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CONTROL PROCESSES

Trajectory of an Observer Tracking the Motion of an Object around a Convex Set in $\mathbb{R}^3$

Academicain of the RAS **V. I. Berdyshev**[a,*]

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Abstract—An object t moving in $\mathbb{R}^3$ goes around a solid convex set along the shortest path $\mathcal{T}$ under observation. The task of an observer f (moving at the same speed as the object) is to find a trajectory closest to $\mathcal{T}$ that satisfies the condition $\delta \le \|f - t\| \le K\delta$ for a given $\delta > 0$. This condition makes it possible to track the object along the entire trajectory $\mathcal{T}$. A method is proposed for constructing an observer trajectory that ensures that the indicated inequality holds with a constant K arbitrarily close to unity and the object can be observed on its trajectory $\mathcal{T}$, except for an arbitrarily small segment of $\mathcal{T}$.

Keywords: navigation, autonomous vehicle, trajectory, observer
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1. An autonomous object t and an observer f that is hostile toward t move in space $\mathbb{R}^3$ containing a closed convex solid set G obstructing motion and visibility. The object follows from an initial point t_* to a final point $t^*(t_*, t^* \notin G)$ and goes around G along the shortest path $\mathcal{T} = \mathcal{T}_t$. It is assumed that the object and the observer have identical velocities V_t, V_f and, at each time τ, the distance between the positions of the mutually visible $t = t_\tau$ and $f = f_\tau$ satisfies the inequalities

$$0 < \delta \le \|t_\tau - f_\tau\| \le K \cdot \delta \quad (1)$$

for given δ and $K \ge 1$. The left inequality "ensures" the mutual safety of the object and the observer, while the right inequality is used to improve the quality of the observation.

The task of the observer is to design a trajectory $\mathcal{T}_f$ such that inequality (1) is satisfied with as small a constant K as possible and the observer $f_\tau \in \mathcal{T}_f$ is able to observe the moving object t_τ on as long a segment of $\mathcal{T}_t$ as possible.

In the absence of constraints on the observer velocity magnitude $|V_f|$, the problem is easy to solve. Specifically, moving along the trajectory $\mathcal{T}_t = \{f_\tau\}$, where $f_\tau = t_\tau - \delta \frac{V_{t_\tau}}{|V_{t_\tau}|}$, the observer f_τ tracks the motion of the object t_τ along the entire trajectory $\mathcal{T}_t$; moreover, $\|t_\tau - f_\tau\| = \delta$ and the observer velocity V_{f_τ} depends on V_{t_τ} and the curvature of $\mathcal{T}_t$. For the observer, it is inexpedient to follow the object along $\mathcal{T}_t$ on its strongly convex segments and near its corner points because of the fear of losing sight of the object t. Since $\mathcal{T}_t$ is the shortest path, whereas $\mathcal{T}_t$ under the conditions $|V_t| = |V_f|$ and (1) is not, there exists a segment of $\mathcal{T}_t$ on which the object moves unseen over the time t_τ.

In this paper, we propose a method for constructing a trajectory $\mathcal{T}_t$ ensuring that inequality (1) holds with a constant K arbitrarily close to unity and the unobseved segment of $\mathcal{T}_t$ has an arbitraruly short length.

2. Since t_* and t^* are not contained in the set G, the initial and final segments of the trajectory $\mathcal{T}$ are straight-line segments. Denote them by $[t_*, \underline{t}]$ and $[t^*, \overline{t}]$. Additinally, we use the notation

$$l_* = \{t_* + \lambda(\underline{t} - t_*): \lambda \ge 0\},$$
$$l^* = \{t^* + \lambda(\overline{t} - t^*): \lambda \ge 0\};$$

$P_{\underline{t}}, P_{\overline{t}}$ are the supporting planes of the set G at the points $\underline{t}$ and $\overline{t}$, respectively. Note that $l_* \subset P_{\underline{t}}$ and $l^* \subset P_{\overline{t}}$.

