq, \quad (15)$$
$$1 \le i \le N, \quad 1 \le j \le N.$$

Here, coefficients a_{ij} cannot be calculated explicitly; however, the integral with respect to φ can be calculated exactly. Indeed, the denominator of the functions to be integrated with respect to the angle φ is a product of certain powers of the cubic trinomials of $\cos(\varphi-\varphi_i)$ and $\cos(\varphi-\varphi_j)$. Denoting the variable of integration by $\varphi' = \varphi - \varphi_i$ and passing to the integration of the function of complex variable z along the unit circle, in the denominator we obtain the product of the powers of the quadratic trinomials in the following form:

$$\left\{a_i\left(z+\frac{1}{z}\right)^2 + b_i\left(z+\frac{1}{z}\right) + c_i\right\}^{n_i}$$
$$\times\left\{a_j\left(\varsigma z+\frac{1}{\varsigma z}\right)^2 + b_j\left(\varsigma z+\frac{1}{\varsigma z}\right) + c_j\right\}^{n_j},$$

where $\zeta = e^{i(\varphi_i-\varphi_j)}$.

Using the factors λ_i, $i = 1, 2, \ldots, N$, which are determined from the solution of system (14) and (15), one can then find the values of the functionals p_s, $s = 1, 2, \ldots, S$ of the elements of gravity.

COMPUTER-AIDED TECHNOLOGIES FOR FINDING THE LINEAR ANALYTICAL APPROXIMATIONS OF THE HARMONIC FUNCTIONS (ELEMENTS OF THE POTENTIAL FIELDS) IN THE LOCAL VERSION

Above we considered the algorithms for constructing the approximations of the anomalous gravity and magnetic fields, which are based on representing the field by the sum of the simple and double layers (S-approximation) or which use the Radon transform. The computer technologies suggested for implementing these approaches comprise three steps.

The first step is the formation of the elements of matrix A. Preliminarily, a certain number (N_{contr}) of the observation points are excluded from the set of the initial points by the programs of sorting and selection.

The matrix elements were calculated by formulas (8). We developed the programs MAVPS2N and MATVEC for finding the elements of the matrix. The linear transformations were constructed with the use of the MATPS2CON program (Strakhov and Stepanova, 2001).

The second step is the solution of SLAEs. The solution of SLAEs is the key computational problem in constructing the approximations based on the linear integral representations. We used the program of the

P-SPPM package (Strakhov, V. and Strakhov, A., 1999a; 1999b) for SLAEs with the symmetric positive-definite matrix and the right-hand side specified approximately, as well as the ORTOG2002-2 program based on the method described by Strakhov and Stepanova (Strakhov et al., 1999; Strakhov and Stepanova, 2001).

The third step is the reconstruction of the field and calculation of its transforms. We have developed the MATPS2CON and MAVPS2N programs for finding the distribution of the elements of gravity, higher order derivatives of the potential, and the analytical extension of the fields.

THE GRAVIMETRICAL METHOD FOR CALCULATING THE PLD

The angle between the direction of the normal gravity γ and the direction of the plumb line at point $M(\bar{g})$ is referred to as the plumb line deflection. The PLD is typically described in terms of its projections on the meridional plane and on the plane of the first vertical (ξ and η, respectively),

$$\xi = -\frac{g_x}{g_z}, \quad \eta = -\frac{g_y}{g_z}. \tag{16}$$

Here g_x, g_y, g_z are the components of the acceleration of gravity in the local coordinate system, in which the Ox axis is directed to the north, Oy axis points to the east, and the Oz axis is perpendicular to the Earth's surface at the point of the observations. According to the Vening Meinesz formula and the formula suggested by L.P. Pellinen (1969), the PLD can be calculated in the following way:

$$\begin{aligned} \xi &= -\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\Delta g_F Q(\psi)\cos A dA d\psi + \Delta\xi_p, \\ \eta &= -\frac{1}{2\pi}\int_0^{2\pi}\int_0^{\pi}\Delta g_F Q(\psi)\sin A dA d\zeta + \Delta\eta_p. \end{aligned} \tag{17}$$

