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Capsule-Type Vibration-Driven Robot with an Electromagnetic Actuator and an Opposing Spring: Dynamics and Control of Motion

N. N. Bolotnik, A. M. Nunuparov, and V. G. Chashchukhin

Institute for Problems in Mechanics, Russian Academy of Sciences, Moscow, Russia

Moscow Institute of Physics and Technology, Dolgoprudny, Russia

e-mail: Bolotnik@ipmnet.ru

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Abstract—A model of a mobile capsule robot that consists of the housing and internal body is considered. The internal body can move relative to the housing along a straight line. The internal body is attached to the housing by a spring. The system motion is excited by a force that acts between the housing and the internal body. The force changes in a pulse-width periodic mode. The robot's motion along a straight line on a rough horizontal plane is investigated. It is assumed that the dry Coulomb friction acts between the housing and the plane. The dependence of the average steady state robot velocity on excitation parameters is analyzed. It is established that it is possible to control the magnitude and direction of the robot motion by changing the period and the duty cycle of the pulse-width excitation signal. The effect of the variation in the direction of the robot motion due to changing the excitation period is observed. This effect is associated with the phenomenon of resonance.

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INTRODUCTION

A capsule robot is a locomotion (mobile) system that moves in a resistive medium without external propelling devices (legs, wheels, continuous tracks, and propeller fins) due to the motion of internal bodies in the presence of the force interaction between the robot housing and the environment. Structurally, the capsule robot consists of a rigid housing (capsule) and internal bodies that can move relative to the housing under the action of actuators. The actuators provide the interaction of the internal bodies with the robot housing. The application of a force to the internal body causes a reaction force applied to the housing. As a result, the the velocity of the housing relative to the medium changes. The housing speed change leads to a change in the resistance friction force applied to the body from the medium. The forces generated by the actuators are internal forces for the mechanical system under consideration (the housing plus internal body), and the medium resistance force is external. Thus by controlling the motion of internal bodies by internal forces, it is possible to control the external force acting on the robot and, therefore, the motion of the system as a whole.

The capsule robot has several advantages over the other types of mobile systems. It is structurally simple, does not require complex mechanisms for the transmission of motion from the drive to the locomotors, easy to miniaturize, its housing can be sealed and smooth without protruding parts. The latter fact makes it possible to use a capsule robot in "vulnerable" media, particularly in medicine for diagnostic tests within the human body or for accurate delivery of medication to the affected area. The capsule robot can also be used for the movement inside thin pipes, for example, in order to inspect their technical state.

Capsule robots belong to the class of vibration-driven mobile systems, which are systems of solid bodies that in the general case interact with one another and with the environment and oscillate relative to each other. In the capsule robot, only one body (housing) interacts with the medium; the internal bodies do not interact with the medium.

Investigations of the dynamics and control processes of locomotion systems moving in resistive due to the motion of internal bodies are used as a scientific basis for the development of capsule robots. In this case, system motions with periodically changing velocity generated by the periodic motion of internal bodies are of major interest. Here, an important aspect is the optimization of motions with respect to the velocity and power consumption.

An optimization problem for the motion of a body moving in a resistive medium and controlled by its interaction with a movable internal body was first stated in [1, 2]. The case where the housing moves along a straight line in the horizontal plane subject to dry Coulomb's dry friction is considered. Periodic modes of control of the internal body's relative motion, at which the housing moves at a periodically changing velocity and passes the same distance in a given direction for one period, are constructed. The internal body is allowed to move within a fixed range. It is assumed that at the beginning and at the end of each period the housing velocity is zero, and the movable body is at rest in one of the extreme positions. Velocity-controlled and acceleration-controlled modes are considered for the internal body. In the first case, the internal body is moving between fixed extreme positions at a constant velocity relative to the housing. The relative velocity of the internal body is different for the forward (in the desired direction of motion of the housing) and backward motions. The magnitudes of the internal body velocities are taken as control variables to be determined. The second mode assumes three intervals of constancy for the relative acceleration of the internal body during one period. A constraint is imposed on the acceleration magnitude. The duration of these intervals and the magnitude of the internal body's acceleration on each of them are taken as the control variables. The optimal parameters that maximize the average velocity of the housing are found for both modes. The above problems are solved in [3] without the assumption that the housing velocity vanishes when the internal body is in one of the extreme positions. The optimal parameters for controlling the internal body velocity are found not only for a medium with dry friction but also for media with piecewise-linear and quadratic laws of resistance of the medium to the housing motion. The optimal parameters for controlling the internal body's acceleration in a medium with a piecewise-linear resistance law were identified numerically in [4].

The optimal control problem for the motion of the above-described mechanical system along a straight line in the horizontal plane provided that the Coulomb friction in [5] acts between the body and the plane was solved. The acceleration of the internal body relative to the housing is taken as the control variable; a constraint on absolute value of this variable is imposed. The periodic control with a zero mean and the corresponding with periodically changing velocity that maximize the displacement of the housing for the period are constructed motion of the housing. The constructed control can be used to restore the periodic law of the internal body's motion that generates the optimal motion of the system. A similar problem is solved for a system with two internal bodies, one of which performs periodic motions along a horizontal line parallel to the line of the motion of the housing, and the other, along the vertical line [6]. The presence of the internal body moving vertically makes it possible to control the normal pressure of the housing on the supporting plane and, thereby, the magnitude of the friction force acting on the body when it moves. In [7], the optimal control problem for the motion of a body with a movable internal mass along a straight line is investigated for a wide class of nonlinear laws of resistance of the environment. The optimality criterion and constraints are the same as in [5]. An algorithm for calculating the optimal control is proposed and the qualitative characteristics of the optimal motions are examined.

The energy-optimal control modes for a system with one internal body moving in a medium with power-law resistance to the motion of the housing are constructed and investigated [8]. The energy consumption is measured by the work of the resistance forces during the period of the system's motion. In constructing the optimal control, the period of the internal body's relative motion and the system's average velocity are assumed to be given. No other constraints are imposed on the system's motion.

The capsule robot, the control of which is based on the principles set out in [1, 2], is described in [9]. The prototype of the robot was built and tested at Tokyo Denki University (Japan). The robot has an electromagnetic (solenoid) actuator. The magnitude and direction of the force applied to the internal body is controlled by changing the magnitude and polarity of the voltage applied to the solenoid coil.

Various aspects of the planning, simulation, and optimization of the motions of capsule robots are also considered in a number of works [10–12].

