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MATHEMATICS

Stability Defect Estimation for Sets in a Game Approach Problem at a Fixed Moment of Time

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Abstract—The game problem of a control system approaching a target set at a fixed point in time is studied. The stability defect of a set in the position space that is weakly invariant with respect to a finite set of unification differential inclusions is estimated from below.

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Given a conflict-controlled system in a finite-dimensional Euclidean space, we consider the game problem of the system approaching a compact set at a finite moment of time [1–8]. The subject of the study is the stability defect of subsets of the position space of the system. This concept was introduced in [9, 10] in order to extend the concept of stability to sets that generally do not have the stability property.

Involved in the definition of stability, the families of differential inclusions induced by the dynamics of the system can have different forms, while identifying the same sets—stable bridges. In this sense, different formulations of the stability property are essentially equivalent. To extend the concept of stability, it was found convenient to use unification definitions of stability [11, 12] based on unification families. These families are infinite, i.e., they consist of infinitely many differential inclusions. In view of this, a certain subset of the position space cannot be checked for stability. Such a check can be made for relatively simple conflict-controlled systems, since a unification family can be replaced by an equivalent (from the point of view of stability) finite set of differential inclusions. For other conflict-controlled systems, the following problem is important. Given a finite subset of a unification family of differential inclusions and a set in the system's position space that is weakly invariant with respect to the subset, the task is to estimate the degree to which this set is close to being stable. In other words, the stability defect of this set has to be estimated from above. An estimate of this kind for the stability defect is derived in this paper.

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1. GAME APPROACH PROBLEM AT A FIXED TIME

On a time interval $[t_0, \vartheta]$, $t_0 < \vartheta < \infty$, we consider a conflict-controlled system

$$\frac{dx}{dt} = f(t, x, u, v), \quad x(t_0) = x^{(0)}, \quad u \in P, \quad v \in Q, \quad (1)$$

where x is the m-dimensional state vector of the system from $\mathbf{R}^m$; u and v are the controls of the first and second players, respectively; and P and Q are compact sets in Euclidean spaces $\mathbf{R}^p$ and $\mathbf{R}^q$.

The right-hand side of system (1) satisfies the following conditions.

Condition A. The function $f(t, x, u, v)$ is defined and continuous on $[t_0, \vartheta] \times \mathbf{R}^m \times P \times Q$ and, for any bounded closed domain $\Omega \subset [t_0, \vartheta] \times \mathbf{R}^m$, there exists a constant $L = L(\Omega) \in (0, \infty)$ such that

$$\| f(t, x^{(1)}, u, v) - f(t, x^{(2)}, u, v)\| \le L\|x^{(1)} - x^{(2)}\|,$$
$$(t, x^{(i)}, u, v) \in \Omega \times P \times Q, \quad i = 1, 2;$$

here, $\|f\|$ is the norm of the vector f in Euclidean space.

Condition B. There exists a constant $\gamma \in (0, \infty)$ such that

$$\| f(t, x, u, v)\| \le \gamma(1 + \|x\|),$$
$$(t, x, u, v) \in [t_0, \vartheta] \times \mathbf{R}^m \times P \times Q.$$

In the game approach problem at a fixed time, the first player needs to ensure that the state vector $x(\vartheta)$ of system (1) hits a given compact set $M \subset \mathbf{R}^m$ for any admissible controls $v = v(t, x), t \in [t_0, \vartheta]$ of the second player. A solution of the approach problem is sought in the class of first player's feedback control procedures [1].

In the evasion problem, which is dual to the approach one, the second player needs to ensure that the state vector $x(\vartheta)$ of system (1) stays away from an ε-neighborhood M_ε of the set M. A solution of the evasion problem is sought in the class of second player's anti-feedback control procedures with a guide [1].

For the differential game consisting of the approach and evasion problems, the following alternative is valid: there exists a closed set $W^0 \subset [t_0, \vartheta] \times \mathbf{R}^m$ such that the approach problem is solvable for all initial positions $(t_*, x_*) \in W^0$ and the evasion problem is solvable for all initial positions $(t_*, x_*) \notin W^0$ [1].

