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ELECTRON AND ION OPTICS

The Paraxial Model of the Electron-Optical System of a Planar Gyrotron with the *T*-Mode Emission

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Abstract—The issues of shaping a ribbon beam with elliptical and rounded-rectangular cross sections in an external magnetic field directed at an angle to the cathode are discussed based on the classical paraxial theory.

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INTRODUCTION

Problems of analysis of magnetron injection guns were discussed in [1–5] both in the context of synthesis [1–3] and with the use of methods for numerical integration of partial differential beam equations. The model problem of a planar magnetron with spatial trajectories in the presence of an arbitrary orientation of the magnetic field at the cathode is examined in [1], and studies [2–5] are focused on narrow tubular fluxes. The authors of [1–3] turned to the known exact solution with the multiplicative separation of variables in spherical coordinates, which characterizes a flux from a cone-type cathode in an inhomogeneous magnetic field (in particular, in a homogeneous field directed along the cone axis) [6–8], and used it to examine scenarios with emission limited by a spatial charge and temperature.

Since the theory of tubular electron beams [9, 10] in the ρ-mode of emission does not allow for the possibility that the magnetic field can be directed at an angle to the cathode, it is not applicable to magnetron injection guns. However, these restrictions are lifted (even for relativistic fluxes) in the case of *T*-mode emission.

The geometrized tubular beam theory, which is free from the limitations of the paraxial approach, is used in [11] to construct a combined model of an axially symmetric magnetron injection gun with a wide emission belt in the ρ-mode of emission. The first approximation of the theory with the detailed characterization of the near-cathode region with the second derivative of the cathode curvature on the beam axis and the third derivative of the emission current density taken into account is used to calculate trajectories (the most conservative model elements).

Purely numerical methods are used in the recent studies of a planar gyrotron [12–14] to calculate the electron-optical system. A model with the detailed characterization of the near-cathode region with the second derivative of the cathode curvature on the beam axis and the third derivative of the electrical field normal to the cathode taken into account (the *T*-mode emission) is constructed in [15] based on the geometrized theory of planar electron fluxes in the second approximation. The geometrized approach has a certain advantage in accuracy over the paraxial approximation used in the present study (see [16]). At the same time, the model based on the paraxial theory of curved ribbon beams, which is a degenerate axially symmetric version of the theory of narrow annular fluxes, deserves attention owing to its simplicity combined with an adequate description of the near-cathode region, which is lacking in the majority of trajectory analysis programs.

In addition to regularization of asymptotic series near a curved cathode, complete singularity extraction, which allows one to use the method of multiple scales to construct the simplest solution in the analysis of shaping electrodes, is performed in the present study. Relativistic fluxes with their basic current tube being an internal surface with zero self-magnetic field are considered.

1. PARAXIAL EQUATIONS OF A CURVED RIBBON BEAM

Let the directrix of a cylindrical surface (the basic current tube, i.e., the beam axis) with arc length l be defined by parametric equations

$$y = Y_0(l), \quad z = Z_0(l). \tag{1}$$

We introduce a curvilinear coordinate system l, s, x (s is the distance along the normal to the axis, and x is the cyclic Cartesian coordinate) that is related to Cartesian coordinates y, z as

$$y = Y_0(l) + Z_0'(l)s, \quad z = Z_0(l) - Y_0'(l)s. \qquad (2)$$

Axis curvature k_1 is written as

$$k_1 = Z_0'Y_0'' - Y_0'Z_0'' = \frac{Y_0''}{Z_0'}, \quad Y_0'^2 + Z_0'^2 = 1. \qquad (3)$$

The equations of a relativistic curved ribbon beam with an inhomogeneous distribution of spatial charge density ρ in the cross section are derived in [10]. The thermal emission conditions can be satisfied only by beams with similar current tubes $s = \xi f(l)$, characterized by the following equations:

$$\begin{gathered}\tilde{V}_l(\tilde{V}_l f')' = \left[\frac{\rho}{1+\tilde{U}} - N(l)\right] f + \bar{P}\omega(l),\\ \tilde{V}_l = (1+\tilde{U})V_l, \quad \tilde{V}_x = (1+\tilde{U})V_x, \quad \omega = 2k_1\tilde{V}_x\\ + \Omega_l + \frac{V_x}{V_l}\Omega_x, \quad N = (1+\tilde{U})U'' + 2k_1^2\tilde{V}_l^2 + \Omega_l^2 \qquad (4)\\ + \Omega_x^2 + 2k_1\tilde{V}_l\Omega_x - \tilde{V}_x\Omega_s' - \tilde{E}_s^2,\\ E_s = (1+\tilde{U})k_1V_l^2 + V_l\Omega_x - V_x\Omega_l, \quad \bar{P} = \text{const}.\end{gathered}$$

Equations (4) and the subsequent relationships are written in the relativistic normalization (the speed of light is the characteristic velocity) that excludes all physical constants of the adopted system of units. The following designations of on-axis quantities are adopted: U is the potential; V_l, V_x are the velocity components; $\tilde{V}_l$, $\tilde{V}_x$ are the components of momentum $\vec{p}$, which tend to V_l, V_x in the nonrelativistic limit (in all other cases, a tilde marks the terms vanishing at nonrelativistic velocities); Ω_l, Ω_s, and Ω_x are the components of magnetic field intensity $\vec{H}$; and E_s is the normal electrical field on the axis.

