ISSN 1064-2269, Journal of Communications Technology and Electronics, 2018, Vol. 63, No. 8, pp. 922–927. © Pleiades Publishing, Inc., 2018.
Original Russian Text © I.N. Kartashov, M.V. Kuzelev, 2018, published in Radiotekhnika i Elektronika, 2018, Vol. 63, No. 8, pp. 852–857.

On the Theory of Beam-Plasma Instability in the Nonstationary Plasma

I. N. Kartashov[a, *] and M. V. Kuzelev[a, **]

[a] *Moscow State University, Faculty of Physics, Moscow, 119991 Russia*

**e-mail: igorkartashov@mail.ru*

***e-mail: kuzelev@mail.ru*

Received October 27, 2017

Abstract—The beam instability in the plasma with density slowly varying with time has been investigated using the potential approximation. It has been shown that a frequency shift occurs in the amplified signal propagating in such an electrodynamic system; analytical relations for determining the value of this shift have been obtained. It is demonstrated that the frequency is shifted most strongly as the plasma density decreases, when the Langmuir frequency of plasma electrons approaches the signal frequency and the instability increment increases with time.

DOI: 10.1134/S1064226918080089

Experiments on microwave plasma electronics have led to the implementation of plasma amplifiers [1, 2] and microwave oscillators [3, 4] with a wide (about an octave) tuning range of the operating frequency, which are promising for an increase in the radiation power and the length of the radiation pulse. Since not all of these advantages were completely implemented, experimental [1–5] and theoretical [6–9] studies on different aspects of the beam-plasma interaction have been continued. Microwave plasma sources represent metal waveguides filled with plasma and penetrated by the electron beam. Development of the beam-plasma instability leads to modulation of the electron beam and excitation of an electromagnetic wave in the system. One of problems of such microwave plasma sources is the plasma nonstationarity, which can be related to either the conventional decay of plasma (in long-pulse systems) or its additional ionization, as well as to the plasma escape from the working region during injection of a high-current electron beam [10]. This study is aimed at investigation of beam-plasma instabilities in nonstationary plasma systems. In real systems, the plasma density becomes nonstationary in the first place, and the characteristic plasma density variation time considerably exceeds the period of excited oscillations.

Studies of excitation and propagation of waves in a nonstationary medium have a long history. A well-known mechanism of radiation of electromagnetic waves by a charge uniformly moving in an inhomogeneous medium is the transition radiation, which was first studied in [11]. A similar phenomenon was observed in a homogeneous medium, but with a time-dependent refractive index [12–14]. Nonstationary processes play an important role in problems of propagation and reflection of electromagnetic waves in the optical range. The permittivity of the medium can sometimes vary with time. This variation can be related to the processes of relaxation in the medium, laser pumping of the medium, or its ionization by an external source. Moreover, characteristic permittivity variation times can be larger than the wave period (slow variation), comparable with it, or stepwise (very fast as compared to the field period). In [15, 16], the theory of wave propagation in media with time-dependent electromagnetic properties was presented. The consideration was based on the exact solution of the Maxwell equations for specially selected time dependences of the permittivity. In particular, in [17], the change in the frequency of a signal reflected from an immobile nonstationary medium was determined. In addition, other cases of wave propagation, reflection, or excitation can be considered as nonstationary, in particular, reflection from a moving ionization front or development of the beam-plasma instability in systems with a short electron beam pulse [18].

Let us consider a homogeneous cold electron plasma. We assume that the unperturbed electron density $n_0(t)$ is a slowly varying function of time. The plasma is penetrated by a homogeneous electron beam moving at constant velocity u directed along the z axis. The dynamics of the electron subsystem of the plasma is described by the system of hydrodynamic equations

$$\frac{\partial n_p}{\partial t} + \vec{\nabla}\left(n_p \vec{v}_p\right) = \frac{\partial n_0(t)}{\partial t}, \quad \frac{\partial \vec{v}_p}{\partial t} + \left(\vec{v}_p \vec{\nabla}\right)\vec{v}_p = \frac{e}{m}\vec{\nabla}\Phi. \quad (1)$$