The set W^0 plays a key role in solving the approach problem. For initial positions $(t_*, x_*) \in W^0$, a solving positional procedure of the first player can be designed as a guided feedback control procedure aiming the state vector $x(t), t \in [t_0, \vartheta]$, of system (1) at a guide evolving in W^0. It is well known (see [1]) that the set W^0 has an important property, namely, W^0 is a maximal u-stable bridge. This property underlies algorithms for approximate calculation of W^0. In [11, 12] W^0 was approximately calculated by applying algorithms based on unification constructions or their modifications.

Below, the u-stability of sets contained in $[t_0, \vartheta] \times \mathbf{R}^m$ is described in terms of a unification family from $\mathbf{R}^m$. Similar families were considered in [11, 12]. This family is associated with a set of differential inclusions used to obtain an infinitesimal description of u-stability. To define the unification family, we introduce the following scalar functions on $[t_0, \vartheta] \times \mathbf{R}^m \times \mathbf{R}^m$:

$H(t, x, l) = \max_{u \in P} \min_{v \in Q} \langle l, f(t, x, u, v) \rangle$ is the Hamiltonian of system (1), $H^*(t, x, l) = \max_{(u,v) \in P \times Q} \langle l, f(t, x, u, v) \rangle$, where $\langle l, f \rangle$ is the scalar product of vectors l and f from $\mathbf{R}^m$.

In view of Condition B, since M is compact, we conclude that there exists a large bounded closed domain Ω in $[t_0, \vartheta] \times \mathbf{R}^m$ that contains W^0 and all state vectors $x(t)$ of system (1) hit a given ε-neighborhood of the set M at the time ϑ.

Let $r = \max_{(t,x,l) \in \Omega \times S} |H(t, x, l)| < \infty$ and $K = \max_{(t,x,l) \in \Omega \times S} |H^*(t, x, l)| = \max_{(t,x,u,v) \in \Omega \times P \times Q} \|f(t, x, u, v)\| < \infty$ here, $S = \{l \in \mathbf{R}^m : \|l\| = 1\}$.

By the definitions of r and K, we have $r \leq K$.

In addition to A and B, the following condition is assumed to hold.

Condition C. It is true that $r < K$.

Define the set $G = B(0; K) = \{b \in \mathbf{R}^m: \|b\| \leq K\} \subset \mathbf{R}^m$. For the ball G thus defined, the vectorgrams $\mathcal{F}(t, x) = \{f(t, x, u, v): (u, v) \in P \times Q\}$, $(t, x) \in \Omega$, and their convex hulls $F(t, x) = \operatorname{co} \mathcal{F}(t, x)$ are contained in G.

Let $\Psi = \{\psi\}$ be a finite set of elements ψ and $\{F_\psi: \psi \in \Psi\}$ be a set of mappings $F_\psi: (t, x) \mapsto F_\psi(t, x) \subset \mathbf{R}^m$, $(t, x, \psi) \in \Omega \times \Psi$, satisfying the following conditions:

Condition A.1. For any $(t, x, \psi) \in \Omega \times \Psi$, the set $F_\psi(t, x)$ is convex and closed in $\mathbf{R}^m$; $F_\psi(t, x) \subset G$; and, additionally,

(a) the mapping $(t, x) \mapsto F_\psi(t, x)$ is equicontinuous on Ω (in the Hausdorff metric) with respect to ψ from Ψ;

(b) for some $\lambda = \lambda(L) \in (0, \infty)$ $(L = L(\Omega))$ and all $\psi \in \Psi$, it is true that

$$d(F_\psi(t, x_*), F_\psi(t, x^*)) \leq \lambda \|x_* - x^*\|,$$
$$(t, x_*) \quad \text{and} \quad (t, x^*) \quad \text{from} \quad \Omega.$$

Condition A.2. For any $(t, x, l) \in \Omega \times S$,

$$\min_{\psi \in \Psi} h_{F_\psi(t,x)}(l) = H(t, x, l);$$

here, $h_F(l) = \max_{f \in F} \langle l, f \rangle$ is the support function of the compact set $F \subset \mathbf{R}^m$ and $d(F_*, F^*)$ is the Hausdorff distance between compact subsets F_* and F^* of $\mathbf{R}^m$.