The flux parameters satisfy the following relationships:

$$\begin{gathered}2U + \tilde{U}^2 = \tilde{V}_l^2 + \tilde{V}_x^2, \quad \tilde{V}_x = \int \Omega_s dl, \quad p_l = \tilde{V}_l\\ + \left(-\frac{V_x}{V_l}\frac{\bar{P}}{f} + k_1V_l + \Omega_x\right)s, \quad p_x = \tilde{V}_x + \left(\frac{\bar{P}}{f} - \Omega_l\right)s,\\ \rho = \frac{J}{V_l f}, \quad H_l = \Omega_l + \left(\Omega_s' - k_1\Omega_l - \tilde{\rho}V_x\right)s, \qquad (5)\\ H_s = \Omega_s + \left(-\Omega_l' + k_1\Omega_s\right)s, \quad H_x = \Omega_x + \tilde{\rho}V_l s,\\ \varphi = U + E_s s + \frac{1}{2}(\rho - \bar{N})s^2, \quad \bar{N} = U'' - k_1E_s.\end{gathered}$$

Here, φ is the potential, and J is the emission current density that is constant in the case of similar current tubes.

2. REGULARIZATION OF PARAXIAL EXPANSIONS

Let us introduce deformed longitudinal coordinate ζ:

$$\zeta = l - \Lambda s^2, \quad \Lambda = \text{const}, \qquad (6)$$

which is used to define a curved emitting surface:

$$\zeta = 0, \quad l = \Lambda s^2. \qquad (7)$$

The solution in the near-cathode region in the case of T-mode emission can be constructed in the form of expansions in half-integer powers of the longitudinal coordinate $t = l^{1/2}$ [10]:

$$\begin{gathered}U = U_2t^2 + U_3t^3 + \ldots, \quad V_l = V_{l1}t + V_{l2}t^2 + \ldots,\\ V_x = V_{x2}t^2 + V_{x3}t^3 + \ldots, \quad \rho = \rho_{-1}t^{-1} + \rho_0t^0 + \ldots,\\ k_1 = k_{10} + k_{11}t + \ldots, \quad f = f_0 + f_2t^2 + \ldots, \qquad (8)\\ \Omega_l = \Omega_{l0} + \Omega_{l2}t^2 + \ldots, \quad \Omega_s = \Omega_{s0} + \Omega_{s2}t^2 + \ldots,\\ \Omega_x = \Omega_{x0} + \Omega_{x2}t^2 + \ldots.\end{gathered}$$

The regularization requirements [17] (the lack of increase in the singularity order of the asymptotic series terms) can be reduced in this case to the requirement that the functions at s and s^2 in formulas (5) should behave near the cathode in the same way as the first terms of these series. The couplings to coefficients from (8) that are involved in regularization and follow from paraxial equation (4), the energy integral, and the current conservation equation in (5) should be added to the corresponding relationships.

The following is then obtained:

$$\begin{gathered}V_{x2} = \Omega_{s0}, \quad V_{x3} = 0, \quad V_{x4} = \frac{1}{2}\Omega_{s3} - \Omega_{s0}\tilde{E}; \quad \Omega_{x0} = 0,\\ \bar{P} = \Omega_{l0}f_0; \quad \kappa_1 = 2\Lambda; \quad U_2 = E, \quad V_{l1} = \sqrt{2E},\\ \rho_{-1} = \frac{J}{\sqrt{2E}}; \quad I \equiv \frac{J}{2E\sqrt{2E}}; \quad \bar{U}_3 = \frac{8}{3}I, \quad \bar{V}_{l2} = \frac{4}{3}I,\\ \bar{\rho}_0 = -\frac{4}{3}I; \quad \bar{U}_4 = -\frac{4}{3}I^2 + \Lambda, \quad \bar{V}_{l3} = -\frac{14}{9}I^2 + \frac{1}{2}\Lambda\\ - \frac{1}{2}\bar{\Omega}_{s0}^2 - \frac{3}{4}\tilde{E}, \quad \bar{\rho}_1 = \frac{10}{3}I^2 + \frac{3}{2}\Lambda + \frac{1}{2}\bar{\Omega}_{s0}^2 + \frac{3}{4}\tilde{E};\\ \bar{U}_5 = \frac{16}{9}I^3 + I\left(\frac{44}{15}\Lambda + \frac{4}{15}\bar{\Omega}_{s0}^2 + \frac{2}{5}\tilde{E}\right),\\ \bar{V}_{l4} = \frac{80}{27}I^3 + I\left(\frac{4}{5}\Lambda + \frac{4}{5}\bar{\Omega}_{s0}^2 - \frac{14}{5}\tilde{E}\right),\\ \bar{\rho}_2 = -\frac{256}{27}I^3 + I\left(-\frac{32}{15}\Lambda - \frac{32}{15}\bar{\Omega}_{s0}^2 + \frac{14}{5}\tilde{E}\right);\\ \bar{V}_{l5} = \frac{1}{2}\bar{U}_6 - \frac{418}{81}I^4 - \frac{13}{45}I^2\Lambda - \frac{83}{45}I^2\bar{\Omega}_{s0}^2\\ - \frac{1}{8}\Lambda^2 + \frac{1}{4}\Lambda\bar{\Omega}_{s0}^2 - \frac{1}{8}\bar{\Omega}_{s0}^4 - \frac{1}{2}\bar{\Omega}_{s0}\bar{\Omega}_{s2}\end{gathered}$$

$$-\left(\frac{23}{30}I^2+\frac{9}{8}\Lambda-\frac{13}{8}\bar{\Omega}_{s0}^2-\frac{23}{32}\tilde{E}\right)\tilde{E},$$