Here, $n_p(t,\vec{r})$ and $\vec{v}_p(t,\vec{r})$ are the density and velocity of plasma electrons, $e = |e|$ and m are the electron charge and mass, and Φ is the potential of the electric field. The right-hand side of the continuity equation is the source of variation in the electron density. As was mentioned above, this variation may be due to the decay of the plasma or its additional ionization, escape to the walls, etc. Let us linearize system (1) with respect to small perturbations of the initial state by representing the unknown functions in the form

$$n_p = n_0(t) + \delta n_p(t,\vec{r}), \quad \vec{v}_p = \delta\vec{v}_p(t,\vec{r}),$$
$$\Phi = \delta\Phi(t,\vec{r}).$$

As a result, we obtain

$$\frac{\partial \delta n_p}{\partial t} + n_0(t)\vec{\nabla}\delta\vec{v}_p = 0, \quad \frac{\partial \delta\vec{v}_p}{\partial t} = \frac{e}{m}\vec{\nabla}\delta\Phi. \qquad (2)$$

Two equations of system (2) can be easily reduced to one equation for the electron density of the plasma

$$\frac{\partial^2 \delta n_p}{\partial t^2} - \frac{n_0'(t)}{n_0(t)}\frac{\partial \delta n_p}{\partial t} = -\frac{e}{m}n_0(t)\vec{\nabla}^2\delta\Phi. \qquad (3)$$

The dynamics of the beam electrons is described by the system analogous to system (1):

$$\frac{\partial n_b}{\partial t} + \vec{\nabla}(n_b\vec{v}_b) = 0, \quad \frac{\partial \vec{v}_b}{\partial t} + (\vec{v}_b\vec{\nabla})\vec{v}_b = \frac{e}{m\gamma^3(v_b)}\vec{\nabla}\Phi. \quad (4)$$

The unperturbed beam density is assumed to be stationary; the relativistic effects in the motion of the beam electrons were taken into account; $\gamma(v_b) = \left(1 - v_b^2/c^2\right)^{-1/2}$ is the relativistic factor, and c is the velocity of light. Specifying perturbations of the beam density and velocity in the form

$$n_b = n_{b0} + \delta n_b(t,\vec{r}), \quad \vec{v}_b = \vec{u} + \delta\vec{v}_b(t,\vec{r}),$$

we write the system of equations of the electron beam linearized relative to perturbations:

$$\begin{aligned} &\frac{\partial \delta n_b}{\partial t} + u\frac{\partial \delta n_b}{\partial z} + n_{b0}\vec{\nabla}\delta\vec{v}_b = 0, \\ &\frac{\partial \delta\vec{v}_b}{\partial t} + u\frac{\partial \delta\vec{v}_b}{\partial z} = \frac{e}{m\gamma^3}\vec{\nabla}\delta\Phi. \end{aligned} \qquad (5)$$

Here, it is assumed that velocity $\vec{u}$ contains only the z component, $\vec{u} = \{0,0,u\}$, and $\gamma = \left(1 - u^2/c^2\right)^{-1/2}$. Similarly to the case of system (2), we reduce system of equations (5) to one equation for dynamics of the beam electron density

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)^2\delta n_b = -\frac{e}{m\gamma^3}n_{b0}\vec{\nabla}^2\delta\Phi. \qquad (6)$$

The potential of the electric field is determined from the Poisson equation

$$\vec{\nabla}^2\Phi = 4\pi e\left(n_p + n_b - n_i\right), \qquad (7)$$

where n_i is the density of ions neutralizing the charge of the electron subsystem. Ions are assumed to be immobile in view of their large mass. Linearizing Eq. (7) with respect to small perturbations, we write the Poisson equations in the form

$$\vec{\nabla}^2\delta\Phi = 4\pi e\left(\delta n_p + \delta n_b\right). \qquad (8)$$

Substituting (8) into (3) and (6), we arrive at the system of two equations describing the dynamics of the beam-plasma instability in the nonstationary plasma

$$\begin{aligned} &\left[\frac{\partial^2}{\partial t^2} - \frac{n_0'(t)}{n_0(t)}\frac{\partial}{\partial t} + \omega_p^2(t)\right]\delta n_p = -\omega_p^2(t)\delta n_b, \\ &\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)^2 + \omega_b^2\gamma^{-3}\right]\delta n_b = -\omega_b^2\gamma^{-3}\delta n_p. \end{aligned} \qquad (9)$$