In all the above-cited papers, it is assumed that the only force that acts on the internal mass of the mobile robot in the direction of its motion is the control force generated by the system actuator. In our paper, a capsule robot in which the internal body is attached to the housing by a spring is addressed. In this case, the oscillatory link "housing–spring–internal body" appears in the system. The link is characterized by a natural frequency, which significantly changes the dynamic behavior of the system. In particular, the resonance phenomenon can be observed in it if the control force changes periodically. A vibration-driven robot, the bodies of which are connected by a spring, is considered in [13–15]. This robot is designed to move inside pipes. It consistsof two bodies (modules), both of which are in contact with the pipe surface. The system is controlled by an electromagnetic actuator that provides the force interaction

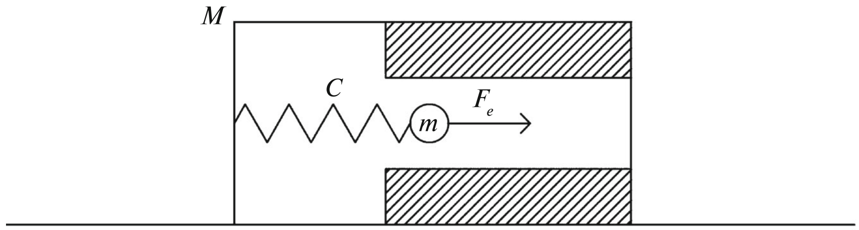

Fig. 1.

between the modules. Contact surfaces of the modules had stops [13] or countings [14, 15] that cause an anisotropy of friction against the surface of the pipe so that the friction force that impedes the robot motion in the desired direction is significantly less than the friction force that impedes its the motion in the opposite direction. In [13], the principle of the robot's motion is described in general terms, the formulas for calculating the magnetic interaction force between the modules are given, the physical parameters of the robot prototype constructed by the authors are presented, and the results of the experimental investigations are briefly described. In [14, 15], a mathematical model of a two-module vibration-driven in-pipe robot with an electromagnetic actuator and an opposing spring is presented; this model allows analyzing the dynamics of the system. The robot dynamics are investigated by numerical simulation, especially the robot's behavior in the steady state when the robot's bodies oscillate periodically relative to each other, if the interaction force of the robot's bodies changes periodically in a pulse-width mode. The dependence of the robot's average velocity on the period and the duty cycle of the pulse-width excitation signal is analyzed. The optimal parameters at which the robot moves inside the pipe at a maximum speed is found. A physical prototype of the vibration-driven in-pipe robot is developed, and experiments are conducted; the experimental data agree with a simulation results. The system is considered in this paper differs from the system discussed in [14, 15] in the fact that one of the bodies is an internal body and does not have come into contact with the surface on which the robot moves and in the fact that the friction between the other body (housing) and the surface is classic Coulomb's dry friction without anisotropy.

1. MECHANICAL MODEL OF THE CAPSULE ROBOT

The robot consists of a rigid body with a shape of a cylinder or parallelepiped and an electromagnetic (solenoid) actuator inside. The actuator consists of an electric coil (solenoid) rigidly secured to the housing and the core made of a ferromagnetic material. It can move axially inside the solenoid. The core is attached to the housing by a spring the axis of which is oriented along the solenoid axis. The solenoid axis is parallel to the housing axis. The housing interacts with a resistive environment in which the robot moves. The robot is driven by the force that acts on the core when an electrical voltage is applied to the solenoid. The actuator is designed so that the force is directed to one side drawing the core into the coil. The core returns to the initial position by the spring when the electromagnet is turned off. The described system is schematically shown in Fig. 1.

In this paper, we consider a model in which the force applied to the solenoid core is the control variable. The solenoid's electrical circuit dynamics are not taken into account. We will investigate the robot's motion along a horizontal line parallel to the axis of its housing.

Let us introduce the following notation: M is the total mass of the housing and a solenoid coil, m is the core mass, F_e is the force with which the solenoid acts on the core, F_{fr} is the force of the medium resistance to the housing motion, c is the spring stiffness coefficient, x is the coordinate that determines the position of the housing center of mass relative to a fixed (inertial) reference system, and ξ is the coordinate that determines the position of the core center of mass relative to the housing. The coordinate ξ is selected so that for $\xi = 0$ the spring is not deformed. Note that the force F_e is internal with respect to the housing–solenoid–core system, whereas the force F_{fr} is external. We assume that the force of the medium resistance to the housing motion depends on velocity of the housing relative to the medium, i.e., $F_{fr} = F_{fr}(\dot{x})$.

By applying Newton's second law separately to the housing and to the core, we obtain the equations of motion of the system under consideration in the following form:

$$M\ddot{x} = c\xi - F_e + F_{fr}(\dot{x}), \quad m(\ddot{x} + \ddot{\xi}) = -c\xi + F_e. \tag{1.1}$$

Let us introduce the new variable

$$X = x + \frac{m}{M+m}\xi, \tag{1.2}$$

which represents the coordinate of the center of mass of the entire system in a fixed reference system, and reduce system of equations (1.1) to the following form:

$$\begin{gathered}(M+m)\ddot{X} = F_{fr}\left(\dot{X} - \frac{m}{M+m}\dot{\xi}\right),\\ \frac{Mm}{M+m}\ddot{\xi} + c\xi = F_e - \frac{m}{M+m}F_{fr}\left(\dot{X} - \frac{m}{M+m}\dot{\xi}\right).\end{gathered} \tag{1.3}$$

Let us proceed to the dimensionless (primed) variables

$$\begin{gathered}X' = \frac{X}{L}, \quad \xi' = \frac{\xi}{L}, \quad t' = \omega t, \quad F_e' = \frac{F_e}{cL},\\ F_{fr}'\left(\frac{dX'}{dt'} - m_2\frac{d\xi'}{dt'}\right) = \frac{1}{cL}F_{fr}\left(\omega L\left(\frac{dX'}{dt'} - m_2\frac{d\xi'}{dt'}\right)\right),\\ \omega = \sqrt{\frac{c(M+m)}{Mm}}, \quad m_1 = \frac{M}{M+m}, \quad m_2 = \frac{m}{M+m},\end{gathered} \tag{1.4}$$

where L is a parameter that has the dimension of length taken and is as the unit of measurement; this parameter is identified below. In the dimensionless variables, Eqs. (1.3) can be represented as follows (the primes are omitted; the dot denotes the derivative with respect to the dimensionless time):

$$\begin{gathered}\ddot{X} = m_1m_2F_{fr}\left(\dot{X} - m_2\dot{\xi}\right),\\ \ddot{\xi} + \xi = F_e - m_2F_{fr}\left(\dot{X} - m_2\dot{\xi}\right).\end{gathered} \tag{1.5}$$

Let dry friction obeying Coulomb's law act between the housing and the medium in which it moves. Then

$$F_{fr}(\dot{x}) = \begin{cases} -N\operatorname{sign}\dot{x}, & \dot{x} \neq 0,\\ -c\xi + F_e, & \dot{x} = 0, \quad |c\xi - F_e| \le N,\\ -N\operatorname{sign}(c\xi - F_e), & \dot{x} = 0, \quad |c\xi - F_e| > N,\end{cases} \tag{1.6}$$

where N is the maximum absolute value of the static friction force.

The force F_e generated by the actuator is modeled by the periodic piecewise constant function

$$F_e = \begin{cases} F_0, & \left\{\dfrac{t}{T}\right\} < \tau,\\ 0, & \left\{\dfrac{t}{T}\right\} \ge \tau,\end{cases} \tag{1.7}$$

where T is the period, F_0 is a positive constant of the dimension of force, and τ is a positive dimensionless constant which belongs to the interval $(0,1)$ and represents a part of the period in which the control force is not zero; the braces denote the fractional part of the enclosed expression. In physics and electronics, the excitation mode of the (1.7) type is called the pulse-width mode, the parameter τ is called the duty cycle of the pulse-width signal Equations (1.6) and (1.7) are represented in the original dimensional variables.

Let us proceed to the dimensionless variables by selecting constant L in Eqs. (1.4) as follows:

$$L = \frac{F_0}{c}. \tag{1.8}$$

This selection means that the spring's static deformation by the force F_0 is taken as the unit of length in the nondimensionalization procedure. Then the expression for F_{fr} that is occurs on the right-hand sides of (1.5) can be represented in the following form:

$$F_{fr}\left(\dot{X} - m_2\dot{\xi}\right) = \begin{cases} -\varepsilon\operatorname{sign}\left(\dot{X} - m_2\dot{\xi}\right), & \dot{X} - m_2\dot{\xi} \neq 0,\\ F_e - \xi, & \dot{X} - m_2\dot{\xi} = 0, \quad |\xi - F_e| \le \varepsilon,\\ -\varepsilon\operatorname{sign}(\xi - F_e), & \dot{X} - m_2\dot{\xi} = 0, \quad |\xi - F_e| > \varepsilon,\end{cases} \tag{1.9}$$

where $\varepsilon = N/F_0$ and the dimensionless expression for the force F_e is defined by formula (1.7) for $F_0 = 1$.