$$\begin{aligned}\rho_3 &= \frac{1}{2}\bar{U}_6+\frac{4486}{81}I^4+\frac{13883}{2160}I^2\Lambda+\frac{401}{45}I^2\bar{\Omega}_{s0}^2\\ &+\frac{19}{8}\Lambda^2-\frac{1}{12}\Lambda\bar{\Omega}_{s0}^2+\frac{3}{8}\bar{\Omega}_{s0}^4+\frac{1}{6}\bar{\Omega}_{s0}\bar{\Omega}_{s2}\\ &+\frac{1}{3}\bar{\Omega}_{l0}\bar{\Omega}_{l2}-\frac{4}{9}I\bar{\Omega}_{l0}+\frac{2}{3}k_{10}\left(2E-\bar{\Omega}_{l0}\bar{\Omega}_{s0}\right)\\ &-\left(\frac{577}{90}I^2-\frac{49}{24}\Lambda+\frac{7}{8}\bar{\Omega}_{s0}^2+\frac{5}{32}\tilde{E}\right)\tilde{E};\quad \bar{f}_2=-2\Lambda,\\ \bar{f}_3 &= -2I\tilde{E},\quad \bar{f}_4=-\bar{U}_6-\frac{256}{81}I^4-\frac{3371}{2160}I^2\Lambda\\ &-\frac{32}{45}I^2\bar{\Omega}_{s0}^2+\Lambda^2+\frac{1}{3}\Lambda\bar{\Omega}_{s0}^2-\frac{1}{3}\bar{\Omega}_{l0}\bar{\Omega}_{l2}\\ &+\frac{1}{3}\bar{\Omega}_{s0}\bar{\Omega}_{s2}-\frac{2}{3}k_{10}\left(2E-\bar{\Omega}_{l0}\bar{\Omega}_{s0}\right)+\left(\frac{152}{45}I^2-\frac{1}{6}\Lambda\right)\tilde{E},\\ \bar{f}_5 &= -\frac{7}{8}\bar{U}_7+\frac{61}{30}\bar{U}_6+\frac{20626}{1215}I^5+\frac{107237}{32400}I^4\Lambda\\ &+\frac{2099}{675}I^3\bar{\Omega}_{s0}^2-\frac{151}{120}I\Lambda^2-\frac{139}{180}I\Lambda\bar{\Omega}_{s0}^2\\ &+\frac{62}{45}Ik_{10}\left(2E-\bar{\Omega}_{l0}\bar{\Omega}_{s0}\right)+\frac{31}{45}I\bar{\Omega}_{l0}\bar{\Omega}_{l2}\\ &-\frac{53}{90}I\bar{\Omega}_{s0}\bar{\Omega}_{s2}+\frac{3}{40}I\bar{\Omega}_{s0}^4+\frac{2}{5}k_{11}\left(\bar{\Omega}_{l0}\bar{\Omega}_{s0}-2k_{10}\right)\\ &-\frac{16}{15}k_{10}^2I+I\left(-\frac{9673}{1350}I^2-\frac{1007}{360}\Lambda+\frac{2}{5}\bar{\Omega}_{l0}^2\right.\\ &\left.+\frac{3}{40}\bar{\Omega}_{s0}^2-\frac{57}{160}\tilde{E}^2\right)\tilde{E};\quad \bar{U}_k\equiv\frac{U_k}{U_2},\quad \bar{V}_{lk}\equiv\frac{V_{lk}}{V_{l1}},\\ \bar{V}_{xk} &\equiv \frac{V_{xk}}{V_{l1}},\quad \bar{\rho}_k\equiv\frac{\rho_k}{\rho_{-1}},\quad \bar{\Omega}\equiv\frac{\Omega}{V_{l1}},\end{aligned} \tag{9}$$

where coefficients U_6, U_7 in the on-axis potential expansion are arbitrary parameters and κ_1 is the on-axis cathode curvature. Regularization governs coefficients up to and including U_5, V_{l4}, ρ_2, and f_3. Subsequent coefficients are defined by antiparaxial expansions of the paraxial beam equations.

Expansions (9) allow one to move away from singularity at $\zeta = 0$ and start integrating Eq. (4), which characterizes the beam shape, in the regular domain.

Coefficient f_0 can be taken equal to unity so as to make parameter ξ determine the initial beam thickness: $s_0 = \xi f_0$.

3. THE COMPLETE SINGULARITY EXTRACTION

Regularization ensures the lack of increase in the singularity order of the asymptotic series for the flux parameters, but does not lead to complete singularity extraction. In the case of the T-mode emission, the singularity is characterized by a two-term formula with regular functions serving as coefficients at the fractional power of l, which sets the point of branching [18]:

$$\begin{aligned}&V_l = l^{1/2}V_1(l)+lV_2(l),\quad V_x=lV_{x1}(l)+l^{3/2}V_{x2}(l),\\ &U=lU_1(l)+l^{3/2}U_2(l),\quad \rho=l^{-1/2}R_1(l)+R_2(l),\\ &k_1=K_0(l)+l^{1/2}K_1(l),\quad f=F_0(l)+l^{3/2}F_1(l).\end{aligned} \tag{10}$$

On-axis magnetic field $\vec{\Omega}$ is a regular function.