Here, we introduced plasma frequencies of beam electrons $\omega_b^2 = 4\pi n_b e^2/m$ and plasma electrons $\omega_p^2(t) = 4\pi n_0(t)e^2/m$. To perform further calculations, it is convenient to pass from system (9) to one equation for perturbations in the beam. Specifically, substituting δn_p from the second equation of system (9) into the first equation, we obtain

$$\begin{aligned} &\left[\frac{\partial^2}{\partial t^2} - \frac{n_0'(t)}{n_0(t)}\frac{\partial}{\partial t} + \omega_p^2(t)\right] \\ &\times\left[\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial z}\right)^2 + \omega_b^2\gamma^{-3}\right]\delta n_b = \omega_b^2\gamma^{-3}\omega_p^2(t)\delta n_b. \end{aligned} \qquad (10)$$

We are interested in the solutions at sufficiently large times t, so that all processes in the investigated region $z > 0$ caused by passage of the beam front were completed and the solution $\delta n_b(t,z)$ was fully determined by boundary conditions at $z = 0$ (the amplifier problem). Since we have the second-order derivative with respect to coordinate in (10), two conditions should be specified at the boundary $z = 0$. Physically, it seems the most natural to specify modulation of the beam velocity. This modulation can be implemented by transmitting the initially homogeneous monovelocity beam through a short accelerating interval with a small-amplitude ac voltage with frequency ω specified at its ends. Then, upon propagation of the electron beam in free space, the velocity modulation will lead to the density modulation. Thus, using Eqs. (5) with zero right-hand part corresponding to free motion of the beam, one can show that, at the boundary of the beam-plasma interaction region at $z = 0$, modulation of the longitudinal component of the beam velocity and density has the form

$$\begin{aligned} &\delta v_{bz}(t, z = 0) = uB\exp(-i\omega t), \\ &\delta n_b(t, z = 0) = -in_bC\exp(-i\omega t). \end{aligned} \qquad (11)$$

Dimensionless coefficient B specifies modulation of the beam velocity and dimensionless coefficient C determines modulation of the beam density; ratio C/B is a real positive number proportional to the length of the free drift space. If immediately after the accelerating interval the beam enters the beam-plasma interaction region, the density modulation does not occur and coefficient C appears to be zero. In order to use boundary conditions (11) together with Eq. (10), it is convenient to transform them by expressing the velocity modulation via the spatial derivative of the density modulation. Using system (5) instead of first boundary condition (11), we obtain

$$\frac{\partial \delta n_b(t, z=0)}{\partial z} = n_b \frac{\omega}{u}(C - iB)\exp(-i\omega t). \quad (12)$$

Applying the Laplace transform to Eq. (10) with respect to the coordinate $z > 0$ and retaining the analogous designation for the image

$$\delta n_b(t,k) = \int_0^{+\infty} \delta n_b(t,z)\exp(-ikz)\,dz, \quad (13)$$

we obtain

$$\left[\frac{\partial^2}{\partial t^2} - \frac{n_0'(t)}{n_0(t)}\frac{\partial}{\partial t} + \omega_p^2(t)\right]\left[\left(\frac{\partial}{\partial t} + iku\right)^2 + \omega_b^2\gamma^{-3}\right]\delta n_b - \omega_b^2\gamma^{-3}\omega_p^2(t)\delta n_b = \left[\omega^2 - \omega_p^2(t) - i\omega\frac{n_0'(t)}{n_0(t)}\right] \times [(\omega - ku)C + i\omega B]\, n_b u \exp(-i\omega t). \quad (14)$$

The integral in (13) is determined for negative values with sufficiently large absolute values of $\mathrm{Im}\,k < k'' < 0$ ($\delta n_b(t,z)$ increases at $z > 0$ not faster than the exponent). In the range $\mathrm{Im}\,k > k''$ in the plane of complex k, values of $\delta n_b(t,k)$ should be considered as the analytical continuation of Eq. (13).

The right-hand part of Eq. (14) corresponds to boundary conditions (11). Since plasma density $n_0(t)$ varies slowly with time, different terms of Eq. (14) have different orders of smallness with respect to the parameter $\varepsilon = T/\tau \ll 1$, where T is the oscillation period and τ is the characteristic time of plasma density variation. We will seek the solution of this equation using the perturbation theory by separating the orders of smallness. Specifically, the forced solution of Eq. (14) can be written in the form

$$\delta n_b = (A_0(t) + A_1(t))\exp(-i\omega t), \quad (15)$$

where $A_0(t)$ has the zero order of smallness $\sim \varepsilon^0$, while $A_1(t)$ has the first order of smallness $\sim \varepsilon^1$, both amplitudes are slow functions of time, i.e., $A_{0,1} = A_{0,1}(\varepsilon t)$. The orders of smallness higher that the first order will be ignored.

