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SYSTEMS ANALYSIS AND OPERATIONS RESEARCH

Game-Theoretic Approach to Managing the Composition and Structure of a Bearing-Only Measurement System in Conditions of a priori Uncertainty

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Abstract—The problem of managing the composition and structure of a bearing-only measurement system (BOMS) in a game-theoretic formulation is considered. An approach for a cooperative search for the placement of BOMS points and a method for estimating the work-time indicator of the system are proposed. The search for the placement of BOMS points uses the toolkit of multiagent potential games. The criteria for selecting the placement of points and the type of potential function are determined. The management of the composition and structure of the BOMS is based on the results of the cluster-variation method (CVM). A structural and functional description of the simulation model is presented. The presented results of simulation modeling confirm the practical effectiveness of the proposed approaches.

Keywords: radiating target, bearing-only measurements, game-theory, multiagent, cluster analysis, passive location

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INTRODUCTION

The creation of multiposition passive location systems is a promising direction for increasing the accuracy and noise immunity of estimating target motion parameters [1–12]. In this case, systems with both stationary and moving positions are considered. Among passive location methods, from the point of view of practical implementation, the bearing-only method is the most popular method [7–12]. In [13–15], a cluster-variation method (CVM) for solving the bearing-only problem is developed, which is an alternative to the known passive location methods, for example [16–20]. The CVM, together with the formation of stable estimates of the target location under conditions of significant a priori uncertainty, provides information about channels with unreliable measurements. Measurement errors can have a different nature of origin: random, caused by the topology of the bearing-only measurement system (BOMS) and target observation conditions, arising as a result of artificial interference (for example, in conflict conditions [1, 5, 6]). It is known [12, 18–20] that, depending on the BOMS topology, the observation conditions, and the chosen numerical optimization algorithm, the task of determining the target coordinates can lead to both correct and incorrect results, i.e., to obtaining poor bearing-only estimates of the target location. An attempt to take into account these factors leads to the formulation of the following tasks: the search for BOMS topologies that ensure maximization of the correct estimates of the target coordinates for a certain area; relocation of BOMS points in order to minimize the impact of artificial interference and maximize the time of the successful operation of the BOMS. In [21, 22], a game-theoretic method is presented for solving the problem of a joint search and surveillance by a group of unmanned aerial vehicles (UAVs) in a certain area. A multiplayer potential game with a limited set of actions is used. Motion is controlled via binary log-linear learning, which provides the optimal coverage of the studied region [23, 24]. Adaptation of the specified game-theoretic approach, taking into account the peculiarities of measuring and processing signals by BOMS points, consists of determining the type of potential function (global utility function). A number of criteria are proposed based on which global and individual utility functions are formed. A comparative analysis of the solutions obtained for some combinations of criteria was carried out. The task of moving BOMS points is formulated in the form of a model of confrontation between the

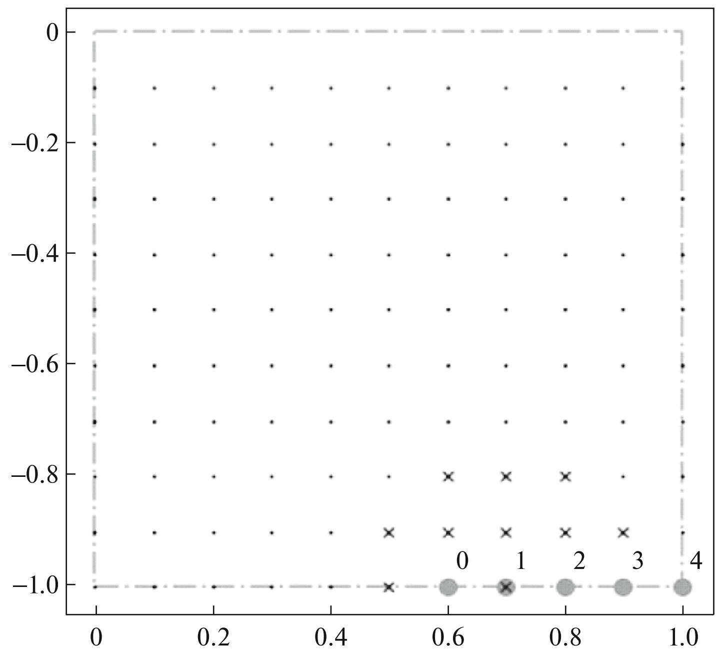

Fig. 1. Positions for moving agent no. 1 at the current step.

observer and the enemy, where the observer controls the topology of the BOMS, and the enemy creates interference in order to disrupt the operation of the BOMS. The enemy has a geographically distributed jamming system (JS), with the help of which it generates a jamming signal. An observer, using a CVM to process measurements, has the ability to detect BOMS points falling within the coverage area of the JS interference signal. An algorithm for countering the actions of the enemy by an observer is proposed. The presented simulation modeling results confirm the practical effectiveness of the counteraction algorithm and make it possible to estimate the operating time of the BOMS.

1. SEARCH FOR THE LOCATION OF BOMS POINTS

Assume that $\{P_n\}_{n=1}^{N} = \left\{[x_n^p, y_n^p]\right\}_{n=1}^{N}$ are BOMS measurement points, where $P_n \in \mathbf{P}$, $\mathbf{P} = \{[x, y] : 0 \le x \le 1, -1 \le y \le 0\}$. The workspace $\mathbf{S}$ is the region in which we expect targets to appear. The task is to find a placement of BOMS points that will ensure the correct observation of targets located in the workspace. Using the notation from [21], we define game $G(I, A, \{U_i, i \in I\})$, where $I = \{\overline{1, N}\}$ is the set of players (agents), $A = A_1 \times A_2 \times \ldots \times A_N$ is the set of joint actions of the agents, and A_i is the set of actions available to the ith agent, and $\{U_i, i \in I\}$ is the set of utility functions, where $U_i : A_i \to \mathbf{R}$ is the utility function of the ith agent. Vector $a = (a_1, a_2, \ldots, a_N)$ consists of the collective actions of agents (observation points), we will write $a = (a_i, a_{-i})$, where $a = (a_i, a_{-i})$, where a_i is the action of the ith point, and a_{-i} are the actions of the remaining points, excluding the ith point. The problem is solved in a discrete variant; to do this, we cover the areas where the points and targets are placed with a grid: $\mathbf{P}_m$ are the grid nodes on $\mathbf{P}$ and $\mathbf{S}_k$ are the grid nodes on $\mathbf{S}$. The set of actions $C_{a_i(t-1)}$ (positions for movement) available to the ith agent at time t depends on its current position and is selected from its neighborhood. The radius of the neighborhood is determined by the constant r_C, which characterizes the agent's ability to move in one step of the game. In this case, $a_i(t-1) \in C_{a_i(t-1)}$, i.e., the agent can choose an action that will result in him staying in his current position. For example, in Fig. 1 the available positions for movement of agent no. 1 are marked for the parameter $r_C = 0.25$ with marker $\times$.