Momentum components $\tilde{V}_l$, $\tilde{V}_x$ are written as

$$\begin{aligned}&\tilde{V}_l=l^{1/2}\tilde{V}_1+l\tilde{V}_2,\quad \tilde{V}_1=\left(1+l\tilde{U}_1\right)V_1+l^2\tilde{U}_2V_2,\\ &\tilde{V}_2=\left(1+l\tilde{U}_1\right)V_2+l\tilde{U}_2V_1;\quad \tilde{V}_x=\int\Omega_s dl,\\ &V_{x1}=\frac{1}{l}\tilde{V}_x\left(1+l\tilde{U}_1-\frac{l^4\tilde{U}_2^2}{1+l^2\tilde{U}_1}\right),\quad V_{x2}=-l^2\frac{V_1\tilde{U}_2}{1+l^2\tilde{U}_1}.\end{aligned} \tag{11}$$

The following relationships are derived from the energy integral:

$$\begin{aligned}&\tilde{V}_1^2=2U_1+l\tilde{U}_1^2+l^2\tilde{U}_2^2-\tilde{V}_x^2,\\ &\tilde{V}_1\tilde{V}_2=U_2+l\tilde{U}_1\tilde{U}_2.\end{aligned} \tag{12}$$

It follows from the current conservation equation that

$$\begin{aligned}&R_1=J\frac{F_0V_1+l^2F_1V_2}{\left(F_0^2-l^3F_1^2\right)\left(V_1^2-lV_2^2\right)},\\ &R_2=-J\frac{F_0V_2+lF_1V_1}{\left(F_0^2-l^3F_1^2\right)\left(V_1^2-lV_2^2\right)}.\end{aligned} \tag{13}$$

Formulas (11) and (12) are simple algebraic connections that provide an opportunity to express functions $\tilde{V}_1$ and $\tilde{V}_2$, (V_1 and V_2) in terms of known functions U_1, U_2, and Ω_s; they also define V_{x1}, V_{x2}.

Let us transform paraxial equation (4) to the form

$$\begin{aligned}&\tilde{V}_l^2\left(\tilde{V}_lf''+\tilde{V}_l'f\right)=J-\tilde{V}_lNf\\ &+\Omega_{l0}\left(2k_1\tilde{V}_l\tilde{V}_x-\Omega_l\tilde{V}_l-\tilde{V}_x\Omega_x\right).\end{aligned} \tag{14}$$

We introduce the following designations for complexes found in Eq. (14):

$$\begin{aligned}&U'=U_{l1}+l^{1/2}U_{l2},\ U_{l1}=U_1+lU_1',\ U_{l2}=\frac{3}{2}U_2+lU_2';\\ &U''=l^{-1/2}U_{ll1}+U_{ll2},\ U_{ll1}=\frac{3}{4}U_2+3lU_2'+l^2U_2'',\\ &U_{ll2}=2U_1'+lU_1'';\ \tilde{V}_l^3=l^{3/2}W_1+l^2W_2,\quad W_1=\tilde{V}_1^3\\ &+3l\tilde{V}_1\tilde{V}_2^2,\ W_2=3\tilde{V}_1^2\tilde{V}_2+l\tilde{V}_2^3;\ \tilde{V}_l^2\tilde{V}_l'=l^{1/2}W_{l1}\\ &+lW_{l2},W_{l1}=\tilde{V}_1\left[\left(1+l\tilde{U}_1\right)U_{l1}+l^2\tilde{U}_2U_{l2}-\tilde{V}_x\tilde{V}_x'\right]\\ &+l\tilde{V}_2\left[\left(1+l\tilde{U}_1\right)U_{l2}+l\tilde{U}_2U_{l2}\right],\quad W_{l2}=\tilde{V}_1\left[\left(1+l\tilde{U}_1\right)\right.\end{aligned} \tag{15}$$

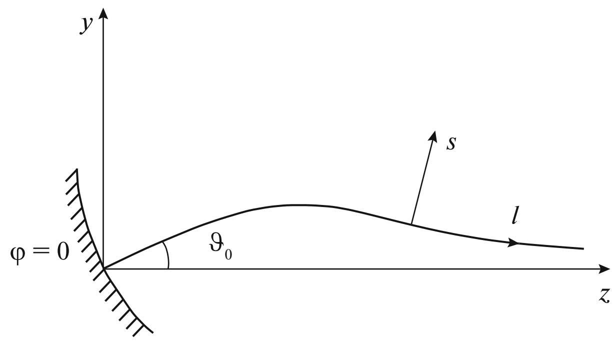

Fig. 1. Coordinate systems of a paraxial beam.

$$\times U_{l2} + l\tilde{U}_2 U_{l1}\Big] + \tilde{V}_2\Big[\big(1 + l\tilde{U}_1\big)U_{l1} + l^2\tilde{U}_2 U_{l2} - \tilde{V}_x\tilde{V}_x'\Big];$$

$$f' = F_0' + l^{1/2}D_1,\ \ D_1 = \frac{3}{2}F_1 + lF_1';\ \ f'' = F_0'' + l^{-1/2}D_2,$$

$$D_2 = \frac{3}{4}F_1 + 3lF_1'l^2F_1''.$$

The external magnetic field is $\Omega_{x0} = 0$ by virtue of the regularization conditions, and self-field is lacking owing to the choice of the beam axis; therefore, $\Omega_x \equiv 0$. In view of this, the following is obtained for field E_s and effective background N:

$$\begin{gathered} E_s = lE_1 + l^{3/2}E_2,\ \ E_1 = \left(V_1^2 + lV_2^2\right) \\ \times\left[\left(1 + l\tilde{U}_1\right)K_0 + l^2\tilde{U}_2 K_1\right] + 2lV_1V_2[(1 + l\tilde{U}_1)K_1 \\ + l\tilde{U}_2K_0],\ \ E_2 = 2V_1V_2\left[\left(1 + l\tilde{U}_1\right)K_0 + l^2\tilde{U}_2K_1\right] \\ + \left(V_1^2 + lV_2^2\right)\left[\left(1 + l\tilde{U}_1\right)K_1 + l\tilde{U}_2K_0\right] - V_{x2}\Omega_l; \\ N = l^{-1/2}N_1 + N_2,\ \ N_1 = \left(1 + l\tilde{U}_1\right)U_{ll1} + l^2\tilde{U}_2U_{ll2} \\ + 4l\left[\left(K_0^2 + lK_1^2\right)\tilde{V}_1\tilde{V}_2 + K_0K_1\left(\tilde{V}_1^2 + l\tilde{V}_2^2\right)\right] \\ + 2l^2\left(K_0\tilde{V}_1 + lK_1\tilde{V}_2\right) - 2l^3\tilde{E}_1\tilde{E}_2,\ \ N_2 = \left(1 + l\tilde{U}_1\right) \\ \times U_{ll2} + l\tilde{U}_2U_{ll1} + 2l\left[\left(K_0^2 + lK_1^2\right)\left(\tilde{V}_1^2 + l\tilde{V}_2^2\right) \right. \\ \left. + 4lK_0K_1\tilde{V}_1\tilde{V}_2\right] + \Omega_l^2 - \tilde{V}_x\Omega_s' - l^2\tilde{E}_1^2 - l^3\tilde{E}_2^2. \end{gathered} \tag{16}$$

Inserting expressions (15) and (16) into Eq. (14), we obtain two relationships for F_0'', D_2:

$$\begin{gathered} W_{l1}F_0' + lW_{l2}D_1 + l\left(W_1F_0'' + W_2D_2\right) \\ = -\left[\left(\tilde{V}_1N_2 + \tilde{V}_2N_1\right)F_0 + l\left(\tilde{V}_1N_1 + l\tilde{V}_2N_2\right)F_1\right] \\ + \Omega_{l0}\left[2\tilde{V}_x\left(K_0\tilde{V}_1 + lK_1\tilde{V}_2\right) - \Omega_l\tilde{V}_1\right], \\ l\left[W_{l2}F_0' + W_{l1}D_1 + lW_2F_0'' + W_1D_2\right] \\ = J - \left[\left(\tilde{V}_1N_1 + l\tilde{V}_2N_2\right)F_0 + l^2\left(\tilde{V}_1N_2 + \tilde{V}_2N_1\right)F_1\right] \\ + \Omega_{l0}\left[2\tilde{V}_x\left(K_0\tilde{V}_2 + K_1\tilde{V}_1\right) - \Omega_l\tilde{V}_2\right], \end{gathered} \tag{17}$$

which can be solved for these differential operators. The terms of the order of $1/l$, which compensate each other by virtue of asymptotics (9), will be present in the right-hand sides of equations obtained this way. Since regular functions F_0, F_1 are written as

$$F_0 = f_0 + lf_2 + l^2 f_4 + \ldots,\ \ F_1 = f_3 + lf_5 + \ldots, \tag{18}$$

pairs (f_0, f_2), (f_3, f_5) from (9) should be used as the initial conditions in integration of equations with F_0'', F_1'' in the left-hand side.

4. THE GOVERNING FUNCTIONS OF THE MODEL

The beam axis shape and the distributions of potential and the magnetic field components on this axis are governing functions of the model. The first two functions, which have a singularity at the cathode, require special consideration.

The beam axis shown in Fig. 1 is plotted using the results of the antiparaxial expansion theory [19]. The basic current tube has the following asymptotics in coordinates X, Y (the normal that is tangent to the cathode at the start point):

$$\begin{gathered} Y = a_4X^2 + a_5X^{5/2} + \ldots,\ \ a_4 = \frac{1}{6}\left(\frac{E'}{E} + 2\bar{\Omega}_l\bar{\Omega}_s\right), \\ a_5 = \frac{4}{3}I\left(\frac{1}{5}\frac{J'}{J} - \frac{1}{3}\frac{E'}{E} - \frac{7}{15}\bar{\Omega}_l\bar{\Omega}_s\right). \end{gathered} \tag{19}$$

Cartesian coordinates z, y are related to X, Y as

$$\begin{gathered} z = X\cos\vartheta_0 - Y\sin\vartheta_0, \\ y = X\sin\vartheta_0 + Y\cos\vartheta_0. \end{gathered} \tag{20}$$

The following is obtained for the beam axis in the z, y system:

$$\begin{gathered} y = Y_2z + Y_4z^2 + Y_5z^{5/2} + \ldots,\ \ Y_2 = \tan\vartheta_0, \\ Y_4 = \frac{a_4}{\cos^3\vartheta_0},\ \ Y_5 = \frac{a_5}{(\cos\vartheta_0)^{7/2}}. \end{gathered} \tag{21}$$

The axis curvature near the cathode is given by

$$k_1 = 2a_4 + \frac{15}{4}a_5X^{1/2} + \ldots. \tag{22}$$

At the start point,

$$k_1(0) \equiv k_{10} = \frac{1}{3}\left(\frac{E'}{E} + 2\bar{\Omega}_l\bar{\Omega}_s\right). \tag{23}$$

If magnetic field Ω_l, Ω_s and electrical field E are given, the setting of a curvature results in the determination of derivative E' along the cathode.

