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SYSTEMS THEORY AND GENERAL CONTROL THEORY

Synthesis of a Discontinuous Control Law for a Step-Down Voltage Converter

S. A. Kochetkov[a,*], O. S. Tkacheva[a,], and A. V. Utkin[a,***]**

[a] *Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, 117997 Russia*

**e-mail: kos@ipu.ru*

***e-mail: tkolga17@gmail.com*

****e-mail: utkin-av@rambler.ru*

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Abstract—This paper presents a nonlinear discontinuous control law that allows stabilizing the output voltage of a step-down voltage converter in conditions when the input voltage and load current are unknown. The main idea is based on the use of the so-called vortex algorithms, which ensure invariance with respect to external unmatched disturbances. The efficiency of the developed algorithms is shown by numerical simulation.

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INTRODUCTION

Voltage converters are widely used in technology as power supplies and voltage stabilizers [1–5]. With the development of modern technologies for generating electricity based on wind generators, solar panels, and tidal power plants, their evolution has received a new direction. The design of the voltage converter consists of a reactive energy storage device (inductive and capacitive elements) and a switching device. With the development of semiconductor technology, it is possible to eliminate mechanical switching devices and use semiconductor diodes, transistors, and thyristors with switching frequencies of up to several hundred kilohertz [1, 6].

The main problem related to the control of semiconductor voltage converters is the stabilization of the output voltage depending on the input voltage and variable load power consumption [1].

This article presents the problem of controlling the output voltage of a step-down converter under the specified conditions. It should be noted that the control input of the converter can only take two discrete values, which corresponds to the on/off state of the switching element. In addition, the movements of currents in different circuits in such devices have a multitemporal character. For these reasons, when synthesizing control algorithms, the theory of discontinuous control and the principles of motion separation can be used [7, 8].

The synthesis of a nonlinear discontinuous control law is considered. The developed control algorithm ensures stabilization of the output voltage when exposed to an unknown input voltage and output load current. The main idea is based on the so-called vortex algorithm, which ensures the property of invariance with respect to external unmatched disturbances. The theoretical results can be realized using modern pulse width modulation converters. The simulation results show the effectiveness of the presented algorithms.

The article is organized as follows. In Section 1, a mathematical model of the control object is introduced and the formulation of the problem is formalized. In Section 2 a nonlinear control law, which makes it possible to stabilize the output voltage under conditions of an unknown input voltage and output load current, is synthesized. In Section 3, the results of numerical simulation in the MATLAB/Simulink environment, demonstrating the performance of the proposed algorithms, are described.

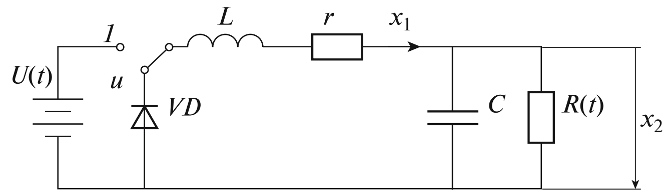

Fig. 1. Simplified step-down converter circuit.

1. MATHEMATICAL MODEL OF THE CONTROL OBJECT: PROBLEM STATEMENT

The main structural elements of a step-down converter are shown in Fig. 1, where L is the inductance of the converter, C is the capacitor, r is the electrical resistance of the inductance winding, $U(t)$ is the input voltage (in general, a function of time), x_1 is the current in the inductance winding, x_2 is the output voltage, $R(t)$ is the unknown variable value of the electrical resistance of the load, and VD is a snap-on diode, which prevents the capacitor from discharging through the inductor and ensures current only in the direction shown in Fig. 1.

The mathematical model of the converter is described by the following system of differential equations [9]:

$$\dot{x}_1 = -\frac{r}{L}x_1 - \frac{1}{L}x_2 + \frac{U(t)}{L}u, \quad \dot{x}_2 = \frac{1}{C}x_1 - \frac{1}{R(t)C}x_2, \tag{1.1}$$

where the control $u(t)$ can take values from the discrete set $\{0, 1\}$.

In the article, the following assumptions are made for the control object.

1. For the unknown load resistance function and its first two derivatives, the following restrictions apply:

$$R(t) \geq R_0, \quad |\dot{R}(t)| \leq R_1, \quad |\ddot{R}(t)| \leq R_2, \tag{1.2}$$

where hereinafter $|\cdot|$ means the absolute value of a number, and R_1 and R_2 are known positive constants.

