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Determination of Parameters of a Thin Plasma Channel Based on Scattered Radiation Data

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Abstract—Approximate analytical expressions have been derived that make it possible to estimate the volume-averaged integral characteristics of a thin (in comparison with the wavelength) plasma channel based on experimental data scattered radiation on. These expressions have been used to find the parameters of a channel formed as a result of microwave discharge in air in the pressure range $P = 70–110$ Torr. These values are compared with the results obtained by the spectral method under similar experimental conditions.

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A method for diagnosing high-pressure microwave-discharge plasma structures using scattered radiation was proposed in [1, 2]. Analysis of the scattered-power dynamics (experiments [1, 3, 4] were carried out in air in the pressure range $P = 70–110$ Torr) revealed specific features of the evolution of both a single microwave streamer [1] and chain of lined-up plasma channels [4]. Data on the dipole-moment amplitude $|\mathbf{d}|$ of the formed channel (plasmoid) were obtained; based on these results, the effective scattering surface and the total charge and current amplitudes averaged over the plasmoid length were determined without any model approximations [3].

In this Letter, we derived simple analytical relations using the integral approach [5], which make it possible to estimate volume-averaged electron concentration $\bar{N}_e$, field amplitude $|\mathbf{E}_c|$ in the central channel region, absorbed power W_j, etc., based on the data on the amplitude $|\mathbf{d}|$ and plasmoid shape and sizes.

The plasmoid was in an external (not perturbed by plasma) electric field $\mathrm{Re}\{\mathbf{E}_0\exp(-i\omega t)\}$. Its longitudinal (along the vector $\mathbf{E}_0 = \mathbf{n}_z E_0$, where $\mathbf{n}_z$ is a unit vector) size is limited by the condition $2b < \lambda$ ($\lambda = 2\pi c/\omega$ and c is the speed of light), and the characteristic transverse sizes are small in comparison with the wavelength λ. Uncompensated opposite space charges arising during electron oscillations against the ion background, which is stationary at the period $T = 2\pi\omega^{-1}$, are symmetric with respect to the plane oriented perpendicular to the axis of the plasmoid (z axis) and passing through its center (the origin is aligned with the center). The parameters and characteristic sizes of the plasma dipole vibrator change only slightly for time T.

Under conditions of weak electromagnetic wave skinning, the complex amplitudes of the dipole moment d_z, electric field E_c, and electronic conductivity σ_{ec} in the center of the channel are related by the following approximate expressions [5]:

$$d_z = \frac{i}{\omega}\int d\mathbf{R} j_z(\mathbf{R}) \approx \frac{i\sigma_{\mathrm{ec}}E_c V_{\mathrm{eff}}}{\omega(1-i\vartheta)}, \tag{1}$$

$$E_c \approx \frac{E_0}{1-\dfrac{i\sigma_*}{1-i\vartheta}\Psi}, \tag{2}$$

where $j_z = \sigma_e E_z/(1 - i\vartheta)$, $\sigma_e = e^2N_e/m\nu$, ν is the electron-collision transport frequency, $\vartheta = \omega/\nu < 1$, $V_{\mathrm{eff}} = \int d\mathbf{R} f(\mathbf{R})$, $R = \sqrt{x^2+y^2+z^2}$, $f = \sigma_e/\sigma_{\mathrm{ec}}$, $\sigma_* = \sigma_{\mathrm{ec}}/\omega\varepsilon_0$,

$$\Psi = \frac{1}{4\pi}\int d\mathbf{R}\left(-\frac{\partial f(\mathbf{R})}{\partial z}\frac{\partial G(R)}{\partial z} + k^2 f(\mathbf{R})G(R)\right), \tag{3}$$

$$G(R) = \exp(ikR)/R, \quad k = \omega/c.$$

The solution to system of equations (1) and (2)

$$\sigma_*(g,h) = g + \sqrt{g^2+h}, \tag{4}$$

$$|E_c| = E_0\frac{|d_*|\sqrt{1+\vartheta^2}}{\sigma_*(g,h)V_*}, \tag{5}$$

where

$$g = \frac{\mathrm{Im}\Psi + \vartheta\,\mathrm{Re}\Psi}{(V_*/|d_*|)^2 - |\Psi|^2}, \quad h = \frac{1+\vartheta^2}{(V_*/|d_*|)^2 - |\Psi|^2},$$

depends only on the normalized dipole-moment amplitude $|d_*| = k^3|d_z|/4\pi\varepsilon_0 E_0$, normalized effective volume $V_* = k^3V_{\mathrm{eff}}/4\pi$, and form factor Ψ. The $|d_*|$ value and the plasma dipole shape and sizes are determined experimentally. However, the spatial distribution of electronic conductivity $f(\mathbf{R})$ can hardly be estimated based on discharge photographs; therefore, below we consider the integral characteristics averaged over volume V:

$$\bar{N}_e, \bar{\sigma}_e = (N_{ec}, \sigma_{ec})\eta, \quad (6)$$

where $\eta = {}^3V_{eff}/V$. The parameter η characterizes the degree of filling of volume V with plasma.

For a plasmoid with a length limited by the condition $2b < \lambda$, it is convenient to use the representation for the form factor Ψ (3) obtained by expanding function G in series in powers of kR. In the axisymmetric case,

$$f = \begin{cases} f(\xi, p_j), & \xi \le 1, \\ 0, & \xi > 1, \end{cases} \quad (7)$$

where $\xi = \sqrt{(r/a)^2 + (z/b)^2}$, $r = \sqrt{x^2 + y^2}$, and p_j (j = 1, 2, ...) are the parameters concretizing the profile $f(\mathbf{R})$, expression (3) can be written as

$$\begin{aligned} \Psi = &-\frac{1-e^2}{e^2}\left(\frac{F_2}{e} - 1\right) \\ &+ \frac