Here, Δg_F is Faye's (free-air) gravity anomaly at a given point and $Q(\psi) = \sin\psi\frac{dS}{d\psi}$, $S(\psi)$ is Stokes' function,

$$\begin{aligned} \Delta\xi_p &= \frac{f\delta R^2}{\gamma}\iint_\omega \frac{h}{r_0}\left(\frac{1}{r_0} - \frac{1}{r}\right)\cos A d\omega, \\ \Delta\eta_p &= \frac{f\delta R^2}{\gamma}\iint_\omega \frac{h}{r_0}\left(\frac{1}{r_0} - \frac{1}{r}\right)\sin A d\omega, \end{aligned} \tag{18}$$

where f is the gravitational constant, δ is the average density of the Earth's rocks, R is the average radius of the Earth, γ is the normal gravity, and h is the difference between the geodetic heights of the observation point and the current point. In the integration by formulas (16), the space around the point of study (at which the PLD is calculated) is subdivided into five areas, just as it is in the template suggested by V.F. Eremeev. The central area is a circle with a radius of 5 km; it is encircled by four ring-shaped areas limited by radii of about 5, 100, 300, 1000, and 2000 km. Since the Vening Meinesz function, $Q(\psi)$, rapidly decreases with increasing distance, the gravity anomalies located in the distant zones exert weaker impacts on the PLD at a given point compared to the impacts from the anomalies located in the nearby zones. Therefore, the effects of the anomalies located in the different zones are taken into account with a different accuracy.

Each ring area is subdivided into a few zones in such a way that their influence on the PLD at the studied point is approximately equal. The coefficients characterizing the impacts of the ring zones on the PLD decrease with the distance from the center of gravity. Each area is partitioned by the radial lines into the segments (spherical trapezoids), which have equal areas and exert different impacts on ξ and η (the impact is proportional to the cosine or sine of the azimuth A). In the integration over each of these spherical trapezoids, we use the Gaussian formula with the highest possible order of accuracy (Krylov and Shul'gina, 1966) with 12 nodes for each measurement. This formula provides an order of accuracy of $2n - 1$, where n is the number of the nodes. Therefore, for the successful calculation of PLD, it is required to obtain the approximation of the anomalous field that is as accurate as possible. The values of the field near the studied point are reconstructed in the nodes of the grid (which is in the general case irregular) by the method of S-approximations with a relative accuracy of up to 10^{-5}, i.e., $\sigma = \sqrt{\frac{\|Ax - f_\delta\|_E^2}{N}} \approx 0.005$ mGal. For this purpose, the gravity field at the points of a certain part of the survey area is represented by the S-approximations (in the examples described below, this area is confined between 36° and 39° E and between 41° and 45° N). We select about 5000 points, which provides quite an accurate approximation and, at the same time, requires a reasonably short time for computing the approximating constructions. After this, the field can be reconstructed at any desired point. In the distant zones from the point of PLD calculation (starting from point 6, with a total of 26 integration zones providing equal influence), we applied the algorithm of finding the value of gravity field that is closest to the value of the field in the current point of the integration. The initial satellite data are suitable for determining the values of Δg on a grid with a step of 1′. The corresponding computerized technologies for this were designed by our team. Taking into account the anomalies of radius $\psi = 20°$ in the area Σ, we obtain a much simpler formula for the Vening Meinesz function. By expanding $Q(\psi)$ into a series, we obtain

$$Q_1 = \frac{1}{2\gamma\sin 1''}\left(\frac{2}{\psi} + \frac{49}{12}\psi + 3\right), \tag{19}$$

$$\xi''_{0-2000} = -\frac{1}{2\pi\gamma\sin 1''} \times \int_0^{r_0=5\,\text{km}} \int_0^{2\pi} \Delta g_F \frac{1}{r}\cos A dA dr - \frac{1}{2\pi R} \times \int_{r_0=5\,\text{km}}^{r=2000\,\text{km}} \int_0^{2\pi} \Delta g_F Q_1 \cos A dA dr. \quad (20)$$

$$\eta''_{0-2000} = -\frac{1}{2\pi\gamma\sin 1''} \times \int_0^{r_0=5\,\text{km}} \int_0^{2\pi} \Delta g_F \frac{1}{r}\sin A dA dr - \frac{1}{2\pi R} \times \int_{r_0=5\,\text{km}}^{r=2000\,\text{km}} \int_0^{2\pi} \Delta g_F Q_1 \sin A dA dr. \quad (21)$$

