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Digital Audio Signal Filtration Based on the Dual-Tree Wavelet Transform

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Abstract—A new method of digital audio signal filtration based on the dual-tree wavelet transform is described. An adaptive approach is proposed that allows the automatic adjustment of parameters of the wavelet filter to be optimized. A significant improvement of the quality of signal filtration is demonstrated in comparison to the traditionally used filters based on the discrete wavelet transform.

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The need for digital filtration of noisy signals arises in solving many scientific and technical problems, in particular, for improving the quality of data reception in communication technology. A relatively new approach to solving this task is offered by wavelet filtration [1]. In recent years, methods based on the discrete wavelet transform (DWT) [2–6] have been widely used due to some advantages in comparison to filtration based on the Fourier transform [1, 2].

A conventional approach to wavelet filtration is based on the use of fast (pyramidal) algorithms of signal expansion in a basis set of wavelet functions, according to which the signal is separated into components corresponding to various (j) time scales and then the coefficients $d_{j,k}$ not exceeding a preset level C are corrected (usually, rejected) because they are most susceptible to the influence of noise. The corresponding threshold filtration is a variant of nonlinear smoothing performed in a wavelet space, which significantly depends on the selected threshold level [7]. In the case of threshold function $p(x)$ "hardly" set as

$$p(x) = \begin{cases} x, & |x| \geq C, \\ 0, & |x| < C \end{cases} \quad (1)$$

only small wavelet coefficients are rejected [7], which leaves the signal amplitude undistorted but introduces additional irregularity caused by the discontinuity of function (1). Discontinuities can be eliminated by "softly" setting the threshold function as [8]

$$p(x) = \begin{cases} x - C, & x \geq C, \\ x + C, & x \leq -C, \\ 0, & |x| \leq C, \end{cases} \quad (2)$$

but this approach leads to the correction of all coefficient and decreases the signal amplitude. This circumstance is not always substantial, since the main task of noisy signal processing is to effectively reduce the noise, after which the signal can be amplified.

An advantage of the aforementioned wavelet filtration methods is their simple implementation (e.g., using Daubechies wavelets family [3]) and fast convergence. Disadvantages include the oscillation of wavelet coefficients in the vicinity of singularities and the lack of invariance with respect to a shift of the wavelet function, which leads to unpredictable changes in the pattern of wavelet coefficients upon a shift of the singularities. In order to eliminate these drawbacks, an alternative approach was proposed based on the use of a dual-tree complex wavelet transform (DTCWT) [9, 10]. The idea of this approach is to supplement the real basis set functions by imaginary parts constructed using the Hilbert transform and pass to a complex analytic low- and high-frequency filter. In particular, complex wavelets $\psi^c(t)$ used in the framework of this approach can be represented as $\psi^c(t) = \psi^r(t) + j\psi^i(t)$, where both functions $\psi^r(t)$ and $j\psi^i(t)$ have their own orthonormalized basis sets. For signal processing, the DTCWT is separately calculated using $\psi^r(t)$ and $j\psi^i(t)$ functions, which leads to complex wavelet coefficients $d^c_{j,k} = d^r_{j,k} + jd^i_{j,k}$ characterizing the signal expansion at various (j) levels of resolution. Since the real and imaginary parts of coefficients $d^c_{j,k}$ are determined separately, the computational process consists of two stages and is implemented in the framework of the DTCWT approach [10]. During signal analysis, this approach is implemented as two independent applications of one-dimensional DWT and its algorithm reduces to two "trees" (pyramidal expansions) [9]. An analogous procedure is carried out for signal restoration using the wavelet coefficients. A characteristic feature of DTCWT is that, since the Hilbert transform for local functions with finite domain does not ensure analytic properties of $\psi^c(t)$ function, the approximately analytic scaling functions and wavelets are obtained by using special methods for constructing mirror filters such as, e.g., in [11].

In the present work, the task of developing methods for audio signal filtration from background noise, various DWT-based approaches with real basis sets of Daubechies wavelets family [7] and DTCWT approach [10] have been tested. The test signal was the short phrase "Hello, how do you do," on which white noise of variable intensity was superimposed. The resulting noisy signal was filtered and compared to the initial audio signal with calculation of the mean-square error E of filtration.

