=== **MATHEMATICS** ===

Kinetic Algorithms for Modeling Conductive Fluids Flow on High-Performance Computing Systems

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Abstract—Processes in the dynamics of electrically conducting fluid flows in complex heat transfer systems are mathematically modeled in detail on high-performance parallel computing systems. The study is based on the kinetically consistent magnetogasdynamic approach adjusted to this class of problems. The kinetically consistent algorithm is well adapted to the architecture of high-performance computing systems with massive parallelism, so that complex heat transfer systems can be effectively studied with a high resolution. The approach, method, and algorithms are described, and numerical results are presented.

INTRODUCTION

In [1], a complex local Maxwellian statistical distribution function and the kinetically consistent approach were used to design a numerical algorithm that effectively uses the capabilities of high-performance computing systems as applied to fundamental and real-world problems. Numerical results obtained for the fundamental astrophysical problem of accreting interstellar matter onto a massive compact astrophysical object were presented in [2]. The three-dimensional system under study consisted of nine equations, including one for the magnetic field and one for the gravitational potential. The magnetic field was described by a three-dimensional equation with magnetic viscosity. Based on this approach, large-scale fundamental problems were effectively solved on spatial grids of more than 10^{10} cells and time intervals of more than 10^4 time steps with the use of high-performance parallel computing systems.

Many important applications require that models be described in detail by the magnetohydrodynamics equations for incompressible electrically conducting fluids. The interaction of an electrically conducting fluid with electric and magnetic fields makes it possible to implement various phenomena in electromagnetic fluid-mechanical energy converters. An advantage of such systems in practical applications under critical conditions is that energy carriers are transferred without using moving mechanical elements. An example is magnetohydrodynamic motors in complex heat transfer systems [3].

The goal of this paper is to adapt an algorithm developed earlier for magnetogasdynamics to the simulation of electrically conducting fluid flows and to apply it to the study of complex distributed heat transfer systems with electromagnetic motors.

As a rule, algorithms used to simulate incompressible flows differ substantially from those intended for simulating the dynamics of viscous compressible heat-conducting gases. In this work, relying on the well-known fact that gases at Mach numbers lower than 0.15 are nearly incompressible, an earlier proposed algorithm is adapted with minimal modifications to the simulation of electrically conducting fluid flows in complex heat transfer systems on high-performance parallel computing systems.

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KINETICALLY CONSISTENT APPROACH TO SIMULATION OF INCOMPRESSIBLE ELECTRICALLY CONDUCTING FLUID FLOWS

The use of the kinetically consistent system of differential equations of magnetogasdynamics for the simulation of electrically conducting fluids is based on the following physical assumptions. The mathematical model and the numerical algorithm to be used have to

take into account the features of fluid behavior, in particular, incompressibility.

The speed of sound in liquids is much higher than the speed of sound in gases. Since the characteristic velocities u in most engineering applications do not exceed tens of meters per second, we can assume that

$$u \leq 0.1c, \tag{1}$$

where c is the speed of sound.

Additional stronger incompressibility conditions can be specified by the phenomenological equation of state

$$p = p_0 + \beta(\rho - \rho_0), \tag{2}$$

where the parameter β reflects larger variations in pressure under minor variations in density, i.e., it specifies a nearly incompressible medium. Large values of β can be used with a sufficient degree of approximation without affecting the simulation results.

In the case of flows characterized by small Mach numbers, a serious difficulty is the requirement that the time step be sufficiently small. In the case of implicit schemes, this requirement is needed for achieving the required accuracy; as a result, the convergence of the solution substantially slows down. In this context, explicit schemes are preferable for parallel computations; moreover, in the case of using hyperbolic systems of magnetohydrodynamic equations, the requirements for stability and, accordingly, for the time step can be reduced significantly [4].

The conducted study was based on a kinetically consistent system of magnetogasdynamics equations of hyperbolic type (for more details, see [5]), which represents a physical model of magnetogasdynamic processes derived from the fundamental kinetic equation [6, 7].