By replacing the angular distance $\psi = r/R$ by the linear distance r, we find that

$$Q_1 = \left(\frac{A}{r} + B + Cr\right), \quad (22)$$

$$A = 1339.6'', \quad B = 0.315'', \quad C = 0.000066''.$$

The surface confined between the circles with radii $r_0 = 5$ km and $r = 2000$ km is subdivided into four concentric ring areas: (1) from 5 to 100 km; (2) from 100 to 300 km; (3) from 300 to 1000 km; and (4) from 1000 to 2000 km. In turn, each ring area is partitioned into concentric zones exerting an equal impact on the PLD. The radii of these zones are determined by the condition $\int_{r_0}^{r_1} Q_1 dr = \int_{r_1}^{r_2} Q_1 dr = \ldots = \int_{r_{n-1}}^{r_n} Q_1 dr = P$, where P is a constant. We selected the radii of these zones in accordance with (Shimbirev, 1975). The first area of integration was divided into 16 equal sectors, the second and third areas, into 24 sectors, and the fourth, into 48 sectors.

For calculating Faye's gravity anomalies at the given points, in formula (8), the coordinates of these points should be substituted instead of x_j, y_j, z_j. Then, the matrices with elements (8) are multiplied by vector λ, which is found from the solution of the following SLAEs:

$$A\lambda = f_\delta,$$

where f_δ is the column vector of the values of gravity in the initial sample (about 3600 points in the segment confined between 29° and 28° W and between 42° and 44° N with a step of 2 minutes and 13 500 points in the segment confined between 48° and 43° W and between 22° and 25° N with a step of 2 minutes).

PLD CALCULATIONS IN THE REAL SURVEY AREA

The real areas for this study were selected in two segments of the territory covered by the detailed gravity survey. These areas are boxes with a size of 16 × 25 degrees along the latitude and longitude (and the corresponding areas on the Krasovsky ellipsoid) with a step of two minutes along each coordinate. The survey was conducted on the sea surface and in the adjacent areas on the shore. For testing the efficiency of the method, we selected the area in each segment, where it is required to calculate the value of the PLD. The sizes of the first and the second areas (central region of the sea) are 2 × 1 and 3 × 5 degrees latitude and longitude, respectively. Using the method of S-approximations, we found the densities of the simple and double layers distributed on the planes, which generate the gravity field in a given area. The layers were distributed on the plane located at the depth l = 15 km below the surface. In this example, we can use the spherical version of the S-approximation because the diameter of the area is greater than 300 km. However, the plane version is applicable. In this case, the support of the simple and double layers is located below the spherical surface at a distance of about 15 km from the top of the spherical segment. In both the real and model examples, the calculations show that the accuracy of the approximation in this case is as high as in the regional version of S-approximations, while the computations are faster. The obtained approximations of the gravity field were then used for finding the values of gravity at the arbitrary points, which is required for the calculations by the Vening Meinesz formulas. Besides, we also carried out the calculations based on the Radon transform by formulas (13)–(15).

Using formulas (7) and (8), we found the solution of the SLAEs. The gravity field was determined with relative accuracy $\Delta = \frac{\|Ax - f_\delta\|_E}{\|f_\delta\|_E} = 2 \times 10^{-4}$. Here, Ax is the gravity field yielded by the approximation; f_δ is the initial gravity field measured at the points of the corresponding sample; $\| \, \|_E$ denotes the Euclidean norm of the vector. Figures 1 and 2 show the maps of the initial height anomalies for the two segments considered.

The accuracy of the field reconstructed on the surface depends on the number and mutual positions of the observation points, as well as on the other factors, among which the ratio $\varepsilon = \Delta_{xy}/r$ is most important. Here, Δ_{xy} is the step along the x and y coordinates; and r is the distance to the nearest topographical features (to the plane on which the simple and double layers are distributed). It is reasonable to specify the value of ε in the range of 0.3 to 0.5. After finding the analytical approximation of the gravity field in the neighborhood of the studied point (where the PLD was calculated), we calculated the PLD components by the Vening Meinesz formulas. The graphs of PLD based on the initial data and calculated by the suggested method for the described areas are shown in Figs. 3–6. The accuracy of reconstructing the PLD is 1.24 arcsec for the