2. For the input and desired output voltages, the following inequalities are satisfied:

$$U_0 \leq U \leq U_1, \quad |\dot{U}(t)| \leq \bar{U}, \quad x_{2d} < \frac{U_0}{1 + (r/R_0)}, \tag{1.3}$$

where U_0, U_1, and $\bar{U}$ are known positive constants.

3. In addition to reverse current protection, there is a protection circuit that forcibly limits the value of the current in the converter coil, and for the variable $x_1(t)$, we can write the inequality:

$$0 \leq x_1 \leq x_{1\max}, \quad x_{1\max} > \frac{x_{2d}}{R_0}. \tag{1.4}$$

Assuming that the variable $x_2(t)$ is available for measurement, the article sets the task of stabilizing the discrepancy (mismatch) in the output voltage:

$$\lim_{t\to\infty} |\bar{x}_2(t)| = 0, \quad \bar{x}_2(t) = x_2(t) - x_{2d}, \tag{1.5}$$

where $\bar{x}_2(t)$ is the voltage mismatch and $x_{2d} = \text{const} > 0$ is the desired output voltage value.

2. SYNTHESIS OF CONTROL ALGORITHM

The parameters of the semiconductor converter are chosen in such a way that in system (1.1) the movements are separated according to the convergence rates. Thus, the current in the inductor can be quickly changed to the desired values, in contrast to the output voltage of the capacitor, which is a fairly inert element designed to filter output voltage ripples. Due to these features, the problem posed can be solved by correspondingly changing the current in the inductance winding. It should be noted that this approach is used in connection with the problem of inconsistent disturbances [10].

According to (1.1) and (1.5), we can write the system equations with respect to errors:

$$\dot{\bar{x}}_2 = -\frac{\bar{x}_2}{R(t)C} + \frac{1}{C}x_1 - \frac{x_{2d}}{R(t)C}, \quad \dot{x}_1 = -\frac{r}{L}x_1 - \frac{1}{L}\bar{x}_2 - \frac{1}{L}x_{2d} + \frac{U}{L}u.$$

To further synthesize the control law, we consider new coordinates in which it is convenient to study the process at the maximum load. We introduce a new variable $\bar{x}_1 = \frac{1}{C}x_1 - \frac{x_{2d}}{R(t)C}$.

Substituting it into the last system, we obtain the following equations:

$$\begin{aligned}\dot{\bar{x}}_2 &= -\frac{\bar{x}_2}{R(t)C} + \bar{x}_1,\\ \dot{\bar{x}}_1 &= -\frac{r}{L}\bar{x}_1 - \frac{\bar{x}_2}{LC} - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC} + \frac{U}{LC}u + \xi(t),\end{aligned} \tag{2.1}$$

where

$$\xi(t) = \left(L\frac{\dot{R}}{R^2(t)} + \frac{r}{R_0} - \frac{r}{R(t)}\right)\frac{x_{2d}}{LC}.$$

To implement one of the variants of the vortex algorithm [11, 12], we select the control input in the form of a discontinuous function:

$$u = \frac{1}{2}[1 - \operatorname{sign}\bar{x}_2]. \tag{2.2}$$

The equations of the closed system, according to (2.1), (2.2), have the form

$$\begin{aligned}\dot{\bar{x}}_2 &= -\frac{\bar{x}_2}{R(t)C} + \bar{x}_1,\\ \dot{\bar{x}}_1 &= -\frac{r}{L}\bar{x}_1 - \frac{\bar{x}_2}{LC} - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC} + \frac{U}{2LC}[1 - \operatorname{sign}\bar{x}_2] + \xi(t).\end{aligned} \tag{2.3}$$

Theorem. *Let the parameters of the converter, load, and input and output voltage be chosen so that the following inequalities are satisfied*:

$$\begin{aligned} &M^- - \left(1 + \frac{1}{\alpha R_0 C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha} > 0,\\ &M^+ - \frac{\bar{U}}{\alpha LC} - \left(1 + \frac{1}{\alpha R_0 C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha} > 0,\\ &\alpha > \frac{1}{2\gamma R_0 C}\left(\frac{1}{\sqrt{LC}} - \gamma\right), \quad \frac{1}{LC} - \frac{r^2}{4L^2} > 0.\end{aligned} \tag{2.4}$$