Thus, the measurement points are considered as agents, and the agents interact and, depending on their capabilities and the environment, form a set of available actions. As a result of the exchange of information, agents agree on actions that ensure the desired state of the entire group.

In [20], the calculation of the uncertainty ellipses of the bearing-only method for two measurement points with fixed errors in angular measurements is given. Based on the calculations obtained, it was concluded that the measurement accuracy is highest if the angle of intersection of the position's lines is sufficiently close to a straight line, and noticeably decreases if the position lines intersect at acute angles. Position lines mean the geometric locus of points of possible location of the radiation source. It is clear that with restrictions on the area of location of BOMS points, it is not possible to obtain the best or close to the best observation conditions for all purposes of the workspace. The task is to find a placement of BOMS points that provides the specified observation conditions for the largest number of targets in the workspace. The search for the location of BOMS points is based on two criteria: maximizing the distances between points and minimizing the cosine of the angle between bearings.

Taking into account these criteria, the global utility function will take the form

$$\Phi(a) = \Phi(\mu_1, \mu_2, \ldots, \mu_N) = \min_{i,j \in \overline{1,N}} (\|\mu_i - \mu_j\|) \sum_{i,j \in \overline{1,N}, g \in \mathbf{S}_k} [1 - f(\mu_i, \mu_j, g)], \tag{1.1}$$

where $\mu_1, \mu_2, \ldots, \mu_N \in \mathbf{P}_m$ are the positions to which BOMS points are moved as a result of actions $a_1, a_2, \ldots, a_N$, $\mathbf{P}_m$ are grid nodes on $\mathbf{P}$, $\mathbf{S}_k$ are grid nodes on $\mathbf{S}$,

$$f(\mu_i, \mu_j, g) = \left| \frac{\|g - \mu_i\|^2 + \|g - \mu_j\|^2 - \|\mu_i - \mu_j\|^2}{2\|g - \mu_i\|\|g - \mu_j\|} \right|$$

is the module of the cosine of the angle between the target bearings g from positions μ_i and μ_j (cosine theorem). In (1.1) the factor

$$\min_{i,j \in \overline{1,N}} (\|\mu_i - \mu_j\|)$$

meets the criterion of maximizing distances between BOMS points; the value of the utility function will increase as the minimum distance between the BOMS points increases. The quantity $[1 - f(\mu_i, \mu_j, g)]$ is responsible for the target observation conditions for the pair of points i, j and varies in the range from 0 to 1. The location of a pair of points that ensures that the angle between the bearings is close to a right angle for the largest number of targets in the workspace corresponds to the maximum value of the sum in (1.1). At the same time, the individual utility function for the ith point takes the form

$$U_i(a_i, a_{-i}) = \Phi(\mu_1, \mu_2, \ldots, \mu_N) - \Phi(\mu_1, \mu_2, \ldots \mu_{i-1}, \mu_{i+1}, \ldots, \mu_N). \tag{1.2}$$

Each agent for positions $\hat{a}_i \in C_{a_i(t-1)}$ calculates the probabilities in accordance with the binary log-linear learning algorithm [21]:

$$\begin{cases} P(a_i(t) = a_i(t-1)) = \dfrac{\exp\left(U_i(a(t-1))/\tau\right)}{\exp\left(U_i(a(t-1))/\tau\right) + \exp\left(U_i(\hat{a}_i, a_{-i}(t-1))/\tau\right)}, \\ P(a_i(t) = \hat{a}_i) = \dfrac{\exp\left(U_i(\hat{a}_i, a_{-i}(t-1))/\tau\right)}{\exp\left(U_i(a(t-1))/\tau\right) + \exp\left(U_i(\hat{a}_i, a_{-i}(t-1))/\tau\right)}, \end{cases} \tag{1.3}$$

where the parameter τ is used to take into account noise by the model and characterizes the probability of the agent choosing an incorrect action [21, 23]. When calculating the probabilities by the ith agent, we assume that the remaining agents do not change positions.

A variant of the global utility function using only the criterion of maximizing distances between points is also considered:

$$\Phi(a) = \Phi(\mu_1, \mu_2, \ldots, \mu_N) = \min_{i,j \in \overline{1,N}} (\|\mu_i - \mu_j\|) \sum_{i,j \in \overline{1,N}} [\|\mu_i - \mu_j\|]. \tag{1.4}$$

Here the second multiplier will increase as the distance between any two points increases, and the multiplier

$$\min_{i,j \in \overline{1,N}} (\|\mu_i - \mu_j\|)$$

carries out compensation so that the growth of the utility function is not achieved only by increasing the distance of any one point from the rest.

Simulation modeling was carried out based on three motion control algorithms.

Algorithm A_1. The choice of the agent to move is random. The final action a_i^* is also selected randomly from a variety of available actions $\hat{a}_i \in C_{a_i(t-1)}$, and the probabilities are calculated for it using formula (1.3). The choice of agents and their actions is made with equal probability. If $P(a_i(t) = \hat{a}_i) > P(a_i(t) = a_i(t-1))$, then the agent performs the selected action, otherwise he remains in place.