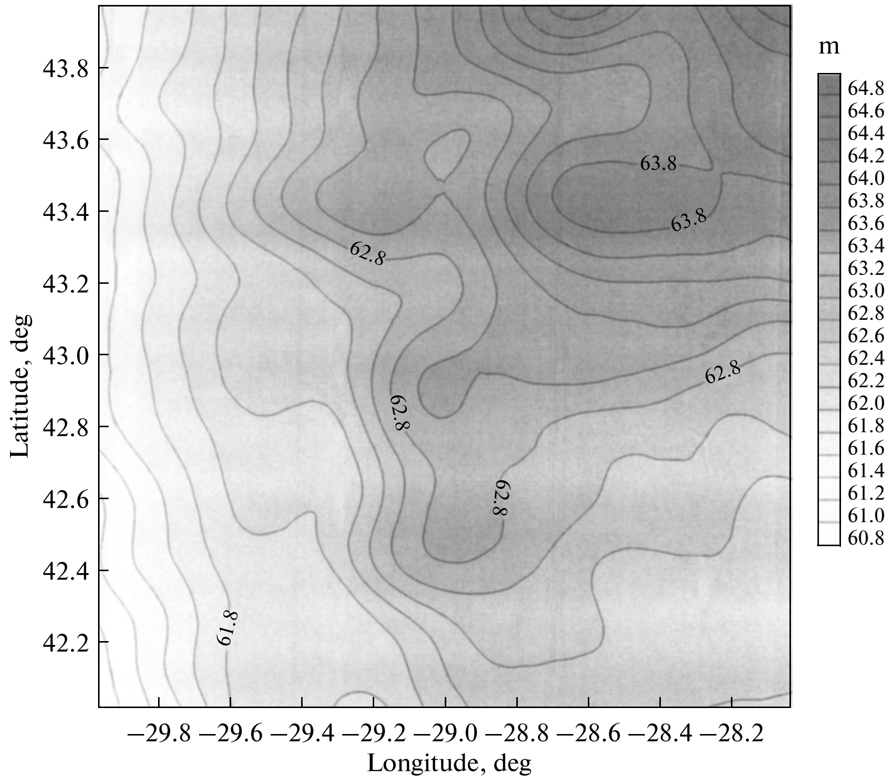

Fig. 1. The contour map of the height anomalies for area 1.

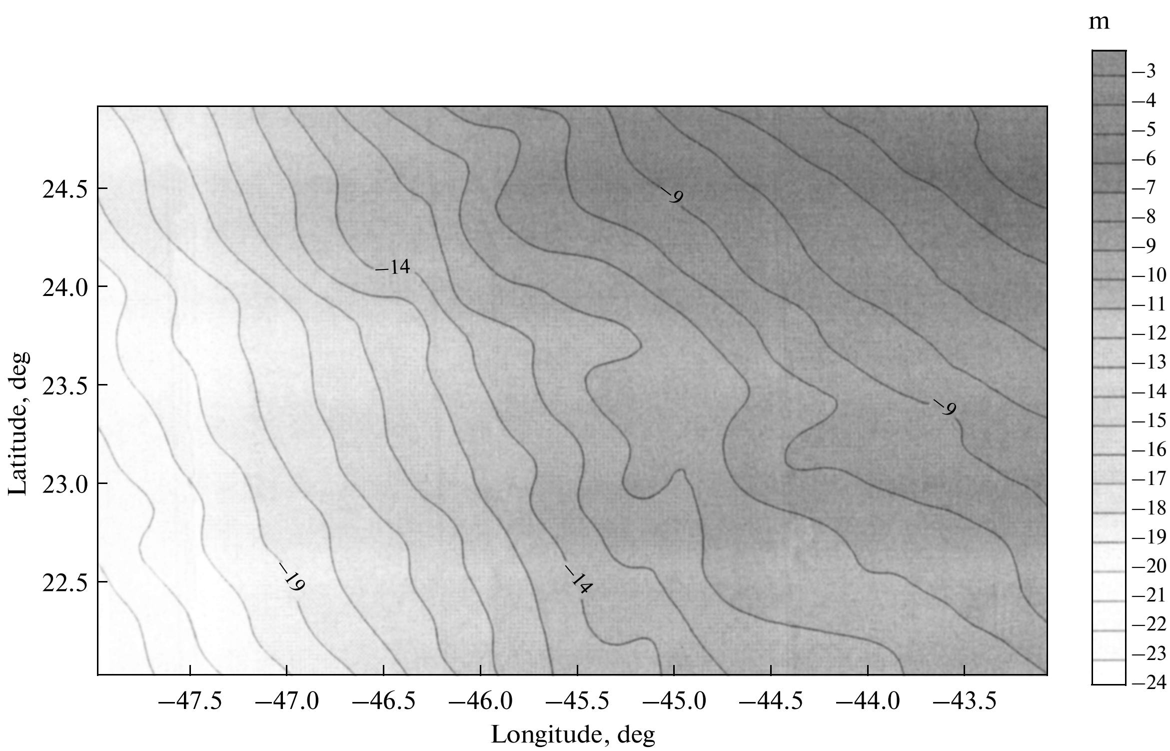

Fig. 2. The contour map of the height anomalies for area 2.