In the zero order of smallness, Eq. (14) is reduced to the equation for amplitude $A_0(t)$

$$\left(\left[\omega^2 - \omega_p^2(t)\right]\left[(\omega - ku)^2 - \omega_b^2\gamma^{-3}\right] - \omega_b^2\gamma^{-3}\omega_p^2(t)\right)A_0 = \left[\omega^2 - \omega_p^2(t)\right]\left[(\omega - ku)C + i\omega B\right] n_b u. \quad (16)$$

Its solution can be easily written as

$$A_0 = \frac{(\omega - ku)C + i\omega B}{(\omega - ku)^2 - \dfrac{\omega_b^2\gamma^{-3}\omega^2}{\omega^2 - \omega_p^2(t)}}\, n_b u. \quad (17)$$

To recover solution $\delta n_b(t,z)$, we perform the inverse Laplace transform

$$\delta n_b(t,z) = \frac{1}{2\pi}\int_{-\infty+ik''}^{+\infty+ik''} \delta n_b(t,k)\exp(ikz)\,dk, \quad (18)$$

where the integration contour lies in the plane of complex k in parallel to the real axis below all singularities of the integrand. According to Eq. (17), $\delta n_b(t,k)$ has two first-order poles at

$$k_\pm = \frac{\omega}{u}[1 \pm i\delta(t)], \quad \delta(t) = \frac{\omega_b\gamma^{-3/2}}{\left(\omega_p^2(t) - \omega^2\right)^{1/2}}. \quad (19)$$

Then, we assume that the external signal frequency $\omega < \omega_p(t)$ and the wave is amplified. For the signs chosen in Eq. (19), the minus sign corresponds to the wave amplified in the positive direction of the z axis. In order to calculate the integral, we close the integration contour by a semi-circle of an infinitely large radius in the domain of $\mathrm{Im}\,k > k''$. The integral over this semicircle is zero. Then, calculation of integral (18) is reduced to calculation of the integrand residues at poles (19). These calculations yield

$$\delta n_b(t,z) = \frac{in_b}{2\delta(t)}\exp\left(-i\omega t + i\frac{\omega}{u}z\right) \times \left([B - \delta(t)C]\exp\left(-\frac{\omega}{u}\delta(t)z\right) - [B + \delta(t)C]\exp\left(\frac{\omega}{u}\delta(t)z\right)\right). \quad (20)$$

Relationship (20) does not differ from the solution of the problem in the case of the stationary plasma, where the plasma frequency is formally considered to be time-dependent. In fact, exactly such solution for the beam-plasma instability was used in [19].

In the first order of smallness, Eq. (14) can be reduced to the equation for amplitude $A_1(t)$:

$$\Big(\big[\omega^2-\omega_p^2(t)\big]\big[(\omega-ku)^2-\omega_b^2\gamma^{-3}\big]-\omega_b^2\gamma^{-3}\omega_p^2(t)\Big)A_1$$
$$+\,2i\Big((\omega-ku)\big(\omega^2-\omega_p^2(t)\big)+\omega\big((\omega-ku)^2-\omega_b^2\gamma^{-3}\big)\Big)$$
$$\times\frac{\partial A_0}{\partial t}-i\omega\big((\omega-ku)^2-\omega_b^2\gamma^{-3}\big)\frac{n_0'(t)}{n_0(t)}A_0 \quad (21)$$
$$=-i\omega\frac{n_0'(t)}{n_0(t)}\big[(\omega-ku)C+i\omega B\big]n_bu.$$

Substituting A_0 from (17), we have for $A_1(t)$

$$A_1=\left((ku)^2-\omega^2-\omega^2\frac{2(\omega-ku)^2-\omega_b^2\gamma^{-3}}{\omega^2-\omega_p^2(t)}\right)$$
$$\times\frac{(\omega-ku)C+i\omega B}{\left((\omega-ku)^2-\dfrac{\omega_b^2\gamma^{-3}\omega^2}{\omega^2-\omega_p^2(t)}\right)^3}\frac{i\omega\omega_b^2\gamma^{-3}n_bu}{\big(\omega^2-\omega_p^2(t)\big)^2}\frac{d\omega_p^2(t)}{dt}. \quad (22)$$