Algorithm A_2. Each agent calculates probabilities using formula (1.3) for each available action $\hat{a}_i \in C_{a_i(t-1)}$ by comparing probabilities and selects the best action:

$$a_i^{\max} = \arg\max_{C_{a_i(t-1)}} \left(\{ P(a_i(t) = \hat{a}_i) \,|\, P(a_i(t) = \hat{a}_i) > P(a_i(t) = a_i(t-1)) \} \right).$$

If there are several such actions, then the final action is selected randomly. Then the best actions of the agents are ranked $\{a_i^{\max}, i \in I\}$. The move is made by the agent who proposed the best action among all agents, i.e., the agent with the index

$$i^* = \arg\max_{i \in I} [P(a_i(t) = a_i^{\max})].$$

Algorithm A_3. The choice of an agent to move is random. The final action a_i^* is selected from the available actions $\hat{a}_i \in C_{a_i(t-1)}$ by comparing probabilities.

Algorithm-based simulation results A_1, A_2, and A_3 using global utility functions (1.1) and (1.4) are given in Section 3.

2. THE TASK OF CONFRONTATION BETWEEN THE OBSERVER AND THE ENEMY

Assume $\{P_n\}_{n=1}^N = \left\{[x_n^p, y_n^p]\right\}_{n=1}^N$ are the BOMS measurement points, where $P_n \in \mathbf{P}$, $\mathbf{P} = \{[x, y] : 0 \le x \le 1, -1 \le y \le 0\}$. $\{S_m\}_{m=1}^M = \{[x_m^s, y_m^s]\}_{m=1}^M$ are the JS points where $S_m \in \mathbf{S}$, $\mathbf{S} = \{[x, y] : 0 \le x \le 1, 0 \le y \le 1\}$, ρ is the height of the interference triangle, γ is half the angle at the vertex of the interference triangle, $\alpha_m \in [-\pi, \pi]$ is the angle of rotation of the interference sector of the mth point (the angle is calculated from the negative direction of the x axis). Angle γ and range ρ are the characteristics of the JS point (we will assume that the characteristics of all points are the same). The angle characterizes the sector in which a point can provide interference of the given level, and the range is the maximum distance from a point within the sector for which the given level of emitted interference is also maintained. We assume that the interference area is an isosceles triangle, one of the vertices of which coincides with the position of the corresponding point, and the coordinates of the two remaining vertices are (Fig. 2)

$$\{B_m\}_{m=1}^M = \left\{[x_m^s - \rho_\gamma \sin(\alpha_m + \gamma), y_m^s - \rho_\gamma \cos(\alpha_m + \gamma)]\right\}_{m=1}^M,$$

$$\{C_m\}_{m=1}^M = \left\{[x_m^s - \rho_\gamma \sin(\alpha_m - \gamma), y_m^s - \rho_\gamma \cos(\alpha_m - \gamma)]\right\}_{m=1}^M, \quad \rho_\gamma = \rho/\cos(\gamma).$$

Assume $p = [x_p, y_p]$ — dot, $[a, b]$ is a line segment ($a = [x_a, y_a]$ and $b = [x_b, y_b]$), and ABC is a triangle ($A = [x_A, y_A]$, $B = [x_B, y_B]$, and $C = [x_C, y_C]$). Then

$$d^{[\,]}(p, ab) = \begin{cases} \sqrt{(x_a - x_p + (x_b - x_a)t)^2 + (y_a - y_p + (y_b - y_a)t)^2}, & 0 < t < 1, \\ \sqrt{(x_a - x_p)^2 + (y_a - y_p)^2}, & t \le 0, \\ \sqrt{(x_b - x_p)^2 + (y_b - y_p)^2}, & t \ge 1 \end{cases}$$

is the distance from point p to the segment $[a, b]$, where

$$t = \frac{(x_p - x_a)(x_b - x_a) + (y_p - y_a)(y_b - y_a)}{(x_b - x_a)^2 + (y_b - y_a)^2},$$

$$d^{\Delta}(p, ABC) = \begin{cases} \left\| -(d^{[\,]}(p, AB), d^{[\,]}(p, BC), d^{[\,]}(p, CA)) \right\|_\infty, S_{ABC} \ne S_{ABp} + S_{BCp} + S_{CAp}, \\ -\left\| -(d^{[\,]}(p, AB), d^{[\,]}(p, BC), d^{[\,]}(p, CA)) \right\|_\infty, S_{ABC} = S_{ABp} + S_{BCp} + S_{CAp} \end{cases}$$

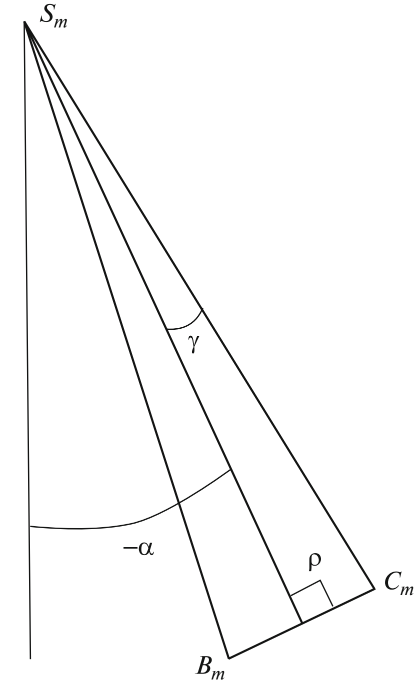

Fig. 2. Geometric representation of the interference zone of a JS point.

is the distance from the point to the triangle, $\|\bullet\|_\infty = \max_i |x_i|$, and S_{ABC}, S_{ABp}, S_{BCp}, and S_{CAp} are areas of the corresponding triangles. The entered distance from the point to the triangle is the minimum distance to points lying on the segments that form the triangle. The distance is positive for points outside the triangle, negative for points inside the triangle, and zero for points on its boundary.

