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Multiscale Analysis of Rhythmic Processes with Time-Varying Characteristics

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Abstract—The problem of diagnostics of structural variations in nonstationary processes is considered in the case of a degree of nonstationarity that is dependent on the frequency range. Multiscale analysis of experimental data on rhythmic processes with time-varying characteristics is carried out by the example of sleep slow wave dynamics. Possibility of improving the quality of diagnostics by selecting proper wavelet basis set functions is discussed.

Keywords: nonstationary process, wavelet analysis, rhythmic dynamics.

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In many system dynamics, nonstationarity is associated with low-frequency trends that can be eliminated by means of high-frequency filtration making possible the application of the classical methods of digital signal processing [1]. However, more complicated situations can be encountered in which the nonstationarity is related to, besides variations of the local average level, variations of the characteristics of rhythmic processes in a certain frequency range [2–6]. In studying this range, it is necessary either to use methods of the analysis of systems with time-varying parameters or estimate the characteristics of system functioning over a small amount of data and assume that the system dynamics is quasi-stationary within sufficiently short time intervals. A widely used tool for solving these problems is offered by wavelet analysis [7].

For digital processing of large amounts of experimental data, it is expedient to use fast algorithms of multiscale analysis involving the expansion over orthonormalized basis set functions, e.g., wavelets of the Daubechies family [7]. A correct choice of the basis set can be important for making effective solution of many practical tasks possible [8–12], but the large variety of wavelet functions makes this choice rather subjective.

The present work considers the problem of diagnostics of structural variations in nonstationary process in cases in which the degree of nonstationarity depends on the frequency range. Multiscale analysis of experimental data carried out by example of sleep slow wave dynamics will show how the quality of diagnostics can be improved by selecting proper wavelet basis set functions.

Multiscale analysis involves the procedure of pyramidal signal expansion with the aid of a set of mirror filters formed by dilations and translations, respectively, of scaling function $\phi(t)$ and wavelet $\psi(t)$ defined as

$$\phi_{j,k} = 2^{j/2}\phi(2^j t - k), \quad \psi_{j,k} = 2^{j/2}\psi(2^j t - k). \quad (1)$$

The expansion is carried out using basis set functions selected from Daubechies wavelet family [7]. The analyzed signal on given resolution level m is presented in the following form:

$$x(t) = \sum_k s_{m,k}\phi_{m,k}(t) + \sum_{j\geq m}\sum_k d_{j,k}\psi_{j,k}(t), \quad (2)$$

where $s_{m,\,k}$ and $d_{j,\,k}$ are the approximation and detalization coefficients, respectively. With allowance for compactness of the Daubechies wavelet carrier, variable t takes values within limits of the wavelet function, the domain of which is scale-dependent. Variability of the detalization coefficients on various scales j is characterized by the signal dispersion defined as

$$\sigma(j) = \sqrt{\frac{1}{M}\sum_{k=0}^{M-1}[d_{j,k} - \langle d_{j,k}\rangle]^2}, \quad (3)$$

which is used for diagnostics of the structural variation of signals [13]. Here, M is the number of detalization coefficients on scale j, which determines the interval of index k variation, and angle brackets denote averaging over all detalization coefficients on the given scale.

Let us consider an example of the task of diagnostic of structural variations in the signal of electric activity of the brain during sleep. The corresponding signal changes are related to the dynamics of sleep slow waves

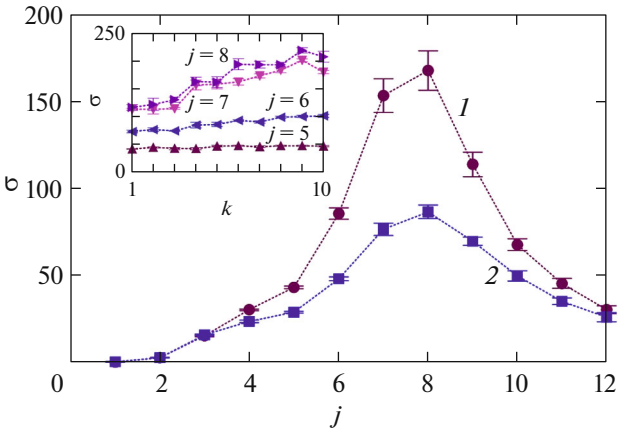

Fig. 1. Plots of the dispersion of detalization coefficients vs. scale j at (*1*) slow- and (*2*) fast-wave sleep stages as determined for D_s^8 wavelet with averaging over 2-min-long EEG segments. The inset presents the plots of dispersion σ vs. segment number k, showing a nonstationary character of signals on scales $j = 7$ and 8.