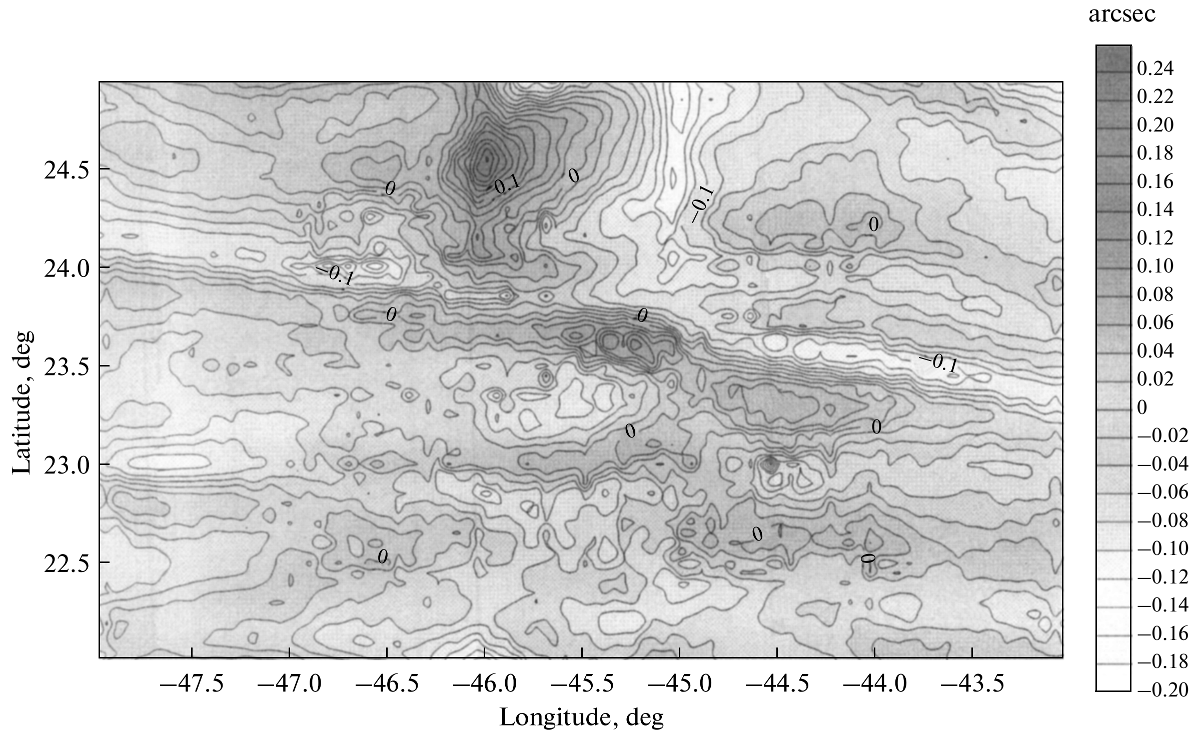

Fig. 3. The contour map of the difference between the PLD values calculated by the method of *S*-approximations and the given PLD values. Area 2, northward deflection.