In the case where the control force and the friction force are modeled by Eqs. (1.7) and (1.9), dimensionless dynamics equations (1.5) involve the five dimensionless parameters, respectively: m_1, m_2, T, τ, and ε. Among these, four parameters are independent because $m_1 + m_2 = 1$.

2. MODELING AND ANALYSIS OF THE ROBOT'S STEADY-STATE MOTION

For the type of robots under considiration, of most interest is the motion mode, in which the core oscillates with a period T relative to the housing and the housing moves relative to the medium at a velocity that changes periodically with the same period T. This motion mode is called a steady state mode. In the case of steady state motion, functions $\xi(t)$ and $\dot{X}(t)$ are T-periodic. One of the most important characteristics of the steady state robot motion is its average velocity V defined as

$$V = \frac{1}{T}\int_0^T \dot{X}(t)dt. \tag{2.1}$$

The quantity V is the average velocity of the robot's center of mass with respect to the medium. It coincides with the average velocity v of the housing relative to the medium, which is defined by the following equation:

$$v = \frac{1}{T}\int_0^T \dot{x}(t)dt. \tag{2.2}$$

The equality of the quantities V and v follows from relation (1.2) and the T-periodicity of the function $\xi(t)$ in the case of the steady state motion.

This section is dedicated to the analysis of the dependence of the robot's average velocity on the excitation parameters T and τ.

The average velocity of the steady state motion was determined on the basis of computer simulation of the robot's motion, which consisted in the numerical integration of Eqs. (1.5), where the function F_{fr} was defined by (1.9). The integration was carried out for zero initial conditions $X(0) = 0$, $\dot{X}(0) = 0$, $\xi(0) = 0$, and $\dot{\xi}(0) = 0$ up to the time at which the relations $\dot{X}(t_*) = \dot{X}(t_* - T)$, $\xi(t_*) = \xi(t_* - T)$, $\dot{\xi}(t_*) = \dot{\xi}(t_* - T)$ hold with a given accuracy. The occurrence of these relations means that the system reaches a steady state motion mode. Thereafter, the average velocity V was calculated from the formula $V = [X(t_*) - X(t_* - T)]/T$. The modeling shows that the steady state mode is reached at any values of the excitation parameters, if the coefficient of friction between the robot's housing and the surface on which it moves is nonzero ($\varepsilon \neq 0$).

The modeling was carried out with the following values of the nonvariable robot parameters:

$$\begin{gathered} M = 0.0213 \text{ kg}, \quad m = 0.0231 \text{ kg}, \quad c = 360 \text{ Nm}^{-1}, \\ F_0 = 0.8 \text{ N}, \quad N = 0.7 \text{ N}, \quad \omega = \sqrt{\frac{c(M+m)}{Mm}} = 180.2 \text{ s}^{-1}. \end{gathered} \tag{2.3}$$

These parameters correspond to the prototype of a miniature vibration-driven in-pipe robot developed at the Institute for Problems in Mechanics of the Russian Academy of Sciences [15]. The in-pipe robot differs from the capsule robot under considiration in the fact that its core has a rod projecting beyond the housing with a contact device at the end, by means of which the core and the housing interact with the pipe's wall. The contact surfaces of the housing and the rod have a coating. The coating ensures an anisotropy for friction of these modules against the pipe's wall; i.e., the force of friction that prevents the motion of the housing and the rod in the desired robot motion direction is significantly less than the force of friction that prevents the motion in the opposite direction.

The dimensionless parameters m_1, m_2, and ε for the model under consideration are as follows:

$$m_1 = 0.480, \quad m_2 = 0.520, \quad \varepsilon = 0.88. \tag{2.4}$$

Let us investigate the dependence of the average steady state system velocity V on the parameter τ that characterizes excitation law (1.7).

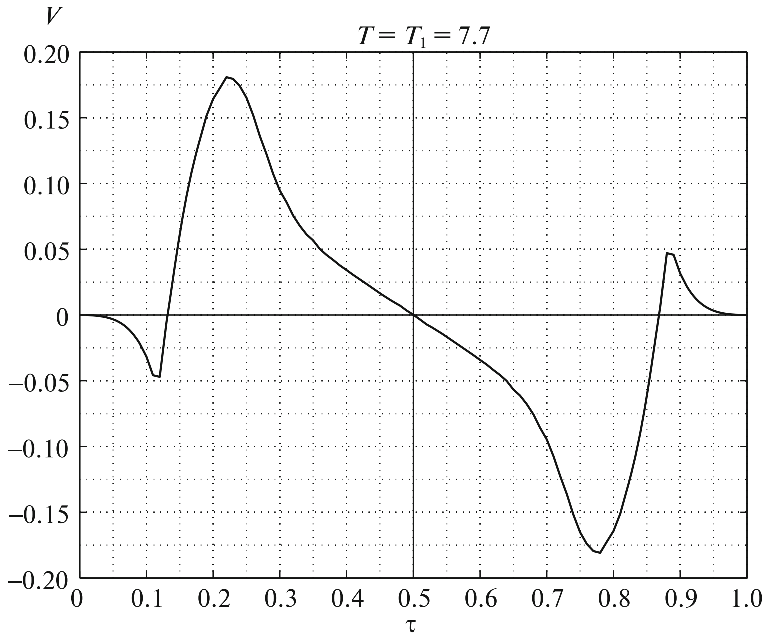

Fig. 2.

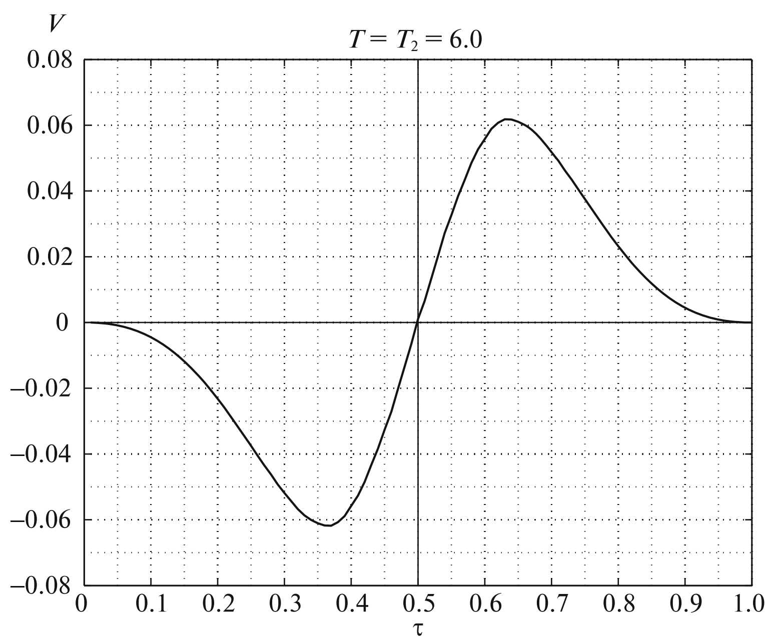

Fig. 3.