Here

$$M^- = \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC}, \quad M^+ = \frac{U_0}{LC} - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC}, \quad \Sigma = \left(L\frac{R_1}{R_0^2} + \frac{r}{R_0}\right)\frac{x_{2d}}{LC},$$

$$\bar{\Sigma} = \left(L\frac{R_2 + 2R_1^2}{R_0^3} + \frac{R_1 r}{R_0^2}\right)\frac{x_{2d}}{LC}, \quad \alpha = \frac{r}{2L}, \quad \gamma = \sqrt{\frac{1}{LC} - \frac{r^2}{4L^2}}.$$

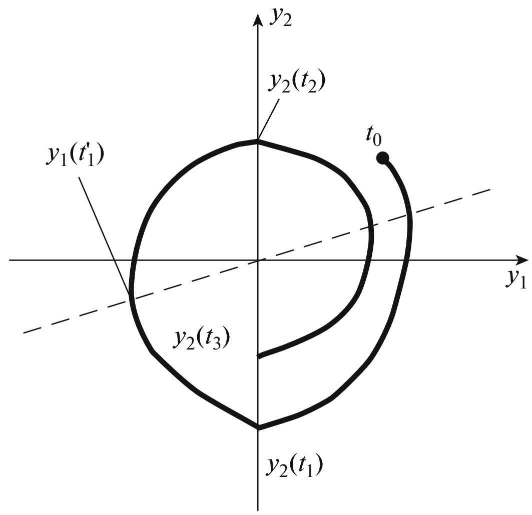

Fig. 2. Phase portrait of a closed system.

Then the variables of the closed-loop system (2.3) asymptotically tend to zero, which guarantees the solution of the posed problem (1.5).

Proof. Let us introduce a change of variables

$$y_1 = \gamma \bar{x}_2, \quad y_2 = \alpha \bar{x}_2 + \bar{x}_1, \tag{2.5}$$

with the help of which system (2.3) can be represented as

$$\begin{gathered}\dot{y}_1 = -\left(\frac{1}{R(t)C} + \alpha\right) y_1 + \gamma y_2, \\ \dot{y}_2 = -\left(\gamma + \frac{\alpha}{R(t)C\gamma}\right) y_1 - \alpha y_2 - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC} + \frac{U}{2LC}[1 - \operatorname{sign}(y_1)] + \xi(t).\end{gathered} \tag{2.6}$$

Using expressions (1.2), (2.1), we write down the restrictions for the disturbance $\xi(t)$ and its derivative:

$$|\xi(t)| \le \Sigma, \quad |\dot{\xi}(t)| \le \bar{\Sigma}.$$

According to the conditions of the theorem, $M^+ > \Sigma$ and $M^- > \Sigma$. Considering the equations of the closed system (2.6), we obtain the phase portrait shown in Fig. 2. Each half-plane of the graph (to the right and left of the vertical axis) corresponds to a different sign of variable y_1. It should be noted that the trajectories of the system cannot belong to the set $y_1(t) = 0$, since the conditions for the existence of a sliding mode are not satisfied on this surface [13, 14]. Having designated t_0 as the initial moment of time, without loss of generality in the proof, we present the case when $y_1(t_0) > 0$. Considering that the discontinuity points of the right-hand side of the differential equations (2.6) belong to a set of zero measure, its solution is understood in the sense of Carathéodory [15].

To analyze the convergence of variables of a closed-loop system, a method based on Lyapunov functions is used together with an analysis of the phase portrait of system (2.6). Let us introduce moments of time $t_i, i = \overline{1,\infty}$, such that $y_1(t_i) = 0$, and denote the interval between them as $\Delta_i = t_{i+1} - t_i$.

Let us first consider the movement in the first and fourth quadrants of the phase portrait (see Fig. 2). Let us define a positive semidefinite function

$$V_1 = M^- \frac{|y_1|}{\gamma} - \frac{\xi}{\gamma} y_1 + \frac{y_1^2}{2} + \frac{y_2^2}{2}, \quad y_1 > 0. \tag{2.7}$$