As applied to our problem, the condition $d^\Delta(P_n, S_m B_m C_m) \le 0$ means that the measurement point P_n is in the interference zone of the point S_m. We will assume that getting into the specified area of the measurement point with probability 1 leads to unreliable measurements. Let $\mathbf{K} = \{n : \exists m, d^\Delta(P_n, S_m B_m C_m) \le 0\}$ be the set of numbers of measurement points that are located inside at least one interference zone; therefore, $|\mathbf{K}|$ is the number of measurement points located in the interference zone, and $N - |\mathbf{K}|$ is the number of working measurement points. The observer's problem for the case of a fixed enemy configuration can be written as follows:

$$\begin{cases} J(u_p, u_s) = |N - \mathbf{K}| \to \max\limits_{u_p}, \\ |\mathbf{K}| < \lfloor N/2 \rfloor + 1, \\ \forall i, j \in \overline{1, N} : \|P_i - P_j\| > B_{\min}, \\ \forall i, j, k \in \overline{1, N} : \dfrac{y_k^p - y_i^p}{y_j^p - y_i^p} \ne \dfrac{x_k^p - x_i^p}{x_j^p - x_i^p}, \end{cases} \tag{2.1}$$

where $u_p = [P_1 \ldots P_N]$, $u_s = [\alpha_1, S_1 \ldots \alpha_M, S_M]$, $\lfloor \bullet \rfloor$ is rounding down to the nearest whole number, and $\|\bullet\|$ is the Euclidean norm. Here fulfillment of the condition $|\mathbf{K}| < \lfloor N/2 \rfloor + 1$ provides the number of working measurement points necessary for the functioning of the BOMS (more than half of the total number). The remaining two restrictions are requirements for the BOMS topology: $B_{\min}$ is the minimum permissible distance between BOMS points; no three BOMS measurement points should lie on the same straight line. The value of the target function $J(u_p, u_s)$ represents the number of working measurement points. The observer maximizes this number subject to the specified constraints. The enemy's task is to minimize it:

$$\begin{cases} J(u_p, u_s) = |N - \mathbf{K}| \to \min\limits_{u_s}, \\ |\mathbf{K}| \geq \lfloor N/2 \rfloor + 1. \end{cases} \tag{2.2}$$

Here the minimization is carried out along the vector u_s for a fixed vector u_p.

The enemy has more parameters for minimization (the position of the interference point and the angle of rotation of the antenna); however, there is only one restriction. Condition $|\mathbf{K}| \geq \lfloor N/2 \rfloor + 1$ requires that for a value that provides the minimum of the target function, the number of BOMS measurement points that are interfered with exceeds half of their total number (the BOMS becomes inoperable only in this case).

Generalized Operating Algorithm

The moving measurement point of the enemy (MPE) carries out radar monitoring of the observer's location area.

The observer, using the BOMS, solves the problem of locating the MPE and, through the jamming point, prevents monitoring of his location area.

The MPE operator reports the fact of interference to the control center.

The enemy control center, having received a message about the fact of interference with its measurement point, makes a decision to interfere with the observer's BOMS.

The interference is carried out from a stationary position. The jamming sector is changed by rotating the emitting antenna without interrupting the jamming process.

If the current configuration of the JS did not allow the enemy to resume solving the radio monitoring problem, then it is necessary to change the configuration of the JS and perform jamming in accordance with the new configuration.

An observer, when assessing the location of an enemy measurement point, detects unreliable measurements and receives information about which BOMS points are within the coverage area of the JS.

Based on the results of detecting unreliable channels and the location of the JS points, the observer makes a decision to move the BOMS points from the interference zone.

End of the Game

The game ends if more than half of the BOMS measurement points are in the interference zone and the observer is not able to change the configuration, or there is no BOMS configuration in which less than half of the points are in the interference zone.

The game ends if less than half of the measurement points are in the jamming zone and the enemy either does not have the ability to change the configuration to suppress more than half of the points, or such a configuration does not exist.

Task: assess the operating time of the BOMS and JS. When solving the problem, we proceed from the following assumptions.

The number of BOMS points N and parameter values $B_{\min}$ are known.

The number of JSs M and the values of parameters ρ and γ are known.

Since the BOMS is a passive radar system, the enemy does not have access to an assessment of the location of BOMS points but only the location area is known.

The initial position of the interference points is not known to the observer, but the area where they are located is known.

An assessment of the location of JS points is available provided that the BOMS is operational.

When jamming more than half of the BOMS points, problems arise with identifying unreliable points. We will assume that after some time t_d (fine) the observer detects the fact that the BOMS (information from external systems) is inoperable without knowing which specific channels are inoperative.

The effectiveness of the applied JS configuration can only be determined indirectly. As a result of suppressing more than half of the BOMS points, solving the problem of locating a moving target will give a false target. The observer will stop influencing the true target, and by the cessation of such influence, the enemy can judge the effectiveness of the configuration used. This means that it takes time t_e to determine the effectiveness of the JS configuration.

The BOMS points can change their location by moving along a linear trajectory at a constant speed. During the movement, there is no solution to the problem of locating the MPE.

The JS points can change their location by moving along a linear path at a constant speed. There is no interference during movement.

We consider a discrete variant of the problem, where the BOMS and JS points are located on nodes $\mathbf{P}_l$ of the grid on $\mathbf{P}$ and nodes $\mathbf{S}_k$ on grids on $\mathbf{S}$, respectively.

Assume N, M, $B_{\min}$, ρ, and γ are fixed values. Solving a problem can be divided into stages.

1. Given the value $B_{\min}$, form set $\mathbf{G}$ of acceptable locations of BOMS points in all possible combinations N of the positions on the grid $\mathbf{P}_l$.