An orthogonal start of particles from the entire cathode surface is not guaranteed automatically in the paraxial theory (this is a well-known result of analysis of the exact beam equations). This requirement comes down to the vanishing of scalar product $\nabla\zeta \cdot \nabla\xi = 0$ identically in s, which results in a zero value of k_{10} and the determination of the field gradient at the cathode in terms of the product $\Omega_{l0}\Omega_{s0}$:

$$\zeta = l - \Lambda s^2, \quad \xi = \frac{s}{f} = \frac{1}{f_0}\left(1 - \bar{f}_2 l\right)s, \quad \bar{f}_2 = -2\Lambda,$$
$$\nabla\zeta \cdot \nabla\xi = \frac{h_1^2 - 1}{h_1^2}\frac{\bar{f}_2}{f_0}s \equiv 0, \quad h_1(0) = 1 - k_{10}s \equiv 1. \tag{24}$$

The formula characterizing the basic current tube configuration and taking asymptotics (21) into account can be written as

$$y = \left\{[1 - \exp(-P)] + [1 - \exp(-S)]^{5/2}\right\}\exp\left(-cz^4\right);$$
$$P = \left(P_1 z + P_2 z^2\right) + P_3 z^3 + \ldots, \quad P_1 = Y_2,$$
$$P_2 = Y_4 + \frac{1}{2}Y_2^2; \quad S = (S_1 z) + S_2 z^2 + \ldots, \tag{25}$$
$$S_1 = Y_5^{2/5}; \quad c = \text{const}.$$

Starting from P_3, S_2, the coefficients of polynomials P and S are arbitrary governing parameters.

Axis arc length l is related to longitudinal coordinate z as follows:

$$l = \int\sqrt{1 + y'^2}dz = \frac{1}{\cos\vartheta_0}$$
$$\times\left[z + \frac{1}{2}\sin 2\vartheta_0\left(Y_4 z^2 + Y_5 z^{5/2}\right) + \ldots\right], \tag{26}$$

thus allowing one to present the on-axis potential in the following way:

$$U = A_2 z + A_3 z^{3/2} + A_4 z^2 + A_5 z^{5/2} + \ldots,$$
$$A_2 = \frac{U_2}{\cos\vartheta_0}, \quad A_3 = \frac{U_3}{(\cos\vartheta_0)^{3/2}}, \quad A_4 = \frac{U_4}{\cos^2\vartheta_0}$$
$$+ \sin\vartheta_0 U_2 Y_4, \quad A_5 = \frac{U_5}{(\cos\vartheta_0)^{5/2}} \tag{27}$$
$$+ \frac{3}{4}\sin 2\vartheta_0\frac{U_3 Y_4}{(\cos\vartheta_0)^{3/2}} + \sin\vartheta_0 U_2 Y_5.$$

As a result, function $U(z)$ is defined by expressions

$$U = c_1\left\{[1 - \exp(-P)] + [1 - \exp(-S)]^{3/2} + \left(c_2\exp\left[-(z - z_*)^{2k}\right] - \exp\left(-z_*^{2k}\right)\right.\right.$$
$$\left.\left.\times\left[1 + 2kz_*^{2k-1}z + kz_*^{2k-2}\left(2kz_*^{2k} - 2k + 1\right)\right]\exp(-5z)^4\right)\right\},$$
$$P = \left(P_1 z + P_2 z^2\right) + P_3 z^3 + \ldots, \quad P_1 = \frac{A_2}{c_1},$$
$$P_2 = \frac{A_4}{c_1} + \frac{A_2^2}{2c_1^2}; \quad S = \left(S_1 z + S_2 z^2\right) + S_3 z^3 + \ldots, \tag{28}$$
$$S_1 = \left(\frac{A_3}{c_1}\right)^{2/3}, \quad \frac{S_2}{S_1} = \left(\frac{2}{3}\frac{A_5}{A_3} + \frac{1}{2}S_1\right);$$
$$c_1, c_2, k, z_* = \text{const}.$$

Similarly to (25), restricted coefficients are put in brackets in the expressions for P and S; the term with factor c_2 can provide the nonmonotonic potential variation in the vicinity of the anode without violating the near-cathode asymptotics.

The above definition of governing functions requires reformulating the basic equation from (4), where a prime mark now denotes differentiation with respect to z, and integration starts from $z = z_0$:

$$\frac{\tilde{V}_l}{\sqrt{1 + y'^2}}\left(\frac{\tilde{V}_l}{\sqrt{1 + y'^2}}f'\right)' = \left[\frac{\rho}{1 + \tilde{U}} - N(z)\right]f$$
$$+ \Omega_{l0}f_0\left(2k_1\tilde{V}_x + \Omega_l\right); \quad \rho = \frac{f_0 J}{fV_l};$$
$$f = f_0\left(1 + \bar{f}_2 l_0 + \bar{f}_3 l_0^{3/2} + \ldots\right), \tag{29}$$
$$f' = f_0\left(\bar{f}_2 + \frac{3}{2}\bar{f}_3 l_0^{1/2} + \ldots\right)\Big/\cos\vartheta_0, \quad z_0 = \bar{l}_0$$
$$- \frac{1}{2}\sin 2\vartheta_0\left(Y_4\bar{l}_0^{\,2} + Y_5\bar{l}_0^{\,5/2} + \ldots\right), \quad \bar{l}_0 = l_0\cos\vartheta_0.$$

Figure 2 presents the results of calculations (performed by V.A. Meleshchuk) for a beam with its parameters being the same (except the start from a concave cylindrical cathode and evolution in a plane magnetic field) as in [14].