For its derivative we can write the following inequalities:

$$\dot{V}_1 = -\frac{M^-}{\gamma}\left(\alpha + \frac{1}{R(t)C}\right)|y_1| - \left(\alpha + \frac{1}{R(t)C}\right)\frac{\xi}{\gamma}y_1 - \frac{\dot{\xi}}{\gamma}y_1 - \left[\left(\alpha + \frac{1}{R(t)C}\right)y_1^2 + \frac{\alpha}{R(t)C\gamma}y_1y_2\right] - \alpha y_2^2$$
$$- \frac{\alpha M^-}{\gamma}|y_1| + \left(\alpha + \frac{1}{R_0C}\right)\frac{\Sigma}{\gamma}|y_1| + \frac{\bar{\Sigma}}{\gamma}|y_1| - y^{\mathrm{T}}Q(t)y \le -\bar{\alpha}_1|y_1| - \frac{1}{M^-}\lambda_{\min Q(t)}[y_1^2 + y_2^2], \tag{2.8}$$

where

$$\bar{\alpha}_1 = \frac{\alpha}{\gamma}\left(M^- - \left(1 + \frac{1}{\alpha R_0 C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha}\right), \quad y^{\mathrm{T}} = (y_1, y_2), \quad Q(t) = \begin{pmatrix} \alpha + \dfrac{1}{R(t)C} & \dfrac{\alpha}{2R(t)C\gamma} \\ \dfrac{\alpha}{2R(t)C\gamma} & \alpha \end{pmatrix},$$

and $\lambda_{\min Q(t)}$ is the minimal eigenvalue of the matrix $Q(t)$.

The expression for the minimal eigenvalue of the matrix $Q(t)$ is

$$\lambda_{\min Q(t)} = \alpha + \frac{1}{2\gamma R(t)C}(\gamma - \sqrt{\gamma^2 + \alpha^2}) = \alpha + \frac{1}{2\gamma R(t)C}\left(\gamma - \frac{1}{\sqrt{LC}}\right),$$

and the lower bound of the minimal eigenvalue

$$\lambda_{\min Q(t)} \ge \alpha - \frac{1}{2\gamma R_0 C}\left(\frac{1}{\sqrt{LC}} - \gamma\right) = \lambda_0. \tag{2.9}$$

According to the last of conditions (2.4) of the theorem and (2.9), the inequality

$$\lambda_{\min Q(t)} = \lambda_0 > 0$$

is guaranteed to be satisfied when system (2.6) moves in the first and fourth quadrants of the phase portrait (see Fig. 2).

According to (2.7), we write the following inequality:

$$V_1 \le |y_1|\frac{M^- + \Sigma}{\gamma} + \frac{1}{2}(y_1^2 + y_2^2) \le c_{01}(|y_1| + y_1^2 + y_2^2), \tag{2.10}$$

where

$$c_{01} = \max\left\{\frac{M^- + \Sigma}{\gamma}, \frac{1}{2}\right\}.$$

Using relations (2.9), (2.10), we can rewrite (2.8) as

$$\dot{V}_1 \le -\bar{\alpha}_1|y_1| - \lambda_0[y_1^2 + y_2^2] \le -c_{11}(|y_1| + y_1^2 + y_2^2) \le -\nu_1 V_1, \tag{2.11}$$

where $\nu_1 = \dfrac{c_{11}}{c_{01}}$ and $c_{11} = \min\{\bar{\alpha}_1, \lambda_0\}$.

To analyze the movement in the second and third quadrants (see Fig. 2), we consider a positive semidefinite function

$$V_2 = \bar{M}\frac{|y_1|}{\gamma} - \frac{\xi}{\gamma}y_1 + \frac{y_1^2}{2} + \frac{y_2^2}{2}, \quad y_1 < 0, \tag{2.12}$$

where

$$\bar{M} = \frac{U}{LC} - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC}.$$

By analogy, citing the case $y_1 < 0$, we obtain the derivative of the function V_2 due to the system taking into account (1.3), (2.9):

$$\dot{V}_2 = -\frac{\bar{M}}{\gamma}\left(\alpha + \frac{1}{R(t)C}\right)|y_1| - \bar{M}y_2 + \frac{\dot{U}}{LC\gamma}|y_1| - \left(\alpha + \frac{1}{R(t)C}\right)\frac{\xi}{\gamma}y_1 - y_2\xi - \frac{\dot{\xi}}{\gamma}y_1$$
$$+ y_1\left(-\left(\alpha + \frac{1}{R(t)C}\right)y_1 + \gamma y_2\right) + y_2\left(-\left(\gamma + \frac{\alpha}{R(t)C\gamma}\right)y_1 - \alpha y_2 + \bar{M} + \xi\right) \le -\frac{M^+}{\gamma}\left(\alpha + \frac{1}{R(t)C}\right)|y_1| \quad (2.13)$$
$$+ \frac{\bar{U}}{LC\gamma}|y_1| + \left(\alpha + \frac{1}{R_0C}\right)\frac{\Sigma}{\gamma}|y_1| + \frac{\bar{\Sigma}}{\gamma}|y_1| - y^{\mathrm{T}}Q(t)y \le -\bar{\alpha}_2|y_1| - \lambda_0[y_1^2 + y_2^2],$$