Introduced with the help of Axioms A.1 and A.2, the sets $F_\psi(t, x)$, $(t, x) \in \Omega$, $\psi \in \Psi$, contained in the ball G for each $(t, x) \in \Omega$ can be appropriately called kernels in G. This set of kernels in G is determined by the structure of the Hamiltonian $H(t, x, l)$ and is connected with the latter by Condition A.2. For some conflict-controlled systems, it may happen that the structure of $H(t, x, l)$ is rather simple, so that, for each $(t, x) \in \Omega$, the ball G contains a finite rather small number of kernels and this number is independent of $(t, x) \in \Omega$. In these cases, given the set $\{F_\psi: \psi \in \Psi\}$, we can effectively develop algorithms for approximate calculation of the set W^0, at least, for systems (1) of low order. In these cases, algorithms for approximate calculation of W^0 are reduced to sets of computational geometry algorithms.

Several such finite sets $\{F_\psi: \psi \in \Psi\}$ are known. We describer one of them that corresponds to a fairly broad class of systems (1). Assume that system (1) has the form

$$\frac{dx}{dt} = f^{(1)}(t,x,u) + f^{(2)}(t,x,v), \quad u \in P, \quad v \in Q,$$

where $f^{(2)}(t,x,v) = f^*(t,x) + C(t,x)v$ and Q is a convex polyhedron in $\mathbf{R}^2$ with a finite set $\Psi = \{v^{(i)}\}$ of vertices $v^{(i)}$.

For this conflict-controlled system, the finite set $\{F_\psi\colon \psi \in \Psi\}$ can be specified as a set of mappings $(t,x) \mapsto F_{v^{(i)}}(t,x) = F^{(1)}(t,x) + f^{(2)}(t,x,v^{(i)}),\ \ v^{(i)} \in \Psi$; here, $F^{(1)}(t,x) = \mathrm{co}\{f^{(1)}(t,x,u) : u \in P\}$.

However, for numerous systems of form (1), the structure of the Hamiltonian $H(t,x,l)$ is not simple enough to be associated with a finite set $\{F_\psi : \psi \in \Psi\}$ satisfying Conditions A.1 and A.2, where Ψ has a small number of elements.

In the study of such systems, it is reasonable to consider an uncountable unification family $\mathcal{F} = \{F_l : l \in S\}$ satisfying Conditions A.1 and A.2 (see [2, 10, 11]). For $(t,x,l) \in \Omega \times S$, it is defined by the relations

$$F_l(t,x) = G \cap \Pi_l(t,x),$$
$$\Pi_l(t,x) = \{f \in \mathbf{R}^m : \langle l, f\rangle \le H(t,x,l)\}.$$

The sets $F_l(t,x)$ are spherical segments in $\mathbf{R}^m$ satisfying A.1 and A.2, where $\Psi = S$ and $\psi = l \in S$.

Let us describe the u-stability property with the help of the set $\{F_l : l \in S\}$.

For a set-valued mapping $t \mapsto W(t)$, let

$$\vec{D}W(t_*,x_*) = \{d \in \mathbf{R}^m : d = \lim_{k\to\infty}(t_k - t_*)^{-1}(w_k - x_*),$$
$$\{(t_k, w_k)\} \text{ from } W,$$
$$\text{where } t_k \downarrow t_* \text{ as } k \to \infty,\ \lim_{k\to\infty} w_k = x_*\}$$

be the derivative set of the mapping $t \mapsto W(t)$ at the point $(t_*,x_*) \in W$, $t_* \in [t_0,\vartheta)$ (see [14]).

Definition 1 [14]**.** A set W is called a *u-stable bridge* in the problem of system (1) approaching M if

(i) $W(\vartheta) \subset M$;

(ii) $\vec{D}W(t_*,x_*) \cap F_l(t_*,x_*) \neq \phi$ for $(t_*,x_*,l) \in [t_0,\vartheta) \times \partial W(t_*) \times S$.

In fact, Definition 1 is an infinitesimal formulation of the u-stability property of the set $W \subset \Omega$. It is an element of derivative-based constructions introduced into the theory of differential games. This definition was found useful in detecting various properties of u-stable bridges.