Figures 2 and 3 show the typical dependences of the quantity V on the parameter τ in dimensionless units. Figure 2 corresponds to $T = T_1 = 7.7$ and Fig. 3, to $T = T_2 = 6.0$. Let us note that for excitation periods T_1 and T_2, the inequalities $T_2 < 2\pi < T_1$ hold, and that 2π is the dimensionless period of the free oscillations of the robot caused by the elasticity of the spring that connects the housing with the internal body. Therefore, the excitation mode with a period of T_1 can be called above resonance mode and the excitation mode with the period T_2, below resonance mode.

Both curves demostrate a significant dependence of the average speed of the robot's steady state motion on the duty cycle of the pulse-width excitation signal making it possible to control the robot's motion by changing only the parameter τ. In the case of $\tau = 0$, $\tau = 1/2$, and $\tau = 1$, the robot's average velocity is zero. Both curves have the property of central symmetry with respect to the point $(1/2, 0)$ of the coordinate plane τV. It means that the change in the duty cycle of the excitation signal from τ to $1-\tau$ at the same period leads to a change in the direction of the capsule robot's motion, while maintaining the absolute value of its velocity.

The latter property holds for all systems that obey Eqs. (1.5) and (1.9) for the pulse-width excitation mode (1.7). Let us prove the corresponding mathematical proposition. We will mark the dependence of the quantities V and $F_e(t)$ on the parameter τ by the superscript in brackets; i.e., instead of V and $F_e(t)$, we will write $V^{[\tau]}$ and $F_e^{[\tau]}(t)$, respectively.

The definition of (1.7) for the function $F_e^{[\tau]}(t)$ for $F_0 = 1$ implies the relation

$$F_e^{[\tau]}(t) = 1 - F_e^{[1-\tau]}(t - \tau T). \tag{2.5}$$

Proposition. The average velocities $V^{[\tau]}$ and $V^{[1-\tau]}$ are related by the following equation:

$$V^{[\tau]} = -V^{[1-\tau]}. \tag{2.6}$$

Proof. Let us subject in Eqs. (1.5), where $F_e = F_e^{[\tau]}(t)$ to the change of variables,

$$Y(t) = -X(t + \tau T), \quad \eta(t) = -\xi(t + \tau T) + 1. \tag{2.7}$$

Given the oddness of the function $F_{\mathrm{fr}}(z)$ with respect to the argument $z = \dot{X} - m_2\dot{\xi}$ and Eq. (2.5), we obtain

$$\begin{aligned} \ddot{Y} &= m_1 m_2 F_{fr}(\dot{Y} - m_2\dot{\eta}), \\ \ddot{\eta} + \eta &= F_e^{[1-\tau]}(t) - m_2 F_{fr}(\dot{Y} - m_2\dot{\eta}). \end{aligned} \tag{2.8}$$

Thus, it is shown that if the functions $X(t)$ and $\xi(t)$ provide the solution of (1.5) for $F_e = F_e^{[\tau]}(t)$, then the functions $Y(t)$ and $\eta(t)$ provide the solution of the same system for $F_e = F_e^{[1-\tau]}(t)$. If the functions $\dot{X}(t)$ and $\xi(t)$ are T-periodic, then the functions $\dot{Y}(t)$ and $\eta(t)$ are also T-periodic. The differentiation of the first equation from (2.7) gives the relation $\dot{Y}(t) = -\dot{X}(t + \tau T)$, from which in the case of the T-periodicity of the function $\dot{X}(t)$ the following equations are obtained:

$$\int_0^T \dot{Y}(t)dt = -\int_0^T \dot{X}(t + \tau T)dt = -\int_0^T \dot{X}(t)dt. \tag{2.9}$$

Since

$$V^{[\tau]} = \frac{1}{T}\int_0^T \dot{X}(t)dt, \quad V^{[1-\tau]} = \frac{1}{T}\int_0^T \dot{Y}(t)dt, \tag{2.10}$$

from Eqs. (2.9) we obtain Eq. (2.6). This completes the proof of the proposition.

Corollary. For $\tau = 1/2$, the average steady state system velocity is zero: $V^{[1/2]} = 0$.

A significant qualitative difference between the curves in Figs. 2 and 3 is that for $\tau \in (0, 1/2)$ the magnitude V in Fig. 2 reaches a maximum, and this maximum is positive, and in Fig. 3 it reaches a minimum, and this minimum is negative. Recall that Fig. 2 corresponds to the above-resonance excitation mode ($T > 2\pi$) and Fig. 3, to the below-resonance mode ($T < 2\pi$). This observation allows us to hypothesize about the resonance effect expressed in the change of the direction of the system's motion when the excitation period T passes certain critical values close to the magnitudes that are multiples of the period of the free elastic oscillations of the system.

The cause of the change in the direction of the system's motion associated with the phenomenon of resonance can be explained based on Eqs. (1.5) and (1.9) for low values of the coefficient of friction

($\varepsilon \ll 1$). In this case, the second term on the right-hand of the second equation in (1.5) can be neglected in comparison with F_e. Let us represent a T-periodic function $F_e(t)$ by its Fourier series:

$$F_e(t) = a_0 + \sum_{n=1}^{\infty} (a_n \cos\omega_n t + b_n \sin\omega_n t), \quad \omega_n = \frac{2\pi n}{T}, \tag{2.11}$$

where a_0, a_n, and b_n are constant coefficients (Fourier coefficients). Then the periodic solution of the second equation in (1.5) for $F_{fr} = 0$ can be represented by the series

$$\xi(t) = a_0 + \sum_{n=1}^{\infty} \frac{1}{1-\omega_n^2}(a_n \cos\omega_n t + b_n \sin\omega_n t), \quad \omega_n \neq 1, \tag{2.12}$$

the differentiation of which provides

$$\dot{\xi}(t) = \sum_{n=1}^{\infty} \frac{\omega_n}{1-\omega_n^2}(b_n \cos\omega_n t - a_n \sin\omega_n t), \quad \omega_n \neq 1. \tag{2.13}$$

For $\omega_n = 1$, resonance is observed in the system. In this case, there is no periodic solution to the second equation in (1.5) for $F_{fr} = 0$. In the vicinity of the resonance, the variable $\dot{\xi}(t)$ can be represented by the asymptotic expression

$$\dot{\xi}(t) = \frac{1}{2(1-\omega_n)}(b_n \cos t - a_n \sin t) + O(1), \quad 0 < |1-\omega_n| \ll 1. \tag{2.14}$$

The first equation in (1.5) is invariant to the change of variables $\dot{X} \to -\dot{X}$, $\dot{\xi} \to -\dot{\xi}$. In the first approximation, from asymptotic expression (2.14) it follows that if the frequency ω_n is replaced by $2-\omega_n$ in expression (1.5), the function $\dot{\xi}(t)$ changes in sign. Consequently, the function $\dot{X}(t)$ also changes in sign, and same occurs for the average velocity V of the steady state motion of the system. The detunings of the frequencies ω_n and $2-\omega_n$ from the resonance frequency, which is equal to unity, are equal in magnitude but opposite in sign.

In terms of the excitation period, the resonance condition is expressed by the relation $T = 2\pi n$, $n = 1, 2, \ldots$ Consequently, at least in the case of low friction ($\varepsilon \ll 1$), it is possible to expect multiple changes in the direction of the system's motion as the excitation period increases.