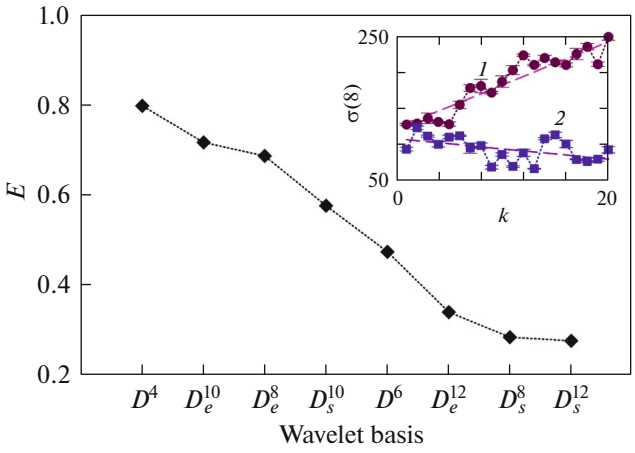

Fig. 2. Deviation E from the averaged value of σ(j) normalized in the interval from 0 to 1 and averaged over expansion levels j. The inset shows plots of dispersion σ(8) for j = 8 vs. number k of a 1-min-long EEG segment, showing different behavior at (*1*) slow- and (*2*) fast-wave sleep stages.

(0.5–4 Hz), which has drawn much interest of researchers in recent years [14, 15]. It is known that the electric activity of the human brain electric exhibits a clear sequence of alternating sleep stages overnight, which depends on the age and is affected by pathologies [16, 17]. However, despite the existence of these variations, they are still insufficiently studied.

In the present study, we have analyzed temporal variations of the characteristics of electroencephalograms (EEGs) measured in a group of seven healthy volunteers at various stages of sleep. At a signal discretization frequency of 250 Hz, scales j corresponding to slow-wave sleep fall in the interval from 6 to 9, in which maximum differences between slow-wave and fast-wave sleep stages are observed (Fig. 1, curves *1*, *2*, respectively). However, there is a peculiarity: variation of the slow-wave characteristics within one stage of sleep (see curves for $j = 7$ and 8 in the inset to Fig. 1) is accompanied by approximately constant values of the dispersion of wavelet coefficients for rhythms of higher frequencies (cf. curve *5* in the same inset). This phenomenon is of interest for diagnostics of structural variations in nonstationary processes, since the temporal variation of dispersion at the fast- and slow-wave sleep stages is different (see the inset to Fig. 2). It can also be suggested that the character of corresponding dependences will also be different between norm and pathology.

A correct choice of the wavelet basis set functions is important for practical application of the proposed approach. Although different wavelets of the Daubechies family lead to qualitatively similar results, the optimum basis set will reduce the spread of characteristics calculated from fragments of experimental data, thus increasing the reliability of the signal expansion. In calculations, we selected different domains of wavelet definition, including Daubechies functions with extremal phases (indicated by subscript "e" in Fig. 2) and least asymmetric wavelets (subscript "s"). We then carried out a search for a wavelet ensuring a minimum spread of estimated characteristics (3) in comparison to values obtained upon averaging the results of calculations using selected basis sets. With allowance for a change in detalization coefficients, these estimations were performed as follows: modulus-averaged deviation E from the averaged value of σ(j) was normalized in the interval from 0 to 1 and then averaged over expansion levels j.

Results are presented in Fig. 2 in the order of decreasing E value. As can be seen from Fig. 2, the maximum spread takes place for the D^4 wavelet having the minimum domain of definition. However, an increase in the domain of wavelet definition (and, hence, in smoothness of the wavelet function) did not always improve the situation. High E values were more frequently observed for asymmetric wavelet functions (D_e^{10}, D_e^8), whereas a decrease in the spread of estimated characteristics (3) as compared to that obtained on the averaging over selected basis set functions took place in case of the choice of the least asymmetric wavelets (D_s^8, D_s^{12}). The latter wavelets seem to be most appropriate for the problem under consideration, judging from the results of comparative analysis of the data obtained for all volunteers. By analogy with the sleep slow waves used for in the above illustrative example, an analysis of processes with nonstationary dynamics in a certain range of frequencies can be performed for various technical systems.

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COMPLIANCE WITH ETHICAL STANDARDS

All studies involving human beings were conducted in accordance with principles for human experimentation as defined in the International Conference on Harmonization Good Clinical Practice Guidelines, the National Committee on Research Ethics, the 1964 Helsinki Declaration, and later amendments or comparable ethical standards. Informed consent was obtained from each participant included in the study.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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