2. STABILITY DEFECT OF SETS IN $[t_0,\vartheta] \times \mathbf{R}^m$

To calculate W^0 approximately in particular game approach problems, we need algorithms based on finite sets of mappings $(t,x) \mapsto F_\psi(t,x)$, $\psi \in \Psi$. In this section, we thin the set $\{F_l : l \in S\}$, namely, pass to a finite set of mappings. This finite set no longer identifies a maximal u-stable bridge W^0 in Ω, but, instead, identifies a closed set $\mathcal{W}^0$ ($\mathcal{W}^0(\vartheta) = M$, $W^0 \subset \mathcal{W}^0$). For $\mathcal{W}^0$ the u-stability property holds with an error, which is called the stability defect.

Let us define the stability defect of a closed set $\mathcal{W} \subset \Omega$ such that $\mathcal{W}(\vartheta) = M$ and, from $t_0 \le t_* < t^* \le \vartheta$ and $\mathcal{W}(t_*) \neq \phi$, it follows that $\mathcal{W}(t^*) \neq \phi$.

Assume that the following condition holds for $\mathcal{W}$.

Condition D. It is true that

$$(t^* - t_*)^{-1} d(\mathcal{W}(t_*), \mathcal{W}(t^*)) \le K,\ t_0 \le t_* < t^* \le \vartheta.$$

Condition **D** implies that

$$\vec{D}\mathcal{W}(t_*,x_*) \cap G \neq \phi, \quad (t_*,x_*) \in [t_0,\vartheta) \times \partial\mathcal{W}(t_*),$$

and this condition is satisfied, for example, by the u-stable bridge W^0.

Each point (t_*,x_*), $(t_* \in [t_0,\vartheta), x_* \in \partial\mathcal{W}(t_*))$ is associated with the number (see [9, 10])

$$\varepsilon(t_*,x_*) = \sup_{l\in S} \rho(\vec{D}\mathcal{W}(t_*,x_*), F_l(t_*,x_*));$$

here, $\rho(F_*,F^*) = \inf\{\|f_* - f^*\| : (f_*,f^*) \in F_* \times F^*\}$, where F_* and F^* are compact subsets of $\mathbf{R}^m$.

The quantity $\varepsilon(t_*,x_*) \ge 0$ is called the stability defect of the set $\mathcal{W}$ at the point (t_*,x_*). It can be treated as a local characteristic of the degree of instability of $\mathcal{W} \subset \Omega$ at the point $(t_*,x_*) \in [t_0,\vartheta) \times \partial\mathcal{W}(t_*)$, i.e., as a measure of the inconsistency, in terms of u-stability, between the dynamics of system (1) and the evolution of the set-valued mapping $t \mapsto \mathcal{W}(t) = \{x \in \mathbf{R}^m\colon (t,x) \in \mathcal{W}\}$ near (t_*,x_*) (on the right in time t). A large value of $\varepsilon(t_*,x_*)$ means that they are strongly inconsistent, while the equality $\varepsilon(t_*,x_*) = 0$ means that the set $\mathcal{W}$ is u-stable at the point (t_*,x_*).

For $t_* \in [t_0,\vartheta)$ (see [9, 10]), let

$$\varepsilon(t_*) = \sup_{(t_*,x_*)\in\Lambda(t_*)} \varepsilon(t_*,x_*),$$
$$\Lambda(t_*) = \{(t_*,x_*) : x_* \in \partial\mathcal{W}(t_*)\};$$
$$\varepsilon(t_*) = 0 \quad \text{at} \quad t_* = \vartheta.$$

The quantity $\varepsilon(t_*)$, $t_* \in [t_0,\vartheta]$, is called the stability defect of the set $\mathcal{W}$ at the time t_*.

N.N. Krasovskii's well-known rule for extremal aiming at $\mathcal{W}$ ($\mathcal{W}(\vartheta) = M$), or the rule for aiming at a guide evolving in $\mathcal{W}$ guarantees that, for initial positions $(t_*, x_*) \in \mathcal{W}$, the state vector $x(\vartheta)$ of system (1) hits M if $\mathcal{W}$ is a u-stable bridge (i.e., $\varepsilon(t) \equiv 0$ on $[t_0, \vartheta]$).

The mapping $(t, x, l) \mapsto F_l(t, x)$ is continuous (in the Hausdorff metric) on $\Omega \times S$ by virtue of Conditions A.1, A.2, and C.