2. We form a set of locations of jamming devices (without taking into account the angle of rotation of the antenna). Based on the resulting set, we select the effective locations of points $\mathbf{H}$. The effectiveness criterion is the set of $\mathbf{G}^h \subset \mathbf{G}, h \in \mathbf{H}$. The BOMS belongs to the set $\mathbf{G}^h$ in the event that its performance may be impaired due to the location of interference points h by implementing any combination of rotation of the radiating antennas. In the process of forming the set of effective configurations of jamming devices $\mathbf{H}$, configurations $h \in \mathbf{H}$, for which $\exists g \in \mathbf{H}$: $\mathbf{G}^h \subset \mathbf{G}^g$, are excluded. Note that the interference configuration h, apart from the set $\mathbf{G}^h$, characterizes the number of combinations $|\mathbf{A}|$ of rotations of the radiating antennas, through the implementation of which the suppression of all the BOMS from $\mathbf{G}^h$ is realized; here $\mathbf{A}$ is the set of effective combinations of antenna rotation angles. Elements of the set $\mathbf{H}$ have the following property: $\forall h_1, h_2 \in \mathbf{H} : \mathbf{G}^{h_1} \Delta \mathbf{G}^{h_2} \neq \varnothing$, where Δ is the symmetric difference of sets

3. Analysis of the resulting sets $\mathbf{G}$ and $\mathbf{H}$. The following conditions are possible.

3.1. There is at least one acceptable arrangement of BOMS points that remains operational for any location of interference points: $\exists g \in \mathbf{G}$: $\forall h \in \mathbf{H}, g \notin \mathbf{G}^h$ is an interference-resistant configuration.

3.2. There is at least one arrangement of interference points for which, by implementing any combination of rotation of the radiating antennas, any BOMS can be disabled: $\exists h \in \mathbf{H} : \forall g \in \mathbf{G}, g \in \mathbf{G}^h$.

3.3. The conditions of subclauses 3.1 and 3.2 are not met.

4. Finding a solution to the adversarial problem taking into account the results of the analysis of sets $\mathbf{G}$ and $\mathbf{H}$.

Under condition 3.1, the observer has a win-win strategy. Having chosen such a configuration, the observer retains its functionality regardless of any interference configurations, i.e., solves the location problem. If any of the BOMS points enter the interference zone, this may slightly reduce the accuracy of location determination. In this case, the observer has the opportunity to change his position to a more advantageous one (if there are several configurations that are resistant to interference). At the same time, for the enemy, failure to solve the location determination problem implies the suppression of more than half of the BOMS points. When the observer uses only stable configurations, this situation is impossible. The enemy has no information about the effectiveness of a particular jamming configuration; i.e., he cannot even influence the accuracy of the observer's location. In this situation, there is no point for the enemy to start the game, since he will not be able, even temporarily, to interfere with the solution of the location determination problem by the observer. By solving this problem, the observer makes it impossible for the enemy to carry out radio monitoring. In conditions 3.2, the enemy, using a single location of jamming points, can disable any BOMS configuration by choosing a combination of rotation of the radiating antennas. Considering the short time required to implement antenna rotation compared to moving points in this situation, there is no point in starting the game for an observer, since the BOMS points will move most of the time and will not be able to counteract enemy radio monitoring. This means that conditions 3.1 and 3.2 allow us to answer the question by analyzing the parameters and the number of means of the opponent and the observer before the start of the game: "Does it make sense to start the game with the current composition of resources?"

Under condition 3.3, both the enemy and the observer must move points to solve their problems, since the observer does not have a BOMS configuration that is resistant to interference, and the enemy does not have a configuration that can disable any BOMS. The enemy based on multiple effective configurations $\mathbf{H}$ looks for those combinations of configurations (pairs, triples, etc.), whose implementation allows dis-

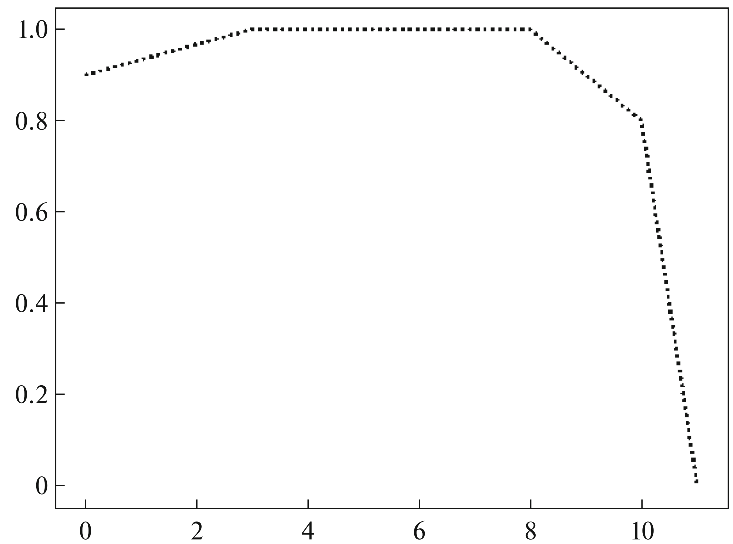

Fig. 3. Performance curve.

abling any BOMS from **G**. He selects the combination that provides the minimum transition time between configurations. He applies configurations from the selected combination until the operation of the BOMS is disrupted. The observer moves the BOMS points to ensure operability. When moving the BOMS or JS points, in addition to the distance between positions, the following parameters should be taken into account: the time required to bring the BOMS point from the working to the transport state, i.e., preparation for movement, speed of movement of a BOMS point, time to bring a BOMS point from a transport state to a working state, and performance of service personnel. The performance curve of the staff of the point may look like in Fig. 3. Since various items can be used as part of one BOMS/JS, due to their characteristics, the above values may differ significantly. The performance curve takes into account the skill of the crew and the effect of fatigue on performance depending on the time spent moving the point. The time of moving point p from position i to position j is determined by the formula $t_{p,i,j} = \rho_{i,j} v_p + (t_i + t_j) E_p(c_p)$, where $\rho_{i,j}$ is the distance between points, v_p is the speed of moving the point, t_i is the time to prepare the point for movement, t_j is the time to prepare the point for work, and $E_p(\bullet)$ is the function of the station crew's performance. Note that the observer solves the problem of finding a new closest BOMS position belonging to the set of safe configurations, taking into account the characteristics of each agent, based on the criterion of minimizing the value $t_{p,i,j}$. The enemy does not have the task of searching for the nearest position, since he must implement all configurations of the combination.