Performing the inverse Laplace transform, as was made to obtain Eq. (20), we determine the first-order correction. Then, we assume that $C=0$ (the beam enters the region of interaction with the plasma without density perturbation) and retain only solutions corresponding to the wave increasing due to the beam-plasma instability. Taking into account the first order of smallness, we have

$$\delta n_b(t,z)=-\frac{in_b}{2\delta(t)}\left(1+\alpha\frac{d\omega_p^2(t)}{dt}\right)B$$
$$\times\exp\left(-i\omega t+i\frac{\omega}{u}z+\frac{\omega}{u}\delta(t)z\right), \quad (23)$$

where

$$\alpha(t,z)=\alpha'(t,z)+i\alpha''(t,z),$$
$$\alpha'(t,z)=\frac{1-\delta(t)\omega z/u}{4\omega\big(\omega^2-\omega_p^2(t)\big)}\omega z/u,$$
$$\alpha''(t,z) \quad (24)$$
$$=\frac{\big(2\omega^2-\omega_p^2(t)\big)\big(1-\delta(t)\omega z/u\big)-\delta^2(t)\omega_p^2(t)(\omega z/u)^2}{4\omega\big(\omega^2-\omega_p^2(t)\big)^2}.$$

At a sufficiently slow plasma density variation with time, we can write (23) in the approximate form

$$\delta n_b(t,z)\approx-\frac{in_b}{2\delta(t)}$$
$$\times B\exp\left(-i\omega t+i\varphi(t)+i\frac{\omega}{u}z+\frac{\omega}{u}\delta(t)z\right), \quad (25)$$

introducing the time-varying increment of the wave phase

$$\varphi(t)=\arctan\left(\frac{\alpha''d\omega_p^2(t)/dt}{1+\alpha'd\omega_p^2(t)/dt}\right)\approx\alpha''\frac{d\omega_p^2(t)}{dt}. \quad (26)$$

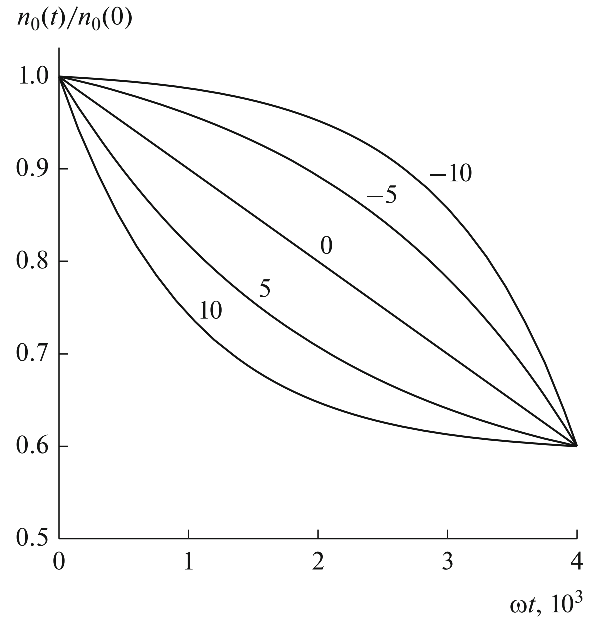

Fig. 1. Time dependence of decreasing plasma density at $\beta = 0, 5, -5, 10$, and -10.

The time variation of the wave phase will be recorded as the frequency shift of the amplified signal by

$$\Delta\omega=-\frac{d\varphi(t)}{dt}\approx-\frac{d}{dt}\left(\alpha''\frac{d\omega_p^2(t)}{dt}\right). \quad (27)$$

To perform the numerical analysis of Eq. (27), we take parameters similar to those used in the experiments on the plasma–beam interaction [1, 2]. We specify the time dependence of the plasma electron density in the form

$$n_0(t)=n_0(0)\left(1\mp\frac{t_f}{\tau}\frac{1-\exp(-\beta t/\tau)}{1-\exp(-\beta t_f/\tau)}\right), \quad (28)$$

where t_f is the observation time; by the time t_f, the plasma density changes by a factor of $1\mp t_f/\tau$. The minus sign corresponds to the decreasing plasma density and the plus sign, to the increasing one. Parameter β characterizes the shape of time profile (28); in particular, at $\beta=0$, we have linear time variation in the plasma density. We choose the following parameter values: the Langmuir plasma and beam frequencies $\omega_p(0)/\omega=1.5$ and $\omega_b\gamma^{-3/2}/\omega=0.3$, the characteristic plasma density variation time $\omega\tau=10^4$, the system length $z/uT=7$, and $\omega t_f=4\times10^3$. These parameter values correspond to the situation when the plasma density noticeably changes in times of several hundreds of oscillation periods; the system length corresponds to seven wavelengths. By the instant of time t_f, the plasma density changes by 40%.