A compact kinetically consistent system of differential equations of magnetogasdynamics was used for numerical computations of the problem under consideration [8]. Brought to hyperbolic form, the three-dimensional system of equations is given by:

$$\frac{\partial \rho}{\partial t} + \frac{\tau}{2}\frac{\partial^2 \rho}{\partial t^2} + \operatorname{div}[\rho(\mathbf{u} - \mathbf{w})] = 0, \tag{3}$$

$$\frac{\partial}{\partial t}\rho\mathbf{u} + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}\rho\mathbf{u} + \operatorname{div}[\rho(\mathbf{u} - \mathbf{w}) \times \mathbf{u}]$$
$$+ B_{i,k} + \nabla\left(p + \frac{B^2}{8\pi}\right) = \operatorname{div}P_{NS}, \tag{4}$$

$$\frac{\partial}{\partial t}E + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}E + \operatorname{div}\left[\left(E + p + \frac{B^2}{2}\right)(\mathbf{u} - \mathbf{w})\right]$$
$$= \operatorname{div}q + \operatorname{div}(P_{NS}\mathbf{u}), \tag{5}$$

$$\frac{\partial}{\partial t}\mathbf{B} + \frac{\tau}{2}\frac{\partial^2}{\partial t^2}\mathbf{B} = \operatorname{curl}[(\mathbf{u} - \mathbf{w}) \times \mathbf{B}] + \nu_m \operatorname{curl}\mathbf{B}, \tag{6}$$

$$\operatorname{div}\mathbf{B} = 0, \tag{7}$$

where ρ is the density, $\mathbf{u}$ is the velocity, p is the pressure,

$$w_{ik} = \frac{1}{\rho}\frac{\partial}{\partial x_i}\left[\left(p + \frac{B^2}{8\pi}\right)\delta_{ik} + \rho u_i u_k - B_i B_k\right]$$

is a regularizer associated with smoothing the solution over a time interval, τ is the characteristic time between molecular collisions: $\tau = 2\mu/p$, $\tau_m = 2\nu_m\rho/\left(p + \frac{B^2}{8\pi}\right)$ is a magnetohydrodynamic time constant, ν_m is the magnetic viscosity, q is the Joule heating, and $\mathbf{B}$ is the magnetic field.

System (3)−(7) with additional conditions (2) was solved numerically using the three-level explicit scheme described in [4]. This scheme is perfectly adaptable to the architecture of computing systems with extramassive parallelism and is a promising direction for ultrahigh-performance parallel computing systems. The asymptotic stability of the considered system is determined by the condition $\Delta t \leq h^{3/2}$.

NUMERICAL EXPERIMENTS

The flow of an electrically conducting fluid was computed in a plane channel of a magnetic dynamic motor having an expansion in the form of a three-dimensional rectangular heat transfer system. The geometry of the problem is shown in Fig. 1. The rectangular channel 30 cm long with a cross section of 2.5×2.5 cm is connected to a rectangular volume of $2.5 \times 30 \times 30$ cm in size, which is connected to a rectangular channel with a cross section of 2.5×2.5 cm. The initial segment of the inlet channel (10 cm long) represents the electromagnetic motor. An electrically conducting sodium fluid (Na) moves in the channel of the magnetic motor under the influence of an external magnetic field applied to the shaded domain. The physical characteristics of the melt are taken from [9]: the temperature is 600 K, the density is 874 kg/m³, and the adiabatic index is 1.2. The viscosity is defined as a function of temperature by the empirical formula

$$\ln\eta = -6.4406 - \ln T + 556.835/T, \tag{8}$$

the thermal conductivity is defined as a function of temperature by the empirical formula

$$k = 124.67 - 0.0011381T$$
$$+ 5.5226 \times 10^{-5}T^2 - 1.1842 \times 10^{-8}T^3, \tag{9}$$

and the electric conductivity is defined as a function of temperature by the empirical formula