The sequence of actions during confrontation is presented in Fig. 4. The systems of the observer and the enemy can be in two states: the system solves the tasks assigned to it and the operation of the system is disrupted. While the enemy is moving points and rotating antennas, the observer system successfully counters the solution of the radio monitoring problem, and vice versa, while the observer is moving points or the BOMS is inoperative, the enemy solves the problem of radio monitoring. Thus, when calculating the time the system remains in each state, in addition to the time required to move points, it must be taken into account that the enemy needs time to rotate the antennas and determine the effectiveness of the implemented configuration. We believe that the time spent on implementing a specific combination of antenna rotation angles is taken into account in time t_e, and if the BOMS becomes inoperable, the observer will face a penalty time of t_d and only move after it has elapsed.

3. MODELING THE SEARCH FOR THE LOCATION OF BOMS POINTS

The simulation was carried out for the BOMS of five measurement points. The initial location of the points is shown in Fig. 1, $r_C = 0.2$, $\tau = 0.1$. The global utility functions (1.1) and (1.4) and workspaces $\mathbf{S}_1 = \{[x, y] : 0 \le x \le 1, 0 \le y \le 1\}$ and $\mathbf{S}_2 = \{[x, y] : -1 \le x \le 2, -2 \le y \le 1\} - \{[x, y] : 0 \le x \le 1, -1 \le y \le 0\}$ were used. Note that for the workspace $\mathbf{S}_1$ and the global utility function (1.1), the initial position of the

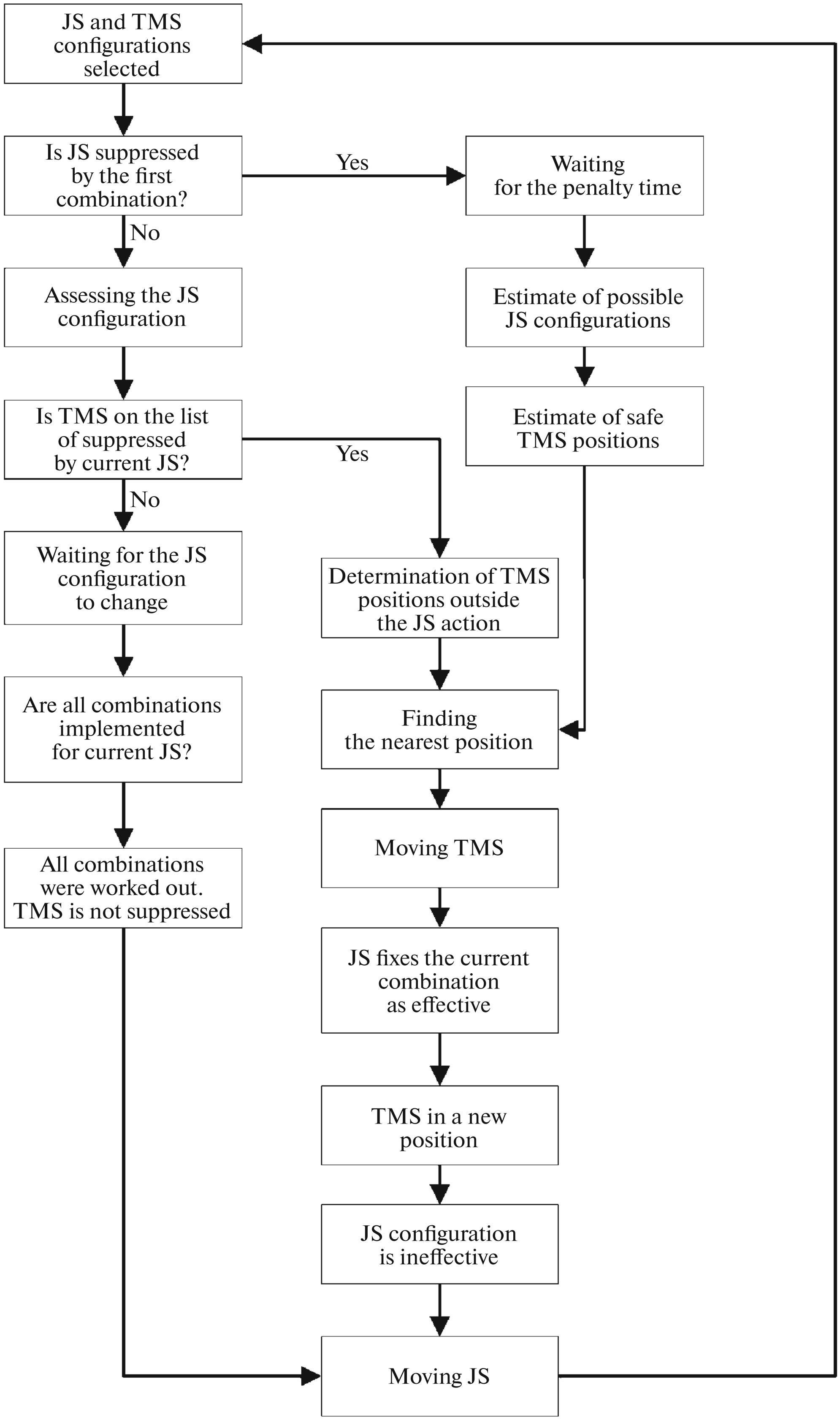

Fig. 4. Counteraction algorithm.