5. THE SOLUTION OF THE EXTERIOR PROBLEM

The paraxial solution of the exterior problem in the higher approximation for an infinite ribbon beam is given by [10]

$$\varphi = U + E_s s + (\rho - \bar{N})s_e\left(s - \frac{1}{2}s_e\right) - \frac{1}{2}\bar{N}(s - s_e)^2$$
$$- \frac{1}{6}\left(3k_1\bar{N} + k_1'U' + k_1^2 E_s + E_s''\right)\left(s^3 - 3s_e^2 s + 2s_e^3\right)$$
$$- \frac{1}{12}\left[\frac{11}{2}k_1^2\bar{N} + \frac{7}{2}k_1 k_1' U' + \frac{5}{2}k_1\left(E_s'' + k_1^2 E_s\right)\right. \tag{30}$$
$$\left. - \frac{1}{2}\bar{N}''\right]\left(s^4 - 4s_e^3 s + 3s_e^4\right), \quad s_e = \xi f.$$

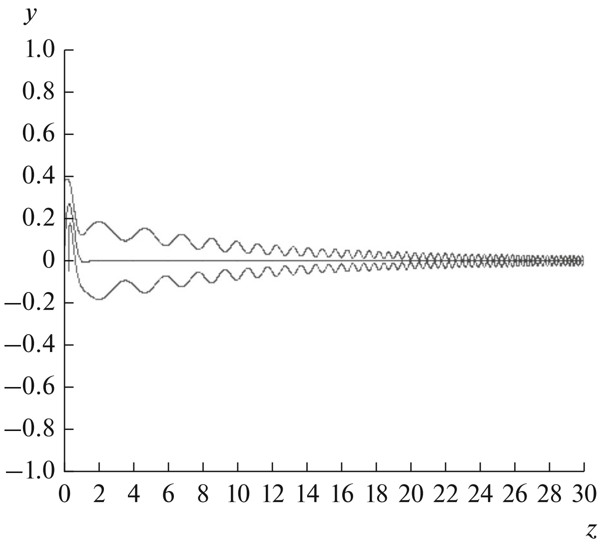

Fig. 2. Configuration of a beam from a concave cylindrical cathode.

It is known that the zero equipotential of the paraxial solution requires the maximum correction. The curve $\varphi = 0$ can be modified in considering the problem locally in the near-cathode region. The transverse gradients of field E are not included into the paraxial theory, but this value can be found by calculating the potential at the beam boundary:

$$\begin{aligned}\varphi_e &= \left[U_2 l + U_3 l^{3/2} + \ldots\right] + \left[E_{s2} l + E_{s3} l^{3/2} + \ldots\right] \\ &\times \xi(1 + f_2 l) + \frac{1}{2}\Big\{\left[\rho_2 - 6U_6 - k_{10}E_{s2}\right] l \\ &+ \left[\rho_3 - \frac{35}{4}U_7 - (k_{11}E_{s2} + k_{10}E_{s3})\right] l^{3/2} + \ldots\Big\} \\ &\times \xi^2 (1 + f_2 l + \ldots)^2.\end{aligned} \tag{31}$$

The formula for the normal electrical field at the flux boundary follows from expression (31):

$$\begin{aligned}E_e &= E + \xi E_{s2} + \frac{1}{2}\xi^2(\rho_2 - 6U_6 - k_{10}E_{s2}), \\ E_{s2} &= 2k_{10}E + \Omega_{l0}\Omega_{s0}.\end{aligned} \tag{32}$$

Note that formula (32) contains a potential expansion coefficient with a relatively high index.

It appears reasonable to use the exact results of the theory of antiparaxial expansions, which define the local equation of the zero equipotential in the following way [19]:

$$\begin{aligned}X &= b_3 Y^{3/2} + b_4 Y^2 + \ldots, \quad b_3 = \frac{4\sqrt{2}}{3} I, \\ b_4 &= \frac{1}{2}\kappa_1 - \frac{28}{3} I^2 - \frac{2}{3}\tilde{\bar{H}}_x^2, \quad \bar{H}_x \equiv \frac{H_x}{\sqrt{2E}} = \frac{J\xi}{\sqrt{2E}}.\end{aligned} \tag{33}$$

Using (23), we obtain the following for field E_e at the periphery of the cathode:

$$E_e = E + E'\xi, \quad E' = 3k_{10}E - \Omega_{l0}\Omega_{s0}. \tag{34}$$

An alternative way to calculate the shaping electrodes involves complete singularity extraction (Section 2) and the zeroth approximation of the method of multiple scales [10]. Let us preserve the terms of the order of ε^2 in formula (29) and regroup them:

$$\begin{aligned}\varphi &= U + E_s s + (\rho - U'' + k_1 E_s) s_e\left(s - \frac{1}{2}s_e\right) \\ &- \frac{1}{2}(U'' - k_1 E_s)(s - s_e)^2 = \left[U - \frac{1}{2}U''(s - s_e)^2\right] \\ &+ E_s\left[s + k_1\left(s^2 - s_e s + \frac{1}{2}s_e^2\right)\right] \\ &+ (\rho - U'') s_e\left(s - \frac{1}{2}s_e\right).\end{aligned} \tag{35}$$

We then introduce deformed longitudinal coordinate ζ and transverse coordinate η:

$$l = \zeta + \varepsilon^2 \Lambda s_e\left(s - \frac{1}{2}s_e\right), \quad \eta = \varepsilon\frac{s - s_e}{\sqrt{2}}. \tag{36}$$

Having expanded function $U(l)$ in (35) with the preserved terms of the order of ε^2 and taking into account the fact that the diverging fragment in the first square bracket represents the first terms of expansion of aggregate $\operatorname{Re} U(\zeta + i\eta)$, we have

$$\begin{aligned}\varphi &= \operatorname{Re} U(\zeta + i\eta) + E_s\left[s + k_1\left(s^2 - s_e s + \frac{1}{2}s_e^2\right)\right] \\ &+ (\rho - U'' + U'\Lambda) s_e\left(s - \frac{1}{2}s_e\right).\end{aligned} \tag{37}$$

Functions E_s, $\rho - U'' + U'\Lambda$ were regularized earlier; with "fast" coordinate z introduced for singularity characterization and "slow" coordinate Z for regular functions, the on-axis potential, by virtue of (10), takes the form that allows for the analytical continuation of just the "fast" power factor:

$$\begin{aligned}U &= ZU_1(Z) + z^{3/2}U_2(Z), \quad \operatorname{Re} U(\zeta + i\eta) \\ &= \zeta U_1(\zeta) + U_2(\zeta)\operatorname{Re}(\zeta + i\eta)^{3/2}.\end{aligned} \tag{38}$$

The above formulas are valid at a sufficient distance of the beam ends. Proposals on shaping the end region are discussed in [15].