The change in the direction of motion of a vibration-driven mobile system associated with the phenomenon of resonance was previously observed in [16]. In this paper, a two-module locomotion system moving along a straight line on a rough horizontal plane is considered. The system consists of two identical modules, modeled as rigid bodies, each of which has an unbalanced vibration exciter. The unbalanced vibration exciter is a rotor the center of mass of which doest not lie on the axis of rotation. The modules are connected by a spring with a linear characteristic. Dry Coulomb's friction acts between the modules of the system and the plane on which it moves. The friction force is assumed to be small. The system is excited by the rotation of the two rotors with the same constant angular velocities but with a phase shift (the perpendiculars dropped from the centers of mass of the rotors onto their rotation axes are not parallel). In the described system, the direction of the motion changes when the excitation frequency passes through the resonant value equal to the frequency of the free oscillations of the modules connected by a spring in the absence of friction. The motion of locomotion systems consisting of two bodies connected by a spring and controlled by interaction forces between adjacent bodies is investigated in [17]. In particular, the behavior in the vicinity of resonance is analyzed. However, unlike the model described in this paper, in [17] it is assumed that both bodies of the system interact with the medium, the control forces change sinusoidally, and the friction between the bodies of the system and the environment is relatively small.

The phenomenon of the resonant change in sign of the average robot velocity manifests itself clearly on the curve representing the dependence of V on the excitation period T. Figure 4 shows this dependence for $\tau = 0.2$. Here, the velocity sign changes at the point $T = 7.14$. In the vicinity of this point, extreme values of the quantity V are reached, a minimum of -0.086 for $T = 6.9$ and a maximum of 0.18 for $T = 8.0$. The shift of the point of change of the velocity V from the resonant period $T = 2\pi$ can be explained by the effect of dry friction between the robot housing and the plane along which it moves on

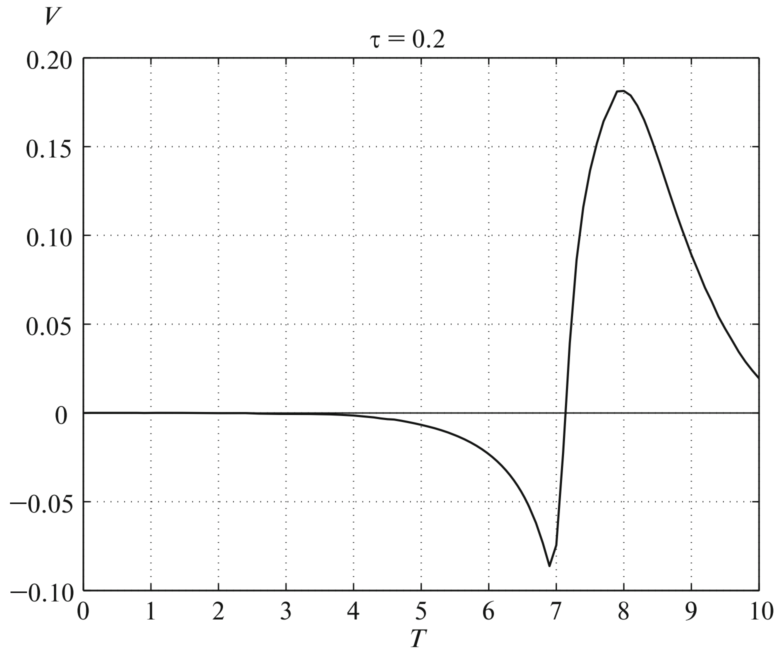

Fig. 4.

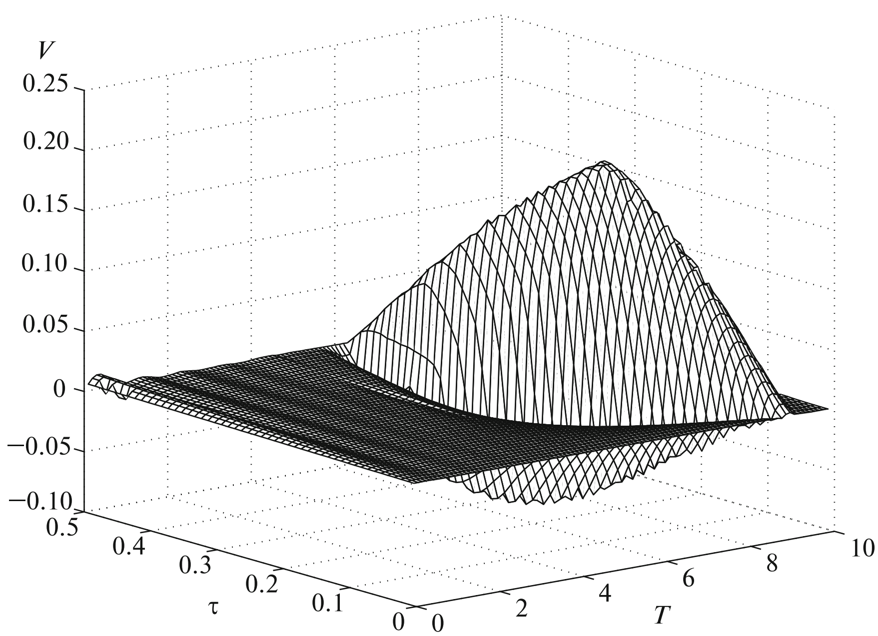

Fig. 5.

the oscillations of the internal body (core). Qualitatively, the same pattern can be observed for other $0 < \tau < 1/2$. According to the proposition proved above, the graphs of the dependence of V on T corresponding to the values τ and $1 - \tau$ of the duty cycle of the pulse-width excitation signal are symmetrical to each other relative to the T axis.

Thus, by changing the excitation period T or the parameter τ, it is possible to control both the magnitude of the velocity and the direction of the motion of the capsule robot. Figure 5 shows a graph for the quantity V as a function of the parameters T and τ in the region $\{T, \tau : 0 < T < 10.0, 0 < \tau < 0.5\}$. According to (2.6), in the region $\{T, \tau : 0 < T < 10.0, 0.5 < \tau < 1\}$, the graph of this function is obtained by rotating the graph shown in Fig. 5 by an angle of 180 degrees about the straight line $V = 0, \tau = 0.5$ in the coordinate space $T\tau V$.

Important characteristics of the capsule robots are the maximum $V_{\max}$ and minimum $V_{\min}$ velocities of the robot that can be provided by an appropriate choice of the excitation parameters.

Calculations for the robot with parameters (2.3) in dimensionless units give the following results (see also Fig. 5):

$$\begin{aligned} V_{\max} &= 0.1868, \quad T_{\max} = 7.95, \quad \tau_{\max} = 0.215, \\ V_{\min} &= -0.1868, \quad T_{\min} = 7.95, \quad \tau_{\min} = 0.785. \end{aligned} \tag{2.15}$$

Here, $T_{\max}$, $\tau_{\max}$, $T_{\min}$, and $\tau_{\min}$ are the values of the parameters at which the extreme values of the average velocity of the robot in the steady state motion mode are achieved.

In the initial dimensional units, Eqs. (2.15) can be represented as follows:

$$\begin{aligned} V_{\max} &= 0.0748 \text{ m/s}, \quad T_{\max} = 0.0441\text{s}, \quad \tau_{\max} = 0.215, \\ V_{\min} &= -0.0748 \text{ m/s}, \quad T_{\min} = 0.0441\text{s}, \quad \tau_{\min} = 0.785. \end{aligned} \tag{2.16}$$

Note that

$$V_{\min} = -V_{\max}, \quad T_{\max} = T_{\min}, \quad \tau_{\max} = 1 - \tau_{\min}. \tag{2.17}$$

These relations hold for the considered type of robot with any parameters, not only with parameters (2.3) for which the modeling was carried out. Relations (2.17) follow from the general relation (2.6).