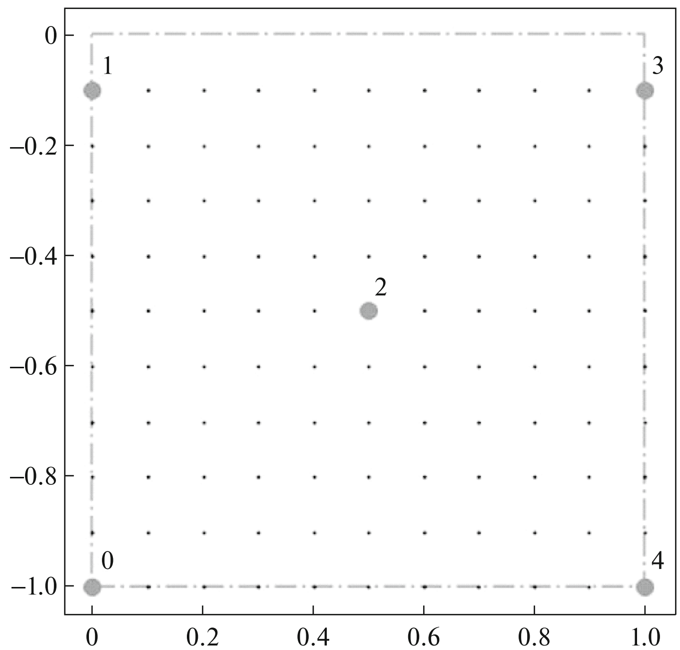

Fig. 5. Final placement of TMS points.

points has a significant impact on the final solution obtained by algorithms A_1, A_2, and A_3. For example, the initial position of the points at the top of the placement area leads to a solution corresponding to a local maximum. The criterion based on the angle between bearings, at positions close to the workspace (the upper part of the placement area), takes values close to the maximum, and the criterion based on the distances between points from the current position cannot compensate its influence. When using the workspace $\mathbf{S}_2$, the application of algorithms A_1 and A_3 also does not provide a stable solution. In the process of moving, such placements of points arise at which the global utility function (1.1) has a local maximum and at further iterations none of the points changes its position, since any movement reduces the value of individual utility (1.2). Consequently, a group of agents reaches an equilibrium state, but the achieved equilibrium does not always provide the maximum of the global utility function. Application of the algorithm A_2 for $\mathbf{S}_2$ allows us to obtain a stable solution at which the utility function reaches a global maximum. At the same time, the algorithm A_2 requires 25 iterations to find a solution, and the final placement is presented in Fig. 5. Note that in the final solution the points are placed at the maximum possible distance from each other. Given this observation, we consider a version of the global utility function (1.4). When using the workspace $\mathbf{S}_1$ and global utility functions (1.4), algorithms A_1 and A_3 also do not provide a stable solution, and the use of algorithm A_2 allows us to obtain a solution at which (1.4) reaches a global maximum, regardless of the initial position of the points. Using the algorithm A_2 for workspace $\mathbf{S}_2$ using function (1.4) also provides a solution at which utility reaches a global maximum. At the same time, algorithm A_2 takes 20 iterations to find a solution. The trajectories of the movement of the BOMS points in the process of searching for a solution are presented in Fig. 6. Note that achieving a global maximum of functions (1.1) and (1.4) in the workspace $\mathbf{S}_2$ is ensured by the placement shown in Fig. 5. The same placement is a solution to the problem with the utility function (1.4) and the workspace $\mathbf{S}_1$, and the placement ensuring the achievement of the global maximum (1.1) for the workspace $\mathbf{S}_1$ is close to the above placement (points 0 and 4 have an ordinate of 0.9). Thus, the solution found by algorithm A_2 using function (1.4) for the workspace $\mathbf{S}_1$ can be used as an initial approximation of the position of points when solving the problem with the global utility function (1.1).

4. SIMULATION MODELING OF CONFRONTATION BETWEEN OBSERVER AND ENEMY

Let us consider a situation where the observer's BOMS consists of five measurement points, and the enemy has two jamming points. In this case, $\rho = 1.0$, $\gamma = 12°$, $B_{\min} = 0.5$, and $\mathbf{P}_l$ and $\mathbf{S}_k$ contain 90 nodes,

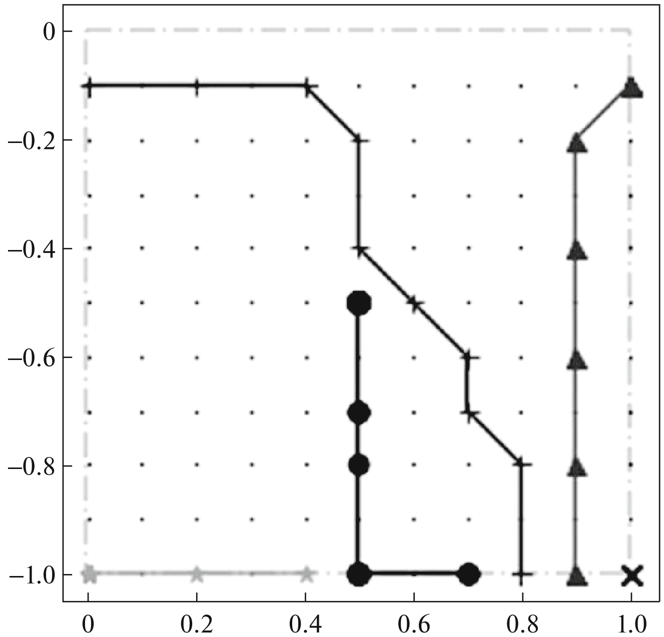

Fig. 6. Trajectories of movement of TMS points for A_2, (1.4).

evenly covering areas **P** and **S**, respectively. The range of antenna rotation angles for the selected location of areas **P** and **S** takes the following form: $[-\pi/2 - \gamma, \pi/2 + \gamma]$. The simulation uses angle values at 20 grid points, uniformly covering the specified range.