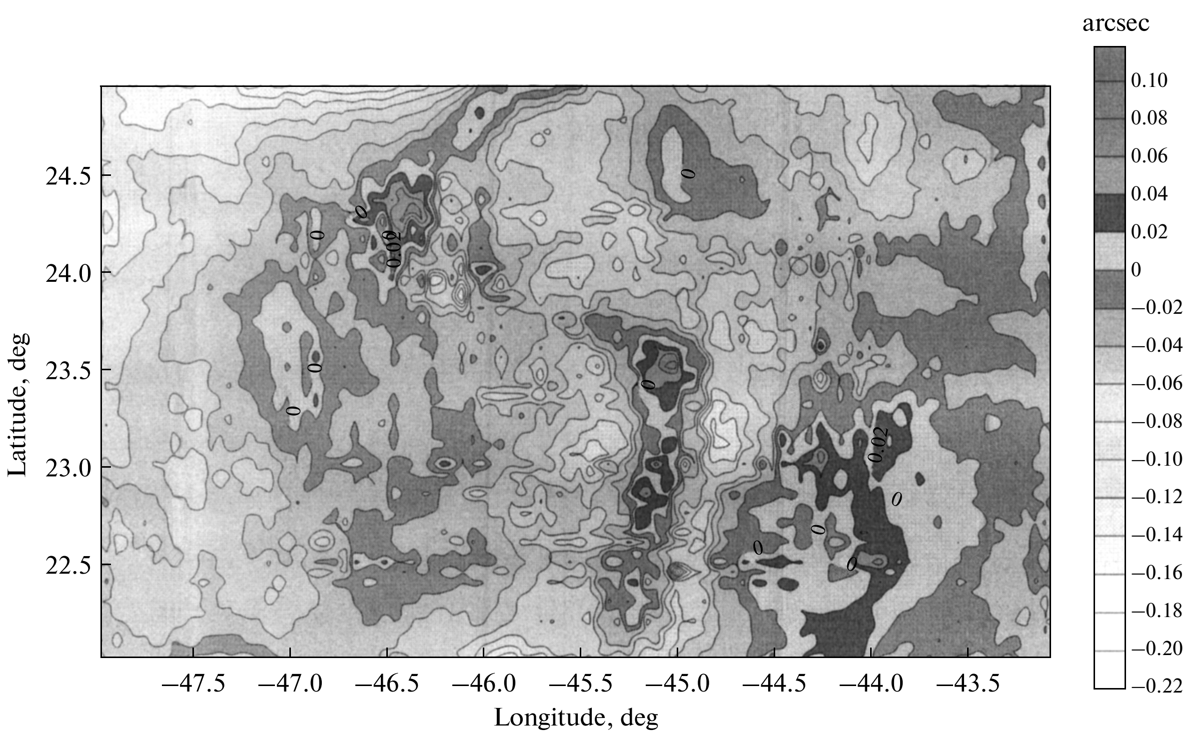

Fig. 4. The contour map of the difference between the PLD values calculated by the method of *S*-approximations and the given PLD values. Area 2, eastward deflection.