The perfomed analysis makes it possible to draw the conclusion about the feasibility of the control of the capsule robot by changing the parameter τ for fixed $T = T_{\max} = T_{\min}$. The variation of the parameter τ within the range from $\tau_{\min}$ to $\tau_{\max}$ makes it possible to implement any possible velocity in the range from $V_{\min}$ to $V_{\max}$.

In addition to the average velocity, the nature of the steady state motion of the housing and the core of the capsule robot is also of interest. Especially interesting is the motion of the housing , in particular, whether there exist time intervals during which the housing is moving "backward" (in the direction opposite to the average velocity). Note that the housing cannot always move in one direction with a nonzero velocity, since, in this case, the dry friction force applied to the housing would be constant and nonzero, and, therefore, the velocity of the system's center of mass would be a linear function of time. This contradicts the T-periodicity of the change in the velocity of the system's center of mass in the steady state motion mode resulting from the T-periodicity of the change in the relative position of the core $\xi(t)$ and absolute velocity $v(t)$ of the housing. Thus, if during the period, the robot's housing never moves backward, it should be at rest relative to the medium for some time.

The simulation shows that the motion of the robot's housing relative to the medium can vary depending on the parameters T and τ as illustrated by Figs. 6–12. In these figures, the dimensionless variables represent the time histories of the velocity of the robot's housing relative to the medium ($v(t)$, solid curves), the velocity of the core relative to the housing ($\dot{\xi}(t)$, dashed curves), and the velocity of the system's center of mass relative to the medium ($\dot{X}(t)$, dashed-dotted curves) for certain values of T and $\tau \in (0, 0.5)$. Let us note the fact that steady state motions are considered for large values of the dimensionless time t. These values can be different in different figures. This fact reflects the real simulation history: a steady state mode is set after a relatively long transient process, the duration of which depends on the excitation parameters. Figures 6 and 7 correspond to the motion for sufficiently small τ. In this case, the housing moves in the direction of the average velocity during one time interval and is at rest during the remaining part of the period. Figure 6 corresponds to the below-resonance mode ($T = 6, \tau = 0.2$) and Fig. 7, to the above-resonance mode ($T = 7.7, \tau = 0.1$). The curves in Figs. 8 and 9 correspond to the motion for τ close to 0.5 under the below-resonance (Fig. 8, $T = 6, \tau = 0.49$) and above-resonance (Fig. 9, $T = 7.7, \tau = 0.49$) modes. Here, it is possible to select the period that is divided into two intervals. On one of these intervals, the housing moves forward and, on the other, backward. There are no intervals where the housing is at rest. Steady state motions are also observed in which at each period the housing has forward and backward motion intervals, as well as one (Fig. 10, $T = 7.1, \tau = 0.2$) or more (Fig. 11, $T = 9.8, \tau = 0.2$) intervals of rest. Figure 12 shows the curves corresponding to the optimal parameters (2.15).

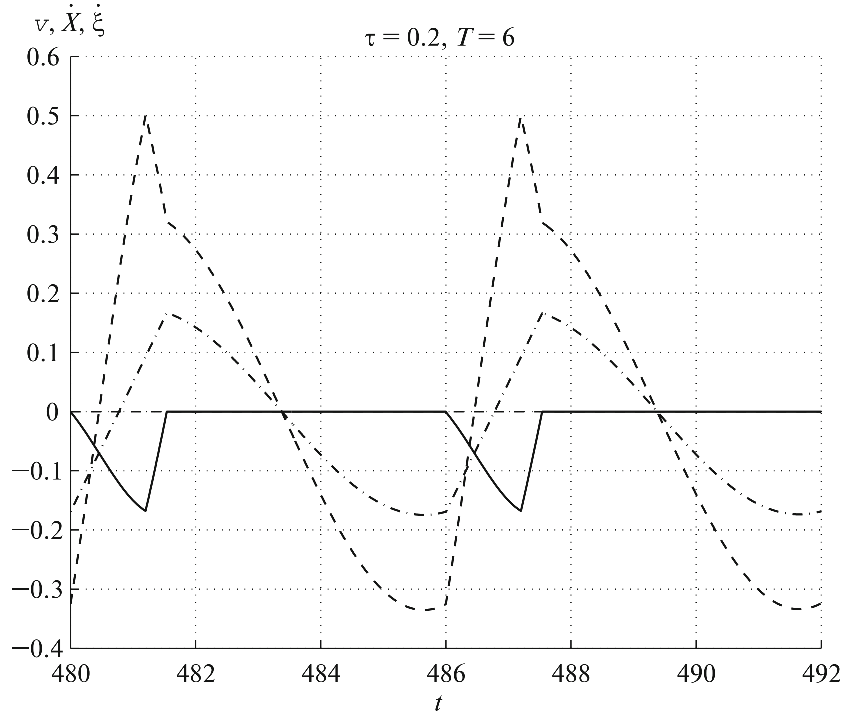
$v, \dot{X}, \dot{\xi}$
$\tau = 0.2,\ T = 6$
0.6
0.5
0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
480
482
484
486
488
490
492
t

Fig. 6.

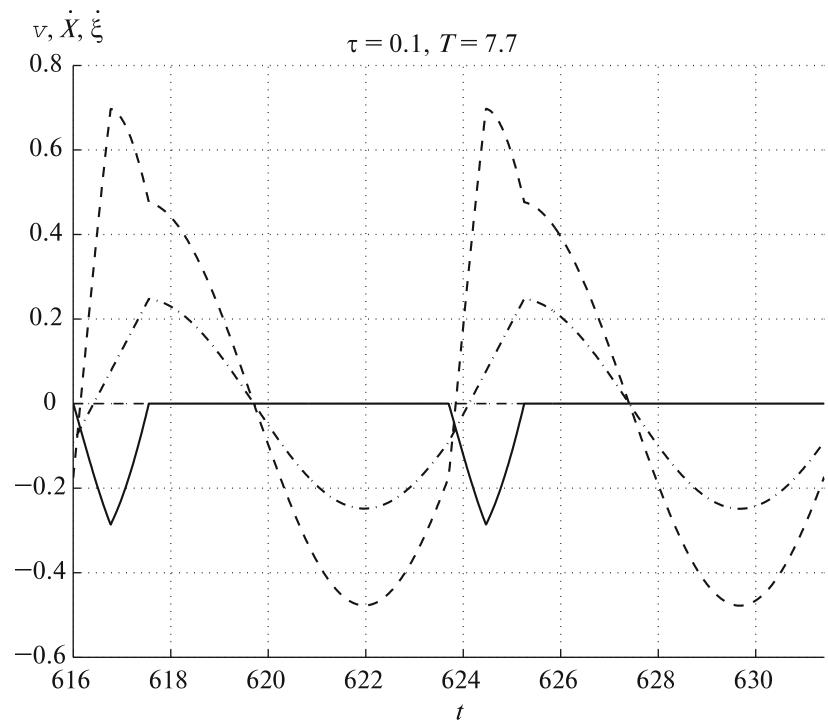
$v, \dot{X}, \dot{\xi}$
$\tau = 0.1,\ T = 7.7$
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
616
618
620
622
624
626
628
630
t

Fig. 7.