The set **G** is formed taking into account $B_{\min}$ by the Monte Carlo method. A choice was made randomly (uniform distribution law) from all possible combinations of N positions on the grid $\mathbf{P}_l$ (total C_{90}^5) of measurement positions of the BOMS. Configurations that do not satisfy the condition $\forall i, j \in \overline{1, N}\ \|P_i - P_j\| > B_{\min}$, dropped out. For the BOMS that passed the screening, the areas of triangles formed by all possible triplets of measurement positions were calculated: if $\exists i, j, k \in \overline{1, N} : S_{P_iP_jP_k} < S_{\min}$, then the BOMS is also eliminated, where $S_{P_iP_jP_k}$ is the area of the triangle formed by points i, j, k, and $S_{\min}$ is the area of an equilateral triangle with side $B_{\min}$. As a result of taking into account the specified restrictions on the location of the BOMS measurement points, 708 configurations passed the screening. The formation of effective configurations of jamming devices occurs on the basis of the resulting set **G**. The location of jamming devices is selected randomly (uniform distribution law) from all possible combinations of M positions on the grid $\mathbf{S}_k$ (total C_{90}^2). For two interference points, the number of possible combinations of angles with the selected grid is 400. Combinations have different efficiencies; thus, the set of possible combinations of rotation angles are also subject to a filtering procedure. Combinations whose implementation does not allow the functionality of any BOMS of the set **G** to be disrupted, as well as duplicating ones (in the sense of a set of suppressed BOMS), are eliminated. Thus, depending on the location of suppression points relative to area **P**, the number of effective combinations **A** will be different. If $\exists g \in \mathbf{H} : \mathbf{G}^h = \mathbf{G}^g$, then the configuration for which the parameter value $|\mathbf{A}|$ is larger is excluded from the set **H**. The resulting set **H** consists of 183 configurations such that $\forall h_1, h_2 \in \mathbf{H}$: $\mathbf{G}^{h_1} \Delta \mathbf{G}^{h_2} \neq \varnothing$, where Δ is the symmetric difference of sets. This means that the sets of suppressed BOMS of each configuration contain at least one BOMS that cannot be suppressed using other configurations. Note that with the selected parameter values, the situation from condition 3.3 is realized, i.e., variants $\exists g \in \mathbf{G} : \forall h \in \mathbf{H}, g \notin \mathbf{G}^h$ and $\exists h \in \mathbf{H} : \forall g \in \mathbf{G}, g \in \mathbf{G}^h$ are not fulfilled. The analysis of various combinations of using effective configurations revealed that in the current situation, to suppress any BOMS from **G**, at least three effective configurations must be used. Based on 183 effective configurations of the suppression of resources, 35 such triplets were obtained. The enemy, sequentially applying configurations from the specified sets, can disable any BOMS from the set **G**. It is clear that these sets of JS configurations suppress BOMS configura-

Table 1. Characteristics of BOMS and JS points

Points	v_p, units/h	$(t_i + t_j)$	t_d	t_e
		h		
BOMS 1–5	1.0	0.5	0.5	–
JS 1.2	2.5	0.3	–	0.015

Table 2. Simulation results

JS configuration set number	Average operating time, h		Number of wins	
	JS	BOMS	enemy	observer
8	15.2	10.8	30	970
6	15.0	10.8	61	939
14	14.9	10.7	121	879
18	16.4	10.6	134	866
7	15.2	10.5	159	841
20	15.2	10.5	164	836
27	14.6	10.3	185	815
13	15.8	10.4	192	808
12	15.4	10.2	231	769
19	16.2	10.3	272	728
25	16.0	10.2	301	699
24	15.2	9.9	348	652
26	14.0	10.0	393	607
2	14.0	9.8	472	528
1	12.2	10.5	502	498
10	12.9	9.7	551	449
16	13.0	9.6	561	439
22	13.4	9.6	567	433
11	12.2	10.0	587	413
4	12.5	9.9	606	394
17	12.3	9.9	613	387
5	12.1	10.0	619	381
23	12.5	10.2	632	368
21	13.1	9.5	664	336
9	12.8	9.4	677	323
3	12.1	9.9	682	318
15	12.8	9.4	699	301

tions sequentially and changing BOMS configurations by an observer can prevent this. To answer the questions "for how long the observer's system prevents enemy radio monitoring" and "for how long the observer system remains idle," simulation modeling was carried out. The characteristics of the BOMS and JS points are presented in Table 1, and the performance curves are identical for all crews and are shown in Fig. 3.

The simulation was carried out as follows: for the selected set of effective configurations, the current configuration is randomly selected, and a combination of antenna rotation angles is also randomly selected for it. From the set of valid BOMS configurations, the current configuration is randomly selected. The step of the counteraction algorithm is carried out (Fig. 4) until the BOMS and JS points have the opportunity to move. In the process of solving the algorithm, the time of the successful operation of the enemy and observer systems is calculated. If, as a result of performing the steps of the algorithm, the BOMS/JS points need to be moved, and the corresponding points cannot carry out the movement, then we record the victory of the enemy/observer. 1000 repetitions of this experiment are carried out. The simulation results show that with the given characteristics of points and crews, a number of sets of JS configurations allow the enemy, at best, to win in 5–6% of cases and, on average, carry out radio monitoring of the observer's deployment area for about 8–9 hours, but the final victory in most cases remains with the observer.

By increasing the angle to $\gamma = 15°$ and leaving the other characteristics unchanged, we obtain that on the set **H** (200 effective configurations) there are 27 sets of two configurations, allowing us to suppress any BOMS from the set **G**. The simulation results are presented in Table 2. Consequently, the enemy can choose a number of sets, whose use allows him to win in 50–70% of cases, and even as a result of a loss, the average operating time of the control system increased from 8–9 to 14–15 hours.

If together with $\gamma = 15°$ we increase the range of the control points to the value $\rho = 1.1$, then we find ourselves in the situation of condition 3.2, where the enemy has such effective configurations that they are capable of suppressing any BOMS from the set **G** without moving points. One such configuration performs suppression using 25 angle combinations. Taking into account the accepted value $t_e = 0.015$, any BOMS will be suppressed within a maximum of 22.5 minutes. In this case, the observer does not have safe positions where he could move the measurement points in advance.

CONCLUSIONS

The simulation shows that the proposed game-theoretic approach makes it possible to find the optimal (in the sense of the specified criteria) placement of BOMS points. The method allows agents to act in accordance with the characteristics of the environment and takes into account their ability to move. In this case, the information interaction of agents ensures the desired state of the entire group. Simulation modeling of the solution of the problem of confrontation between an observer and an enemy shows that the proposed approach allows us to estimate the operating time and the probability of victory of the observer (enemy) or indicate the insufficient means for counteraction. This assessment is also available in the case of different technical characteristics of points and/or levels of crew training, and also takes into account the performance of BOMS/JS crews.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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