$$R = -9.9141 + 8.2022 \times 10^{-2}T$$
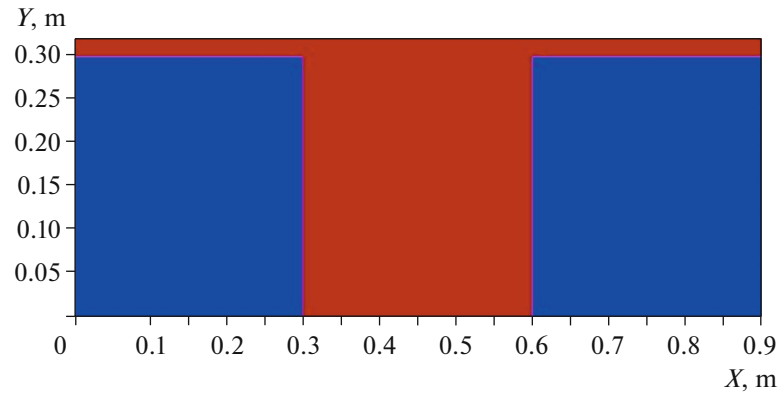$$- 1.3215 \times 10^{-4}T^2 + 1.7212 \times 10^{-7}T^3. \tag{10}$$

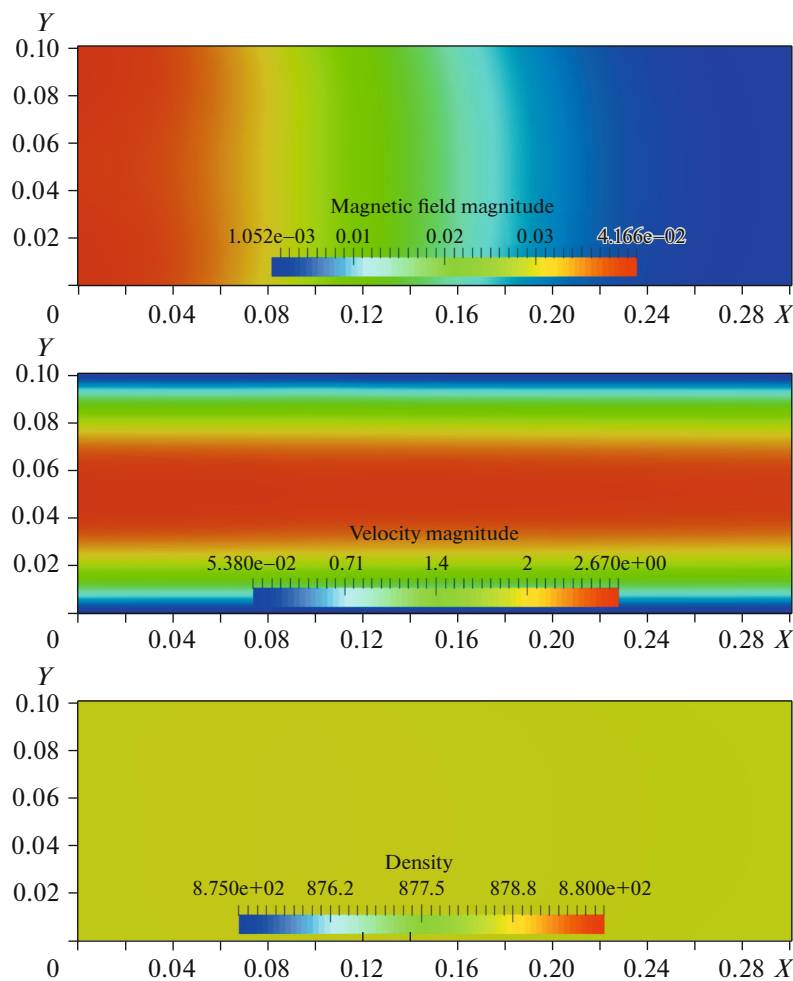**Fig. 1.** 2D projection of the geometry of the numerical experiment.

Fig. 2. Computation of the flow of the electrically conducting fluid (Na) in the magnetic dynamic motor: (a) magnetic field strength distribution, (b) flow velocity distribution, and (c) density distribution.

Three-dimensional numerical experiments were performed on a grid consisting of $200 \times 2000 \times 2000$ cells, so that the flow in the complicated geometric system was simulated in detail.