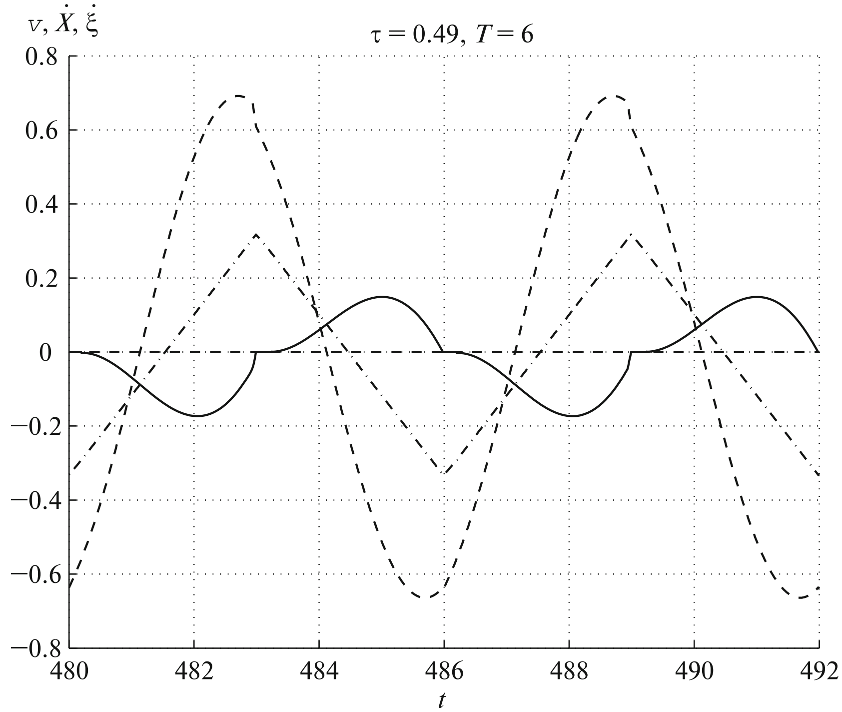
$v, \dot{X}, \dot{\xi}$
$\tau = 0.49,\ T = 6$
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
480
482
484
486
488
490
492
t

Fig. 8.

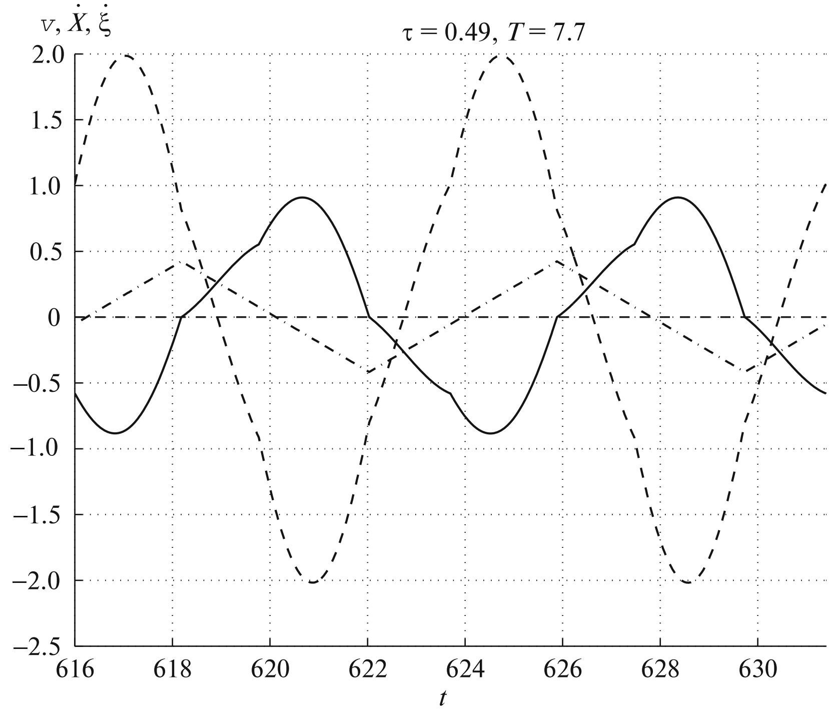
$v, \dot{X}, \dot{\xi}$
$\tau = 0.49,\ T = 7.7$
2.0
1.5
1.0
0.5
0
−0.5
−1.0
−1.5
−2.0
−2.5
616
618
620
622
624
626
628
630
t

Fig. 9.

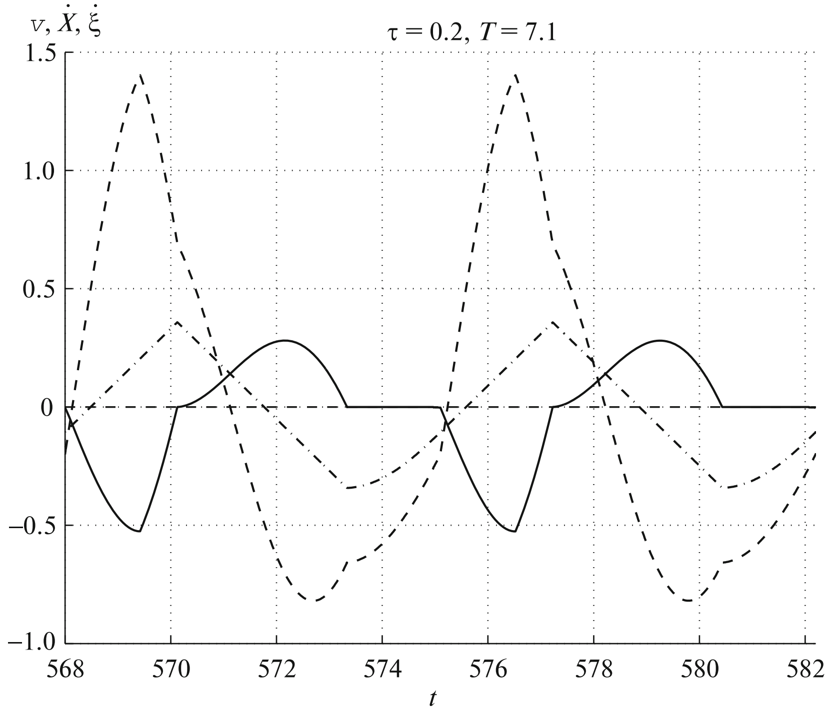

Fig. 10.

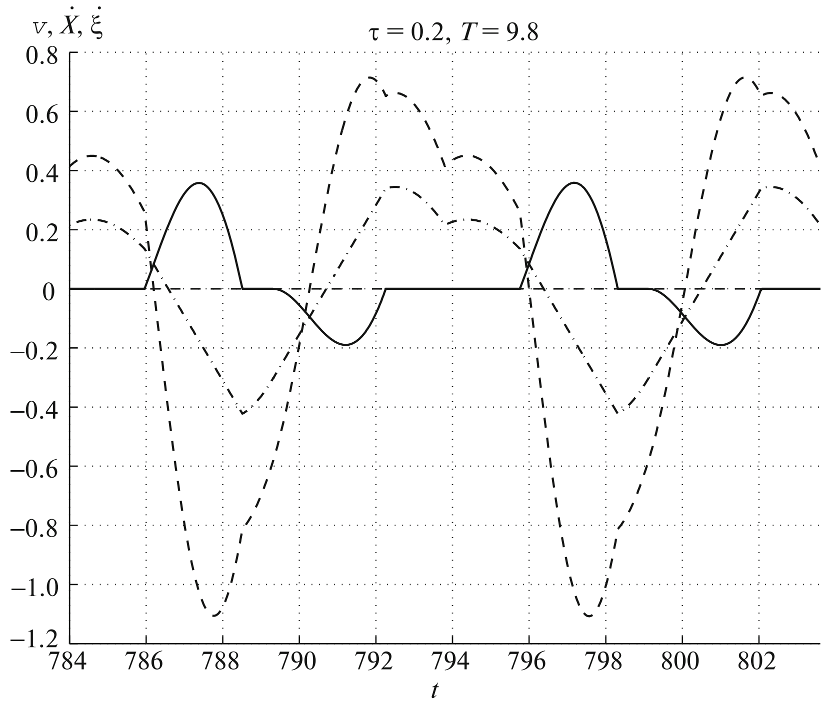

Fig. 11.