3. Suppose that the planes $P_{\underline{t}}, P_{\overline{t}}$ are parallel or intersect, $l = P_{\underline{t}} \cap P_{\overline{t}}$, and

[a] *Krasovskii Institute of Mathematics and Mechanics, Ural Branch, Russian Academy of Sciences, Yekaterinburg, 620219 Russia*
* *e-mail: bvi@imm.uran.ru*

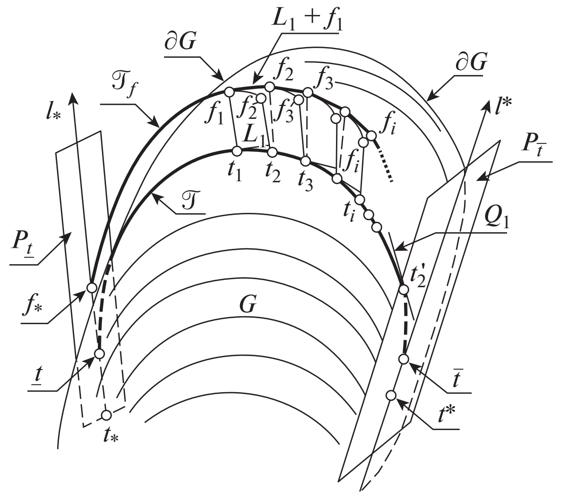

Fig. 1. Trajectories $\mathcal{T}$, $\mathcal{T}_f$ (thick curves) of the object and the observer; and the set G and the supporting planes $P_{\underline{t}}$, $P_{\overline{t}}$ (thin curves).

$$\rho(t_*, l) > \rho(\underline{t}, l), \quad \rho(t^*, l) > \rho(\overline{t}, l). \tag{2}$$

Moving along the trajectory

$$\mathcal{T}_f = \mathcal{T} + b, \quad \text{where} \quad b = \delta \frac{\underline{t} - t_*}{\|\underline{t} - t_*\|}, \tag{3}$$

the observer f_τ is able to see the object $t_\tau = (f_\tau - b) \in \mathcal{T}$ without changing the direction of observation b.

4. Consider the case when the reverses of inequalities (2) hold (see Fig. 1). In what follows, $\widehat{t,t'}$ is the arc of $\mathcal{T}$ lying between the points t, t' and $|\widehat{t,t'}|$ is the length of this arc. We introduce a sequence of points $t_i \in \mathcal{T}$ and define the corresponding sequence of points f_i. Later, the piecewise linear arc with nodes at f_i will be included in the trajectory $\mathcal{T}_f$.

Since $\mathcal{T}$ is the shortest path, there exists a pair of tangent vectors at each point t of $\mathcal{T}$ (see, e.g., [1, 2]). Let L_i denote the tangent vector to V_{t_i} at the point t_i, which is the velocity of the object t. The arc $\widehat{t_i, t_{i+1}}$ of the trajectory $\mathcal{T}$ is denoted by Δ_i.

The point $t_1 \in \mathcal{T}$ is such that the tangent vector to $\mathcal{T}$ at this point is orthogonal to the ray l_*. Let $f_1 = t_1 + b_1$, where $b_1 = b$ (see (3)). As an initial segment of the trajectory $\mathcal{T}_f$, we use the arc $(\widehat{\underline{t}, t_1}) + b_1$. To determine $t_2 \in \widehat{t_1, \overline{t}}$, we find a point $t_2' \in \widehat{t_1, \overline{t}} \subset \mathcal{T}$ such that the straight line Q_1 containing t_2' and parallel to the vector b_1 does not intersect G°, where G° is the interior of the set G. The point t_2 must lie on the arc $\widehat{t_1, t_2'}$ at a small distance from t_1. We construct the arc $\widehat{f_1, f_2'} = \Delta_1 + b_1$ and, on the ray $L_1 + f_1$, mark the point f_2 for which $\|f_1 - f_2\| = |\Delta_1|$. A continuous one-to-one mapping of the segment $[f_1, f_2]$ to the arc $\widehat{t_1, t_2}$ supplies the observer $f_\tau \in [f_1, f_2]$ with a way of tracking the moving object t_τ. This completes the first step. It is easy to see that an increase in the distance from t_1 to t_2 leads to an an increase in $\|f_2 - t_2\|$ and in the constant K in inequality (1). At the second step, by analogy with the first one, we find a point $t_3' \in \mathcal{T}$ such that the straight line Q_2 containing the point t_3' and parallel to the vector $b_2 = f_2 - t_2$ does not intersect G°. The point t_3 is taken on the arc $\widehat{t_2, t_3'}$ at a small distance from t_2, etc. Increasing the number of steps, we construct sequences $\{t_i\} \in \mathcal{T}$, $t_i \to \overline{t}$, and $\{f_i\}$, $\rho(f_i, P_{\overline{t}}) \to 0$ $(i \to \infty)$. The trajectory of the observer f has the form