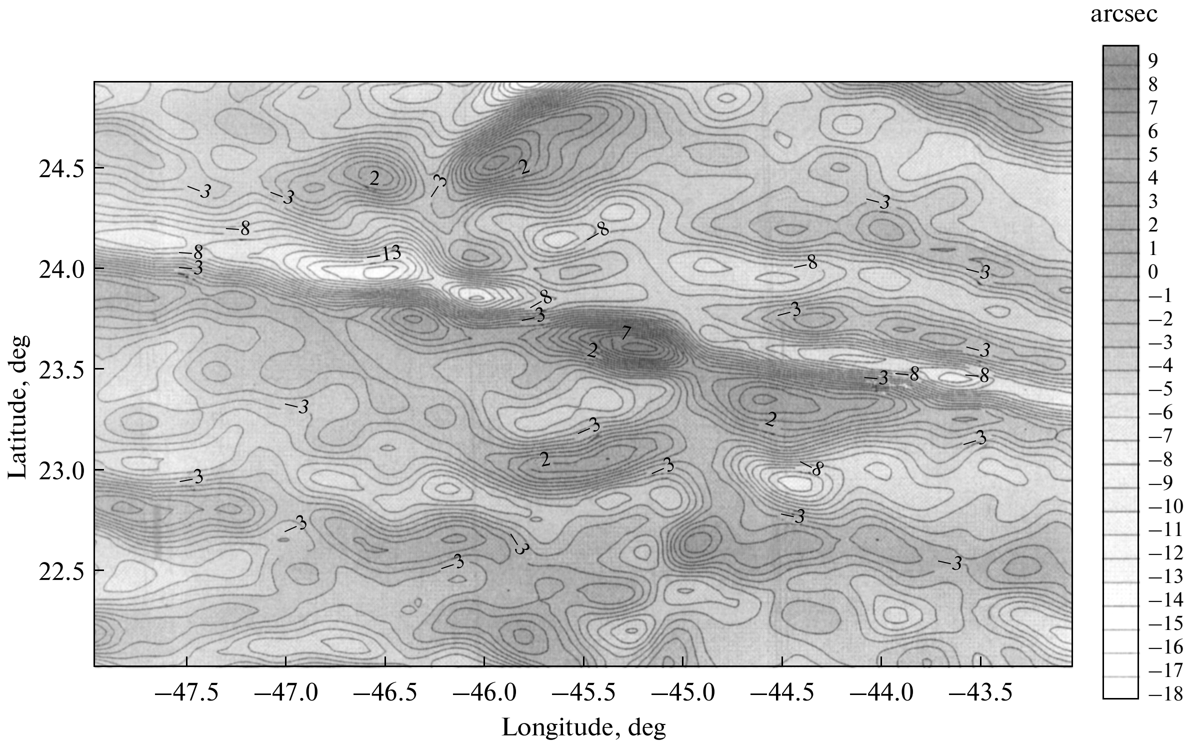

Fig. 5. The contour map of the northward PLD. Area 2.

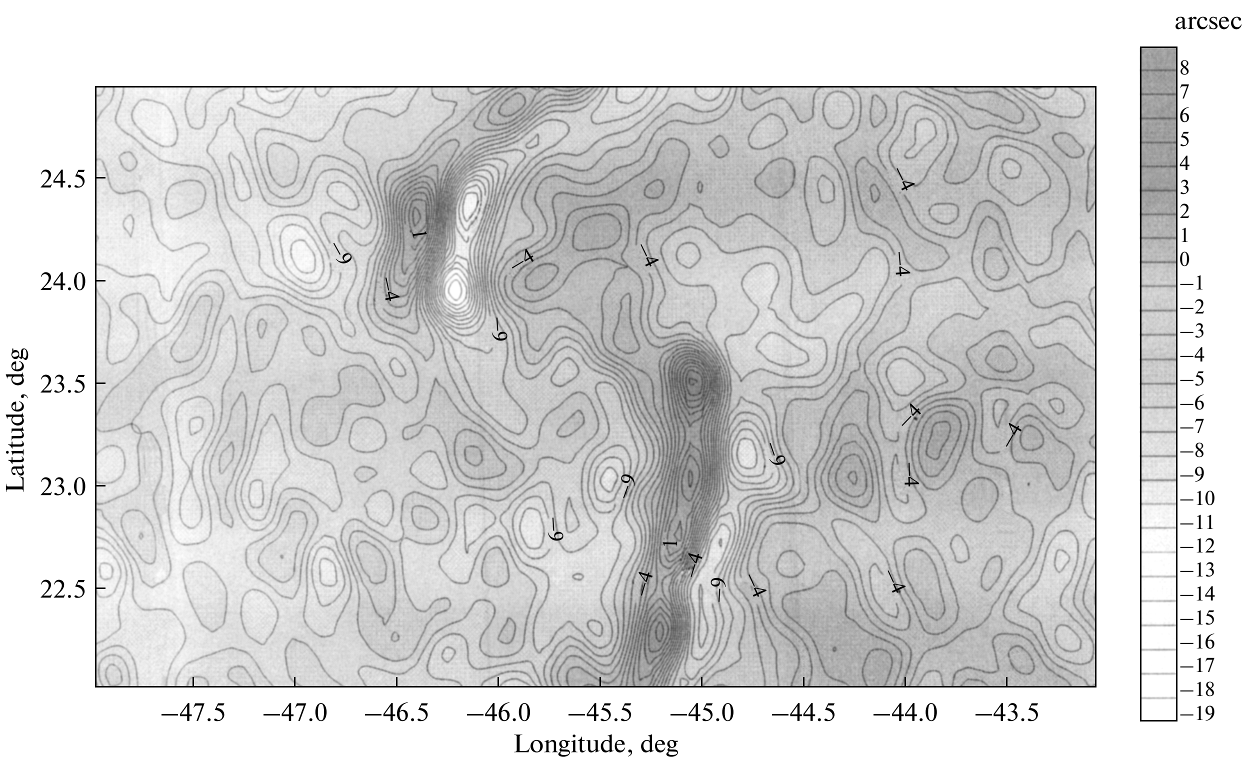

Fig. 6. The contour map of the eastward PLD. Area 2.

first area and 2.1 arcsec for the second area, which is a larger one.

CONCLUSIONS

A new technique for calculating the PLD is described, in which the *S*- and *R*-approximations are applied in the calculations by the Vening Meinesz formulas. The suggested method makes it possible to estimate the PLD at an arbitrary point of the Earth with reasonable accuracy (1.5–2 arcsec).

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