In many scenarios, an ellipse providing a fill factor of $\pi/4 \approx 0.785$ is an acceptable approximation of a rectangular contour. The initial contour

$$\frac{x^2}{a_0^2} + \frac{s^2}{b_0^2} = 1 \tag{39}$$

is transformed into an ellipse owing to the drift velocity and compression along axis s:

$$\frac{\bar{x}^2}{a_0^2} + \frac{s^2}{b_0^2 f^2} = 1, \quad \bar{x} = x - \frac{1}{1 + \tilde{U}(l)} \int_0^l \Omega_s dl. \tag{40}$$

Under such gradients that are justified in the paraxial approximation, a beam in the l, s, $\bar{x}$ system can be regarded as a quasi-cylinder. The solution for this quasi-cylinder in the Laplace domain is given by

$$\varphi = \varphi_i + S, \quad S = -\rho c^2 \left\{ \frac{1}{4} c_1^2 [\exp(2v) - 1] + \frac{1}{2}(1 - c_1^2) v + \frac{1}{4}[\exp(-2v) - 1] + \frac{1}{2} c_1 (1 - \cosh 2v) \cos 2u \right\}, \quad c(z) = \frac{1}{2}[a_0 + b_0 f(z)], \tag{41}$$
$$c_1(z) = \frac{a_0 - b_0 f(z)}{a_0 + b_0 f(z)},$$

where φ_i is the potential in the beam.

Curvilinear coordinates u, v are related to $\bar{x}$, s as follows:

$$\bar{x} = c[\exp(-v) + c_1 \exp(v)] \cos u, \quad s = c[\exp(-v) - c_1 \exp(v)] \sin u. \tag{42}$$

CONCLUSIONS

The presented model of an electron-optical system of a planar gyrotron with the T-mode emission is simpler than the model based on the geometrized approach [15]. The higher complexity of the latter model translates into a potentially more accurate and detailed description (especially in constructing the second approximation of the theory). Both models serve as alternatives to the use of purely numerical methods in problem analysis [12–14], are based on the integration of ordinary equations, and are free from problems related to the inclusion of near-cathode singularity.

REFERENCES

1. Sh. E. Tsimring, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **20**, 1550 (1977).
2. V. N. Manuilov and Sh. E. Tsimring, Radiotekh. Elektron. (Moscow) **23**, 1486 (1978).
3. V. N. Manuilov, Radiotekh. Elektron. (Moscow) **26**, 2425 (1981).
4. V. N. Manuilov, B. V. Raiskii, and Sh. E. Tsimring, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **35**, 846 (1992).
5. V. N. Manuilov, B. V. Raiskii, E. A. Soluyanova, and Sh. E. Tsimring, Radiotekh. Elektron. (Moscow) **40**, 648 (1995).
6. P. T. Kirstein, J. Electr. Contr. **7**, 417 (1959).
7. V. W. Dryden, J. Appl. Phys. **33**, 3118 (1962).
8. V. W. Dryden, in *Proc. 4th Int. Congr. Microwave Tubes, "Microwaves", 1963* (Centrex Publ. Comp., Eindhoven, 1963), p. 587.
9. V. A. Syrovoi, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **33**, 1391 (1990).
10. V. A. Syrovoi, *Introduction to a Theory of Intense Charged-Particle Beams* (Energoatomizdat, Moscow, 2004) [in Russian].
11. V. A. Syrovoi, J. Commun. Technol. Electron. **47**, 1015 (2002).
12. V. N. Manuilov, V. Yu. Zaslavskii, M. Yu. Glyavin, et al., in *Problems of Theoretical and Applied Electronic and Ionic Optics (Proc. XI All-Rus. Seminar, Moscow, 2013)* (GNTs RF "NPO ORION", Moscow, 2013), p. 55.
13. S. A. Kishko, A. N. Kuleshov, M. Yu. Glyavin, I. V. Zotova, I. V. Zheleznov, N. S. Ginzburg, V. N. Manuilov, and V. Yu. Zaslavskii, J. Commun. Technol. Electron. **59**, 777 (2014).
14. V. N. Manuilov, V. Yu. Zaslavsky, N. S. Ginzburg et al., Phys. Plasmas **21**, 023106 (2014).
15. V. A. Syrovoi, J. Commun. Technol. Electron. **61**, 272 (2016).
16. T. M. Sapronova and V. A. Syrovoi, J. Commun. Technol. Electron. **55**, 679 (2010).
17. V. N. Danilov, *Ill-Posed Problems*, (Nauka, Moscow, 1974), Part 1, p. 46.
18. V. A. Syrovoi, J. Commun. Technol. Electron. **50**, 804 (2005).
19. V. A. Syrovoi, *A Theory of Intense Charged-Particle Beams* (Energoatomizdat, Moscow, 2004) [in Russian].

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