Figure 1 shows time dependences of the plasma density at $\beta = 0, 5, -5, 10$, and -10. In all cases, the

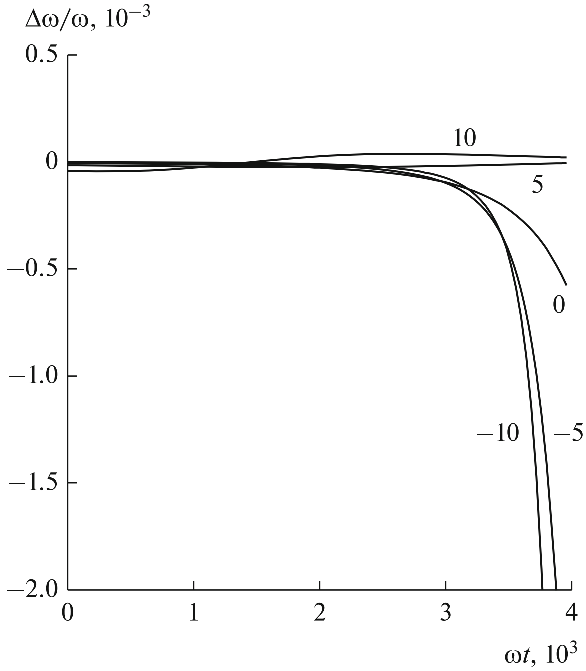

Fig. 2. Time dependence of the frequency shift of the amplified signal for decreasing plasma density at $\beta = 0$, 5, −5, 10, and −10.

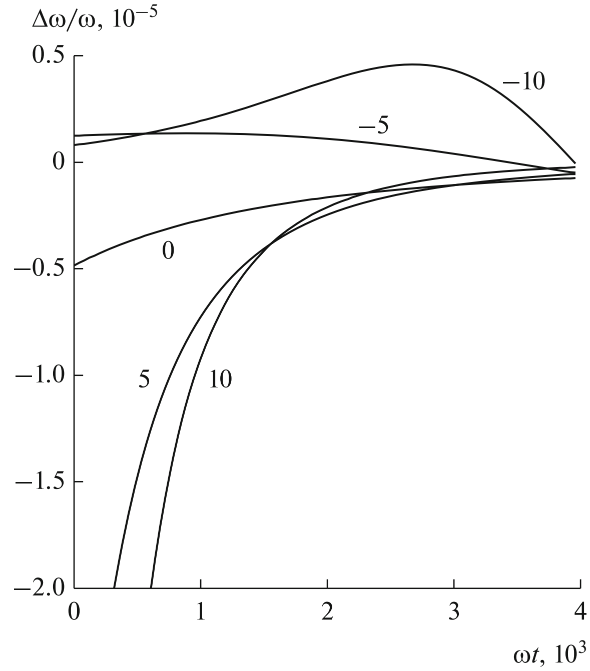

Fig. 4. Time dependence of the frequency shift of the amplified signal for increasing plasma density at $\beta = 0$, 5, −5, 10, and −10.

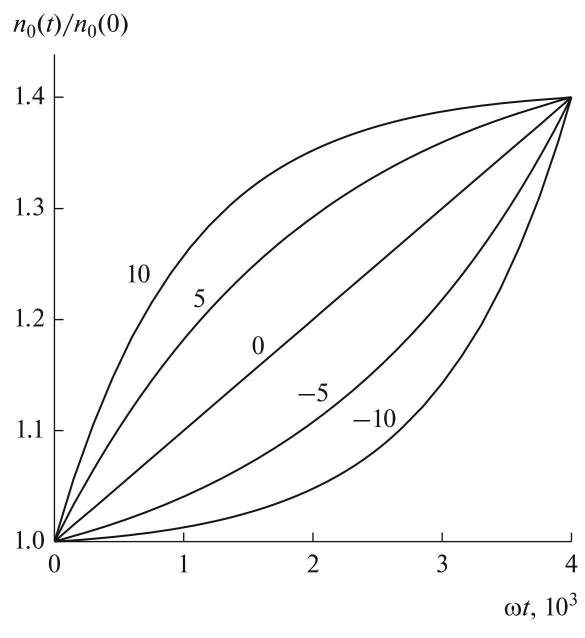

Fig. 3. Time dependence of increasing plasma density at $\beta = 0$, 5, −5, 10, and −10.