where

$$\bar{\alpha}_2 = \frac{\alpha}{\gamma}\left(M^+ - \frac{\bar{U}}{\alpha LC} - \left(1 + \frac{1}{\alpha R_0C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha}\right).$$

According to (1.3), (2.12), for V_2 we can write

$$V_2 \le \frac{|y_1|}{\gamma}(M_{\max} + \Sigma) + \frac{1}{2}(y_1^2 + y_2^2) \le c_{02}(|y_1| + y_1^2 + y_2^2), \quad (2.14)$$

where

$$M_{\max} = \frac{U_1}{LC} - \left(1 + \frac{r}{R_0}\right)\frac{x_{2d}}{LC}, \quad c_{02} = \max\left\{\frac{M_{\max} + \Sigma}{\gamma}, \frac{1}{2}\right\}.$$

Using expressions (2.15), relation (2.14) can be written as

$$\dot{V}_2 \le -\bar{\alpha}_2|y_1| - \lambda_0[y_1^2 + y_2^2] \le -c_{12}(|y_1| + y_1^2 + y_2^2) \le -\nu_2V_2, \quad (2.15)$$

where

$$\nu_2 = \frac{c_{12}}{c_{02}}, \quad c_{12} = \min\{\bar{\alpha}_2, \lambda_0\}.$$

Considering the phase portrait (see Fig. 2), using (2.10)–(2.11) and (2.14)–(2.15), we obtain the following estimates:

$$V_1(t_1) = \frac{y_2^2(t_1)}{2} \le V_1(t_0)e^{-\nu_1\Delta_0} \le c_0[|y_1(t_0)| + y_1^2(t_0) + y_2^2(t_0)]e^{-\nu_1\Delta_0} \Rightarrow y_2^2(t_1) \le Y_0e^{-\nu\Delta_0},$$

where $c_0 = \max\{c_{01}, c_{02}\} = c_{02}$, $\nu = \min\{\nu_1, \nu_2\}$, $\Delta_0 = t_1 - t_0$, and $Y_0 = 2c_0\,|y_1(t_0)| + y_1^2(t_0) + y_2^2(t_0)$.

After examining the phase portrait in the second quadrant using the function $V_2(t)$ from (2.12), (2.15), we write the majorant:

$$V_2(t_2) = \frac{y_2^2(t_2)}{2} \le V_2(t_1)e^{-\nu_2\Delta_1} \le \frac{y_2^2(t_1)}{2}e^{-\nu\Delta_1} \Rightarrow y_2^2(t_2) \le y_2^2(t_1)e^{-\nu\Delta_1} \le Y_0e^{-\nu(\Delta_0+\Delta_1)}.$$

Let us set the inequality, by analogy, at some point in time t_i:

$$y_2^2(t_i) \le Y_0e^{-\nu\sum_{k=0}^{i-1}\Delta_k}.$$

Taking into account the oscillatory nature of the transition process and the last relation, we can conclude that the variable $y_2(t)$ is limited by the majorant

$$|y_2(t)| \le \sqrt{Y_0}e^{-\frac{\nu}{2}(t-t_0)}, \quad t \ge t_0. \quad (2.16)$$

It is obvious that the fluctuations of the variable $|y_1(t)|$ reach the maximum provided $\dot{y}_1(t) = 0$ at moments in time t, for which

$$y_1(t) = \gamma\left(\frac{1}{R(t)C} + \alpha\right)^{-1}y_2(t).$$

Assume this equality holds at moments of time t_i' ($t_i < t_1' < t_{i+1}$) (see Fig. 2). Then for the value $y_1(t_i')$, taking into account (1.2), the following estimates are valid:

$$|y_1(t_i')| \le \frac{\gamma}{\alpha}|y_2(t_i)|.$$

Thus, taking into account expression (2.16), the amplitude of oscillations (maxima) of the variable $|y_1(t)|$ decay exponentially, and the variables of system (2.6) tend to zero as time tends to infinity:

$$\lim_{t\to\infty}|y_2(t)| = 0, \quad \lim_{t\to\infty}|y_1(t)| = 0.$$

From the last relations and expressions (2.5), it follows that

$$\lim_{t\to\infty}|\bar{x}_2(t)| = 0.$$

The theorem is proved.