As initial conditions, the parameters of the magnetohydrodynamic motor were specified so that the velocity of the electrically conducting fluid (Na) in the electromagnetic motor was 360 m/s ($M = 0.15$).
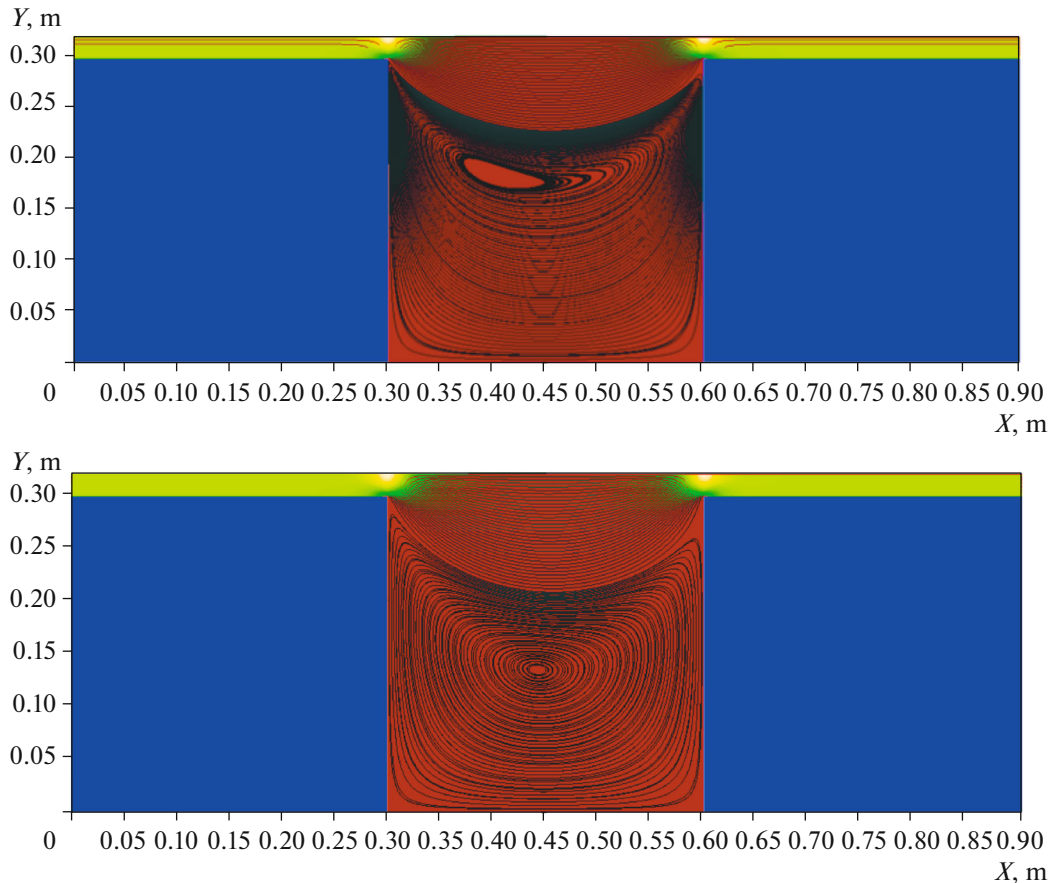
Fig. 3. 2D projection of the flow of the electrically conducting fluid (Na) in the cavity at $t =$ (a) 0.02 and (b) 0.3 s; steady state.

Figure 2 shows the basic magnetohydrodynamic parameters on the inlet segment for steady laminar flow: (a) the magnetic field strength, (b) the velocity in the middle of the channel (reaches 270 m/s), and (c) the density in the inlet channel; the variation in the density is at most 0.2%.

Figure 3 presents 2D-projections of the flow of the electrically conducting fluid (Na) in the system: the inlet channel with the magnetic dynamic motor, the cavity, and the outlet channel. The dynamics of the flow in the system is characterized by several basic stages. Figure 3a shows the transitional flow regime at the time $t = 0.02$ s; we can see a stable central vortex shifted toward the forward wall of the cavity, which is caused by the reaction of the flow directed from the back wall of the cavity. Figure 3b presents the steady flow regime at $t = 0.3$ s. The flow dynamics is stabilized with the formation of a constant central vortex in the cavity. Its expected parameters are comparable with those given in [10]. Over the entire time interval, the inlet and outlet flows remain equivalent up to high accuracy, which is typical for incompressible flow.

CONCLUSIONS

A kinetically consistent model earlier applied to the simulation of ionized viscous compressible gas flows can be successfully used to compute flows of electrically conducting incompressible fluids in complex heat transfer systems with magnetohydrodynamic motors. The proposed algorithm is important for the simulation of processes in engineering devices on high-performance computing systems.

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