As can be seen from this figure, the period of the robot's steady state motion can be defined so that each period contains a time interval during which the housing moves forward, two time intervals during which the housing moves backward, and a time interval during which the housing rests.

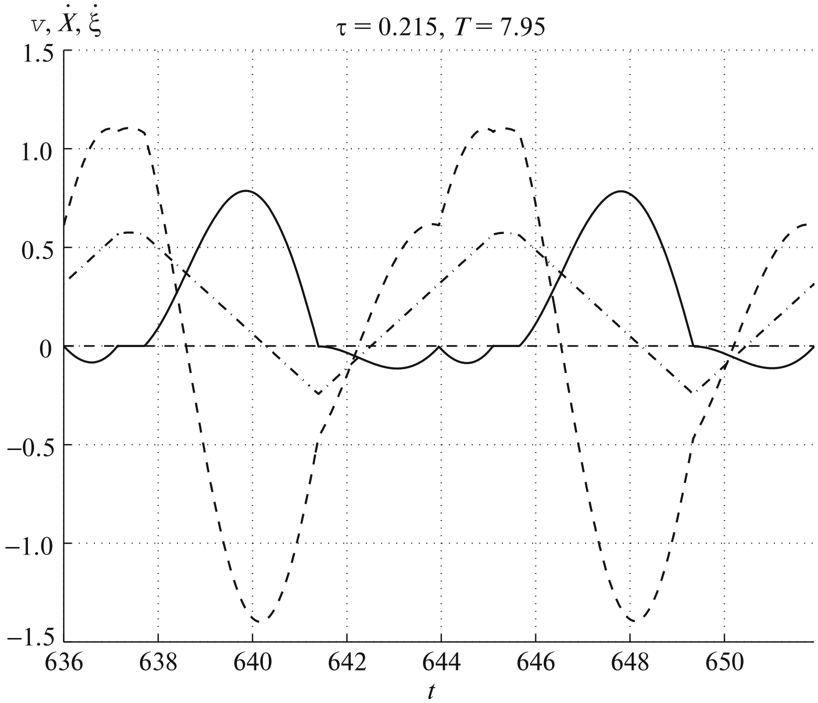

Fig. 12.

CONCLUSIONS

The direction and magnitude of the capsule robot velocity, in which the internal body is attached to the housing by a spring, can be controlled by periodically changing the interaction force between the housing and the internal body in the pulse-width mode. In this mode, the control force is constant in sign, and the period and duty cycle of the control signal are adjustable parameters. The duty cycle is defined as the relative duration of the active portion of the period for which the control force is nonzero. The monotonic change of the period with a constant duty cycle implies a continuous change in the average velocity of the robot in the steady state motion. At certain critical values of the period, the velocity changes in sign. The change in the velocity sign is associated with the resonance phenomena in the oscillatory link of the system. The change of the duty cycle for a constant period also leads to a change in the magnitude and direction of the velocity of the robot's motion. If the duty cycles are τ and $1 - \tau$, the robot moves at the same speed but in opposite directions. Thus, the system can be controlled by both a change in the period and a change in the duty cycle of the control signal. The control perfomed by changing the duty cycle for a fixed period corresponding to the absolute maximum of the robot's average velocity seems to be the most reasonable way. In this case, the change of the duty cycle makes it possible to cover the entire range of possible robot velocities. The dependence of the velocity of the robot housing relative to the medium and the core velocity relative to the housing on time in the steady state motion mode has a different shapes for different values of the excitation parameters. In particular, the robot housing may or may not have motion intervals in the direction opposite to the average velocity, and in the presence of motion intervals in both directions, it may or may not have the intervals during which the housing is at rest relative to the medium.

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REFERENCES

1. F. L. Chernous'ko, "On the motion of a body containing a movable internal mass," Dokl. Phys. **50**, 593 (2005).
2. F. L. Chernous'ko, "Analysis and optimization of the motion of a body controlled by means of a movable internal mass," J. Appl. Math. Mech. **70**, 819 (2006).

3. F. L. Chernous'ko, "The optimal periodic motions of a two-mass system in a resistant medium," J. Appl. Math. Mech. **72**, 116 (2008).
4. H. B. Fang and J. Xu, "Dynamic analysis and optimization of a three-phase control mode of a mobile system with an internal mass," J. Vibrat. Control **17** (1), 19–26 (2011).
5. T. Yu. Figurina, "Optimal motion control for a system of two bodies on a straight line," J. Comput. Syst. Sci. Int. **46**, 227 (2007).
6. N. N. Bolotnik and T. Yu. Figurina, "Optimal control of the rectilinear motion of a rigid body on a rough plane by means of the motion of two internal masses," J. Appl. Math. Mech. **72**, 126 (2008).
7. N. N. Bolotnik, T. Yu. Figurina, and F. L. Chernous'ko, "Optimal control of the rectilinear motion of a two-body system in a resistive medium," J. Appl. Math. Mech. **76**, 1 (2012).
8. A. G. Egorov and O. S. Zakharova, "The energy-optimal motion of a vibration-driven robot in a resistive medium," J. Appl. Math. Mech. **74**, 495 (2010).
9. H. Y. Li, K. Furuta, and F. L. Chernousko, "Motion generation of the capsubot using internal force and static friction," in *Proceedings of the 45th IEEE Conference on Decision and Control, USA, San Diego, 2006,* pp. 6575–6580.
10. Y. Liu, H. Yu, and T. C. Yang, "Analysis and control of a capsubot," in *Proceedings of the 17th IFAC World Congress, Seoul, Korea, July 6–11, 2008,* pp. 756–761.
11. A. A. Farahani, A. A. Suratgar, and H. A. Talebi, "Optimal controller design of legless piezo capsubot movement," Int. J. Adv. Robotic Syst. **10**, 126 (2013).
12. M. N. Huda, H. Yu, and S. O. Wane, "Self-contained capsubot propulsion mechanism," Int. J. Automat. Comput. **8** (3), 348–356 (2011).
13. L. Sun, P. Sun, and X. Qin, "Study on micro robot in small pipe," in *Proceedings of the United Kingdom Automatic Control Council UKACC International Conference on Control'98, Swansea, UK, Sept. 1–4, 1998* (IET, 1998), pp. 1212–1217.
14. V. G. Chashchukhin, "Simulation of dynamics and determination of control parameters of inpipe minirobot," J. Comput. Syst. Sci. Int. **47**, 806 (2008).
15. V. G. Gradetskii, M. M. Knyaz'kov, L. F. Fomin, and V. G. Chashchukhin, *Mechanics of Miniature Robots* (Nauka, Moscow, 2010) [in Russian].
16. K. Zimmermann, I. Zedis, N. Bolotnik, and M. Pivovarov, "Dynamics of a two-module vibration-driven system moving along a rough horizontal plane," Multibody Syst. Dyn. **22**, 199–219 (2009).
17. K. Zimmermann, I. Zedis, and C. Behn, *Mechanics of Terrestrial Locomotion with a Focus on Nonpedal Motion Systems* (Springer, Heidelberg, 2009).

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