The results of [11, p. 168] imply that $\lambda = \lambda(L)$ in Condition A.1 can be specified by the equality $\lambda = \lambda(L) = \dfrac{KL}{\sqrt{K^2 - r^2}} < \infty$.

In addition to Condition D, we assume that the set $\mathcal{W}$ and the function $\varepsilon(t)$, $t \in [t_0, \vartheta]$, obey two more conditions.

Condition H. There exists a function $\varphi^*(\delta) \downarrow 0$ as $\delta \downarrow 0, \delta \in (0,\ \vartheta - t_0)$, such that

$$h\left(\bar{D}\mathcal{W}(t_*, x_*) \cap B(0, 3K), \frac{\mathcal{W}(t_* + \delta) - x_*}{\delta}\right) \le \varphi^*(\delta),$$

$$(t_*, x_*) \in [t_0, \vartheta) \times \partial\mathcal{W}(t_*), \quad \delta \in (0, \vartheta - t_0);$$

here, $B(0, 3K) = \{b \in \mathbf{R}^m : \|b\| \le 3K\}$; $\dfrac{\mathcal{W}(t_* + \delta) - x_*}{\delta} = \{h^{(\delta)}:\ h^{(\delta)} = \dfrac{w^{(\delta)} - x_*}{\delta},\ w^{(\delta)} \in \mathcal{W}(t_* + \delta)\}$; $\partial\mathcal{W}(t_*)$ is the boundary of the set $\mathcal{W}(t_*)$ in $\mathbf{R}^m$; $h(W_*, W^*)$ is the Hausdorff deviation of W_* from W^*; and W_* and W^* are compact sets in $\mathbf{R}^m$.

Condition E. The function $\varepsilon(t)$ is Lebesgue measurable on $[t_0, \vartheta]$.

$$\text{Let } \mathcal{W}^* = \bigcup_{t \in [t_0, \vartheta]} (t, \mathcal{W}^*(t)),$$
$$\text{where } \mathcal{W}^*(t) = \mathcal{W}(t)_{æ(t)},$$

$$æ(t) = \int_{t_0}^{t} e^{\lambda(t - \tau)} \varepsilon(\tau)\, d\tau \text{ is the Lebesgue integral;}$$

here, $\mathcal{W}(t)_{æ(t)}$ is a $æ(t)$-neighborhood of the set $\mathcal{W}(t)$ in $\mathbf{R}^m$.

It is true that $\mathcal{W}^*(t_0) = \mathcal{W}(t_0)$, and $æ(t)$ is an increasing function of t on $[t_0, \vartheta]$.

Theorem 1 [10]. *The set $\mathcal{W}^*$ is a u-stable bridge in the problem of system* (1) *approaching the set $M_{æ(\vartheta)}$ at the time ϑ.*

The number $\varepsilon_{\mathcal{W}} = æ(\vartheta)$ is called the stability defect of the set $\mathcal{W}$ [10].

Theorem 1 implies that, if $(t_0, x^{(0)}) \in \mathcal{W}$, then the rule for extremal aiming at $\mathcal{W}^*$ (see [10]) guarantees that the state vector $x(\vartheta)$ ($x(t_0) = x^{(0)}$) of system (1) for the first player hits the set $M_{\varepsilon_{\mathcal{W}}}$. Moreover, if $\varepsilon_{\mathcal{W}}$ is small, this means that the problem of approaching M has been solved with a small error.

3. UPPER ESTIMATE FOR THE STABILITY DEFECT OF SETS IN Ω

Given a particular problem of approaching M, if the stability defect $\varepsilon_{\mathcal{W}}$ of a set $\mathcal{W} \subset \Omega$ is small, then it is reasonable to replace this problem by a milder one, namely, by the problem of system (1) approaching the set $M_{\varepsilon_{\mathcal{W}}}$.

In game problems of approaching M, it is reasonable to consider various remarkable sets $\mathcal{W} \subset \Omega \subset [t_0, \vartheta] \times \mathbf{R}^m$, $\mathcal{W}(\vartheta) = M$. A task of much interest is to determine to which degree these sets $\mathcal{W}$ are close to being u-stable in the game problem of system (1) approaching the set M at the time ϑ. Examples of such sets are the set of programmed absorption in the game problem of approaching M at the time ϑ and the set of positional absorption in the game problem of approaching M by the time ϑ (see [1]).