Note that the transient process for a closed system (2.3) can occur, in the general case, at negative current values of x_1 through an inductor. The system's model does not take into account the physical restrictions that were provided for in assumption (1.4). According to these limitations, during the transient process, the current will be limited to a certain range, which is specified during the design. However, even if the trajectories of the system reach the specified limits, after a certain period of time they will fall into the region where the proof given above is valid. The transient curve in a real device in this case will be different, and the phase portrait shown in Fig. 2, will be cut off by the values included in inequality (1.4).

3. NUMERICAL MODELING

We consider the simulation results for the following parameters of the semiconductor converter: $L = 2 \times 10^{-5}$ Gn, $C = 3 \times 10^{-4}$ F, and $r = 0.2$ Ohm. The input voltage and limitations for it, according to (1.3), are

$$U(t) = 90 + 10\cos(10t)\ \text{V}, \quad U_0 = 80\ \text{V}, \quad U_1 = 100\ \text{V}, \quad \bar{U} = 100\ \text{V/s}.$$

The desired output voltage value $x_{2d} = 63$ V. The unknown load is modeled by the periodic function

$$R(t) = 6 - 4\sin(100t),\ \Omega.$$

According to the formulation of the problem, for this unknown function only the restrictions specified in (1.2) are known. By performing simple calculations, we can get

$$R_0 = 2\ \Omega, \quad R_1 = 400\ \Omega/\text{s}, \quad R_2 = 4\times 10^4\ \Omega/\text{s}^2. \tag{3.1}$$

For numerical modeling, according to inequality (1.4), the following physical restrictions on the current were chosen: $0 \le x_1 \le 35$ A. By calculating the values given in the theorem in accordance with the converter's parameters and the load function (3.1), the following constants can be obtained:

$$M^- = 1.155\times 10^{10}\ \text{V/(G F)}, \quad M^+ = 1.783\times 10^9\ \text{V/(G F)}, \quad \Sigma = 1.071\times 10^9\ \text{V/(G F)},$$
$$\bar{\Sigma} = 1.155\times 10^{10}\ \text{V/(G F s)}, \quad \gamma = 1.19\times 10^4\ \text{rad/s}, \quad \alpha = 5\times 10^3\ \text{s}^{-1}.$$

The validity of the parameters of the control law according to the conditions of the theorem can be checked by the following calculations:

$$M^- - \left(1 + \frac{1}{\alpha R_0 C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha} = 10^{10}\ \text{V/(G F)},$$
$$M^+ - \frac{\bar{U}}{\alpha L C} - \left(1 + \frac{1}{\alpha R_0 C}\right)\Sigma - \frac{\bar{\Sigma}}{\alpha} = 3.077\times 10^8\ \text{V/(G F)},$$
$$\alpha > \frac{1}{2\gamma R_0 C}\left(\frac{1}{\sqrt{LC}} - \gamma\right) = 70.54\ \text{s}^{-1}, \quad \frac{1}{LC} - \frac{r^2}{4L^2} = 1.42\times 10^8\ \text{rad}^2/\text{s}^2 > 0.$$

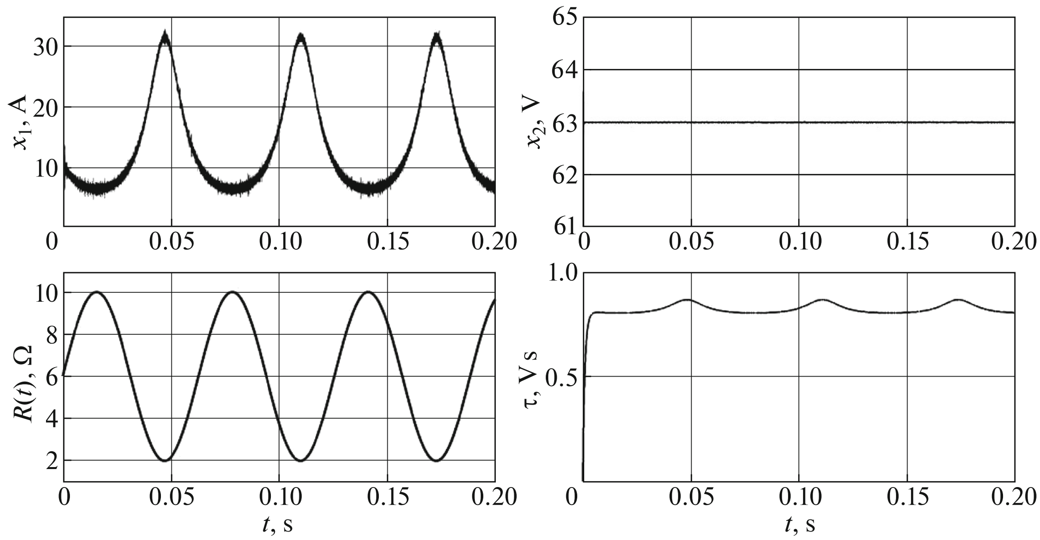

Fig. 3. Simulation results of the first experiment.

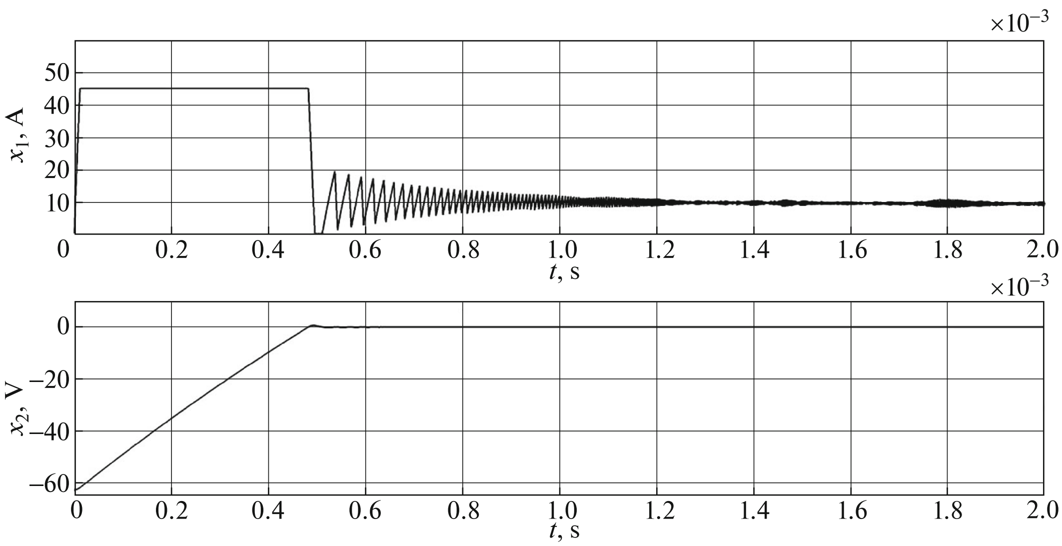

Fig. 4. Transient response of the system.

To demonstrate the slow component of the control law (2.2), corresponding to the duty cycle of the switching element [1], a new variable is introduced, which is actually the output of a low-pass filter,

$$\mu\dot{\tau} = -\tau + u(t),$$

where μ is the filter's time constant.

In Figs. 3, 4 the results of modeling the developed control law in the MATLAB/Simulink environment are presented. In the first experiment, the Dorman–Prince method (ode5) with a fixed integration step is used for numerical integration $t_s = 10^{-7}$ s.

In the second experiment, the results of which are shown in Figs. 5 and 6, several integration steps are used:

$$t_s = 10^{-6} \text{ s}, \quad t_s = 10^{-7} \text{ s}, \quad \text{and} \quad t_s = 10^{-8} \text{ s}.$$

A consequence of the proven theoretical result is that the switching frequency of the control input tends to infinity over time. In practice, the switching frequency is limited, resulting in a steady-state control