plasma density decreases. Figure 2 shows corresponding time dependences of the amplified signal frequency $\Delta\omega/\omega$. The increment of the wave phase $\varphi(t)$ is determined by both the plasma density variation rate and the value of α'', which, being negative, decreases as ω_p approaches ω. At positive values of β, the plasma density first rapidly decreases; in this case, the value of α'' is relatively small; then, at the end of the pulse, the plasma density decreases slowly, while the value of $|\alpha''|$, on the contrary, grows. This behavior determines a sufficiently slow variation in the increment of the wave phase $\varphi(t)$ and, consequently, small values of $\Delta\omega/\omega$. At negative values of β, the plasma density rapidly varies at the end of the pulse along with rapid variation in $|\alpha''|$, which leads to a strong decrease in frequency.

Figures 3 and 4 show similar dependences for the increasing plasma density (the plus sign in Eq. (28)). In general, the frequency shift in Fig. 4 is smaller than in Fig. 2. This difference is explained by the fact that, at the same depth of variation in the plasma density, the value of α'' varies slower than in the case of decreasing density of plasma electrons. The frequency drops the most abruptly at the beginning of the pulse for positive values of parameter β, when the plasma electron density increases most rapidly.

Thus, the change in the frequency of the signal amplified in the case of the beam-plasma instability, which is caused by the minor nonstationarity of the plasma, is significant and can be measured by available experimental techniques [19]. This circumstance should be taken into account in studies of microwave plasma radiators and, especially, their spectral characteristics.

REFERENCES

1. P. S. Strelkov, I. E. Ivanov, and D. V. Shumeiko, Plasma Phys. Rep. **38**, 488 (2012).
2. P. S. Strelkov, V. P. Tarakanov, I. E. Ivanov, and D. V. Shumeiko, Plasma Phys. Rep. **40**, 640 (2014).
3. D. K. Ul'yanov, R. V. Baranov, O. T. Loza, S. E. Ernyleva, and I. L. Bogdankevich, Tech. Phys. **58**, 1503 (2013).
4. O. T. Loza, D. K. Ul'yanov, and R. V. Baranov, Tech. Phys. **56**, 413 (2011).
5. P. S. Strelkov, V. P. Tarakanov, I. E. Ivanov, and D. V. Shumeiko, Plasma Phys. Rep. **41**, 492 (2015).
6. I. N. Kartashov, M. V. Kuzelev, A. A. Rukhadze, and N. Sepehri Javan, Tech. Phys. **50**, 298 (2005).
7. I. N. Kartashov, M. V. Kuzelev, and N. Sepehri Javan, Plasma Phys. Rep. **31**, 244 (2005).
8. I. N. Kartashov, M. V. Kuzelev, and A. A. Rukhadze, Tech. Phys. **51**, 151 (2006).
9. I. N. Kartashov, M. V. Kuzelev, and A. A. Rukhadze, Plasma Phys. Rep. **35**, 169 (2009).
10. V. P. Tarakanov and E. G. Shustin, Plasma Phys. Rep. **33**, 130 (2007).
11. V. L. Ginzburg and I. M. Frank, J. Exp. Theor. Phys. **16**, 15 (1946).
12. V. L. Ginzburg, Radiophys. Quantum Electron. **16**, 386 (1973).
13. V. L. Ginzburg and V. N. Tsytovich, Sov. Phys. JETP **38**, 65 (1974).
14. N. S. Erokhin, S. S. Moiseev, and L. A. Nazarenko, Sov. Phys. JETP **42**, 67 (1975).
15. A. B. Shvartsburg, Physics – Uspekhi **43**, 1201 (2000).
16. A. B. Shvartsburg, Physics – Uspekhi . **48**, 797 (2005).
17. A. B. Shvartsburg, Quantum Electron. **28**, 193 (1998).
18. E. V. Rostomyan, Plasma Phys. Controlled Fusion **50**, 085019 (2008).
19. I. N. Kartashov, M. V. Kuzelev, P. S. Strelkov and V. P. Tarakanov, Plasma Phys. Rep. **44**, 289 (2018).

Translated by E. Bondareva