$$\mathcal{T}_f = (\widehat{\underline{t}, t_1} + b_1) \cup_i [f_i, f_{i+1}]. \tag{4}$$

The constructed sequences t_i, f_i $(i = 1, 2, \ldots)$ satisfy the relations

$$\begin{aligned} \|t_{i+1} - f_{i+1}\| - \|t_i - f_i\| &= \|t_{i+1} - f_{i+1}\| - \|t_{i+1} - f_{i+1}'\| \\ &\le \|f_{i+1}' - f_{i+1}\| = o(\|f_{i+1}' - f_i\|) = o(\|t_i - t_{i+1}\|), \end{aligned} \tag{5}$$

$$\begin{aligned} \|t_{i+1} - f_{i+1}\| &\le \|t_i - f_i\| + \|f_{i+1}' - f_{i+1}\| \\ &\le \|t_{i-1} - f_{i-1}\| + \|f_i' - f_i\| \\ &+ \|f_{i+1}' - f_{i+1}\| \le \ldots \le \delta + \sum_2^{i+1} \|f_k' - f_k\|, \end{aligned} \tag{6}$$

moreover, $\|t_i - f_i\|$ is an increasing sequence.

Theorem 1. *Let* $\{t_i\}_1^\infty \subset \widehat{t_1, \overline{t}}$ *be the sequence of points generated according to the rule described above, and* $\widehat{t_{i+1}, \overline{t}} \subset \widehat{t_i, \overline{t}}$ $(i = 1, 2 \ldots)$. *Moving along the trajectory* $\mathcal{T}_f$ (4), *the observer f is able to track the motion of the object* $t = t(f)$ *on* $\mathcal{T}$, *where* $t(f) = f - b_1$ *for* $f \in (\widehat{\underline{t}, t_1}) + b_1$ *and* $t(f^\lambda) \in \Delta_i$, $|\widehat{t_i, t(f^\lambda)}| = \lambda|\Delta_i|$ *for* $f^\lambda = (1 - \lambda) f_i + \lambda f_{i+1}$ $(0 \le \lambda \le 1)$.

The number of segments in (4) can be limited by projecting the point f_j for a sufficiently large i onto the plane $P_{\overline{t}}$. Denote this projection by $\overline{f}$. While moving along the segment $[t_j, \overline{t}]$, the observer does not see the object following the arc $\widehat{f_j, \overline{f}}$, but, while moving on the plane $P_{\overline{t}}$ from the position $f_\tau = \overline{f}$, the observer tracks the motion of the object over the segment $[\overline{t}, t^*]$.

By using inequalities (5) and (6), it is easy to prove the following result.

Theorem 2. *Suppose that, for any index* $n = 1, 2, \ldots,$ *the rule described above generates an ordered grid of nodes* $\{t_i^n\}_{i=1}^{k(n)} \subset \widehat{t_1, \overline{t}}$ *such that* $t_{k(n)}^n \to \overline{t}$ *as* $k(n) \to \infty$ *and*

$$|\widehat{t_i^n, t_{i+1}^n}| \le \frac{|\widehat{t_1, \overline{t}}|}{n}.$$

Then, for the sequence of trajectories

$$\mathcal{T}_{f^n} = (\widehat{\underline{t}, t_1} + b_1) \bigcup_{i=1}^{k(n)-1} [f_i^n, f_{i+1}^n] \quad (n = 1, 2, \ldots),$$

where $f_i^n = t_i^n + b_i$, *it is true that*

$$\max_i \|t_i^n - f_i^n\| \to \delta \quad as \quad n \to \infty.$$

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CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

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