Below, we consider a closed set $\mathcal{W} \subset \Omega$ and obtain an upper estimate for the corresponding stability defect $\varepsilon_{\mathcal{W}}$.

Specifically, given a set $\mathcal{F}$ of set-valued mappings $(t, x) \mapsto F_l(t, x)$, $(t, x) \in \Omega, l \in S$, we choose a finite subset $\mathcal{F}^* = \{(t, x) \mapsto F_{l_\rho}(t, x) : \rho = 1, 2, \ldots, N\}$.

Assume that $\mathcal{W}^0$ ($\mathcal{W}^0(\vartheta) = M$) is a maximal (in the sense of inclusion) subset of Ω that is weakly invariant with respect to the set $\mathcal{F}^*$ of differential inclusions

$$\frac{dx}{dt} \in F_{l_\rho}(t, x), \quad \rho = 1, 2, \ldots, N, \quad t \in [t_0, \vartheta], \tag{2}$$

corresponding to mappings from $\mathcal{F}^*$.

The closed set $\mathcal{W}^0$ satisfies the inclusion $W^0 \subseteq \mathcal{W}^0$, along with the quality $\mathcal{W}^0(\vartheta) = M$. In particular game problems, as a rule, $\mathcal{W}^0 \neq W^0$ and, hence, $\mathcal{W}^0$ is not u-stable. Accordingly, the following question arises: to which degree is $\mathcal{W}^0$ defined with the help of $\mathcal{F}^*$ far from being u-stable?

To answer this question, we employ two estimates used in the proof of the main result of this paper.

1. The function $l \mapsto H(t, x, l), l \in S$ satisfies the inequality

$$|H(t, x, l_*) - H(t, x, l^*)| \le K\|l_* - l^*\|,$$
$$(t, x) \in \Omega, \quad l_* \text{ and } l^* \text{ are from } S.$$

2. It holds that

$$d(F_{l_*}(t,x), F_{l^*}(t,x)) \leq \frac{2K}{\sqrt{K^2 - r^2}} \|l_* - l^*\|$$

for $(t,x) \in \Omega$, l_* and l^* are from S, and $\|l_* - l^*\| \leq \delta$, where $\delta \in \left(0, 1 - \dfrac{r}{K}\right)$.

Below is the main result.

Theorem 2. *Let a finite set* $S^{(\delta)} = \{l_\rho \in S : \rho = 1, 2, \ldots, N\}$ *be a* δ*-network in the sphere* $S \subset \mathbf{R}^m$*, where* $\delta \in \left(0, 1 - \dfrac{r}{K}\right)$*. Then the stability defect* $\varepsilon_{\mathcal{W}^0}$ *of the set* $\mathcal{W}^0 \subset \Omega$ *defined with the help of* $\{F_{l_\rho} : l_\rho \in S^{(\delta)}\}$ *satisfies the inequality*

$$\varepsilon_{\mathcal{W}^0} \leq \frac{2K}{L}\left(e^{\frac{KL}{\sqrt{K^2 - r^2}}(\vartheta - t_0)} - 1\right) \cdot \delta. \tag{3}$$

It follows from (3) that a set $\mathcal{W}^0$ ($\mathcal{W}^0(\vartheta) = M$) satisfying the u-stability property to a sufficiently high degree can be constructed using the set $\{F_{l_\rho} : l_\rho \in S^{(\delta)}\}$ corresponding to a small $\delta > 0$. It is also of interest to identify the spectrum of the parameters $K, L, r, (\vartheta - t_0)$ for which

$$\frac{2K}{L}\left(e^{\frac{KL}{\sqrt{K^2 - r^2}}(\vartheta - t_0)} - 1\right) \leq \varepsilon,$$

where $\varepsilon \in (0, \infty)$ is a small number.

Obviously, this issue is reduced to an analysis of the inequality

$$e^{\gamma_1 L} \leq 1 + \gamma_2 L, \ L \in (0, \infty), \tag{4}$$

where $\gamma_1 = \dfrac{KL}{\sqrt{K^2 - r^2}}(\vartheta - t_0)$ and $\gamma_2 = \dfrac{\varepsilon}{2K}$.

A further study of inequality (4) is reduced to comparing the coefficients γ_1 and γ_2.

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