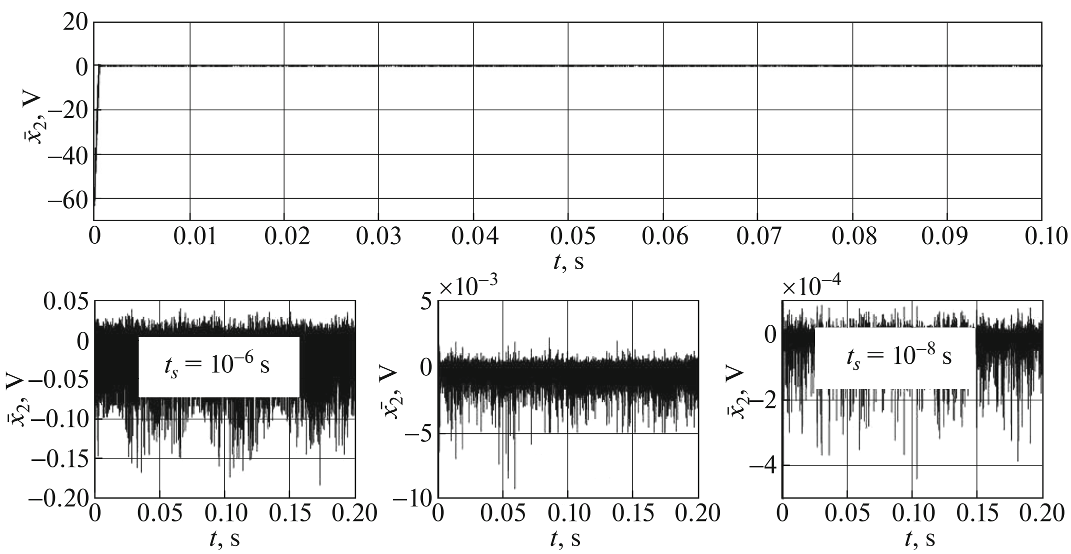

Fig. 5. Steady-state error at various integration steps.

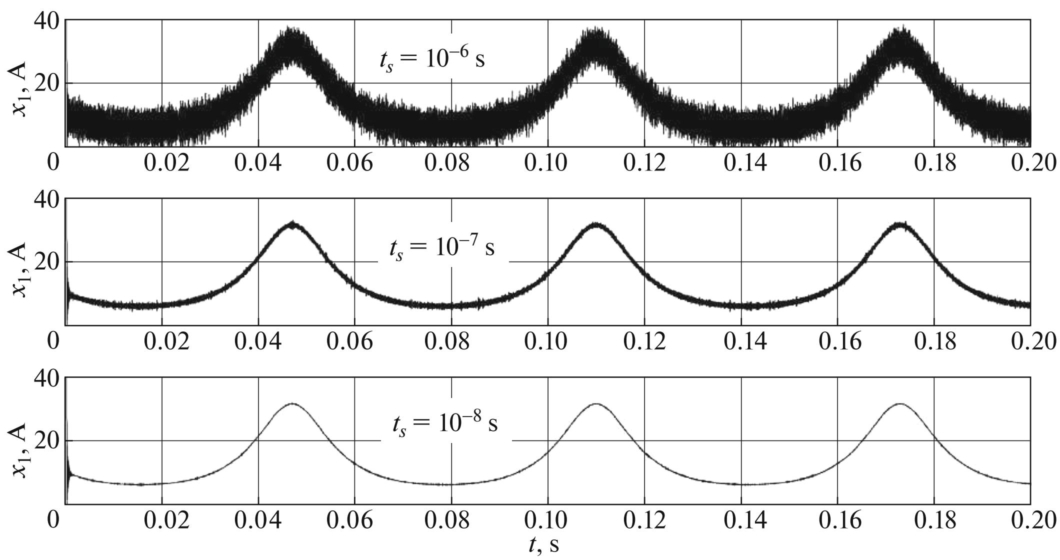

Fig. 6. Current graphs at various integration steps.

error. From Figs. 5 and 6 it is clear that this error depends on the switching frequency (integration step): the higher the frequency the smaller the error and vice versa. Such limitations must be taken into account when implementing the described approach in practice; however, this issue requires further study and this case is not considered in this article.

CONCLUSIONS

A new control algorithm for a semiconductor step-down converter has been studied. Assuming that the load function can be described by a continuous bounded function with two bounded first derivatives, the problem of stabilizing the given output voltage was solved. For the practical implementation of the developed algorithm in further research, it is necessary to study the adaptation of the resulting control law for working with converters with pulse-width modulation.

The performance of the proposed algorithm was confirmed both analytically and using simulation in the MATLAB–Simulink environment.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

CONSENT TO PARTICIPATE

Informed consent was obtained from all individual participants included in the study.

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