Figure 1a compares the results of signal filtration (at noise level of 20 dB) by two methods using a real basis with "soft" and "hard" setting of threshold function in the wavelet space. As can be seen, a minimum error E is achieved for the "soft" threshold, but the positions of $E(C)$ minimum correspond to different C values. From this it follows that, in order to ensure effective wavelet filtration, it is important to optimize the choice of the threshold level. A particular form of the $E(C)$ curve and the position of minimum also depend on the signal to noise ratio (SNR), but in all cases the wavelet filtration with "soft" threshold (2) provided higher quality of processing than that achieved with threshold function (1).

At the next stage, wavelet filtration results were compared for the methods employing real and complex bases. For this purpose, analogous calculations were performed using DTCWT with a basis proposed in [11]. In Fig. 1b, the results are compared to those obtained using the standard DWT with threshold function (2). It is clearly seen that an increase in the noise level to 30 dB is accompanied by a significant change in the optimum threshold level. Thus, the C value for wavelet filtration should be selected taking into account the SNR in particular signals. However, during the digital processing of audio signals recorded at the input of a receiving device, it is not possible to compare the signal to that in the absence of noise and calculate the mean-square error. This problem can be solved using the following approach. Since the main parameter for adjustment of the wavelet filtration algorithm is SNR, it is necessary to estimate the level of noise in the input signal. This estimation can be performed by calculating the power spectrum and determining the "noise pedestal" height. Then, the optimum threshold level has to be determined that corresponds to a minimum of $E(C)$ for the DTCWT based filtration. For this purpose, a test signal (e.g., harmonic function) is generated and mixed with a noise so as to obtain the same SNR as that in the received audio signal. The C value corresponding to the $E(C)$ minimum for the test signal is used for the wavelet filtration of received audio data. This procedure of wavelet filter tuning can be performed automatically.

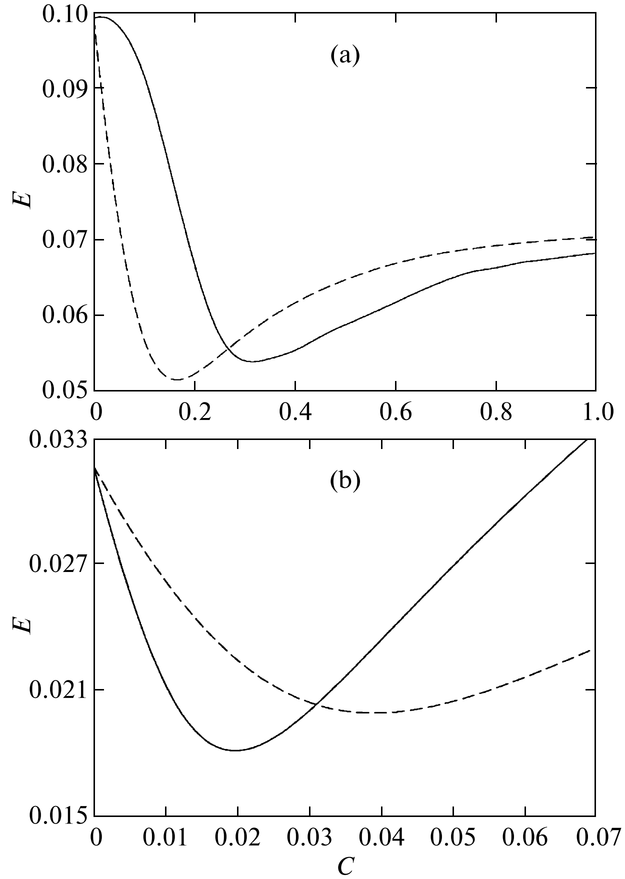

Fig. 1. Plots of mean-square error E of wavelet-based filtration of the test signal vs. threshold level C: (a) with "hard" (solid curve) and "soft" (dashed curve) threshold function for Daubechies wavelet D^8 at a noise level of 20 dB; (b) with complex basis [11] (solid curve) and real basis of Daubechies wavelet D^8 with "soft" threshold function (dashed curve) at a noise level of 30 dB.

Thus, a variant of adaptive wavelet filter capable of automatic adjustment to a particular recorded audio signal is proposed. This adaptation ensures a significant decrease in the mean-square error of reconstruction of informative signals in a receiving device. Investigations performed for a large number of test audio signals with the admixture of noise with variable intensity and statistics showed the advantage of adaptive DTCWT filtration approach in comparison to the conventional DWT-based filtration. On average, a decrease in the mean-square error of DTCWT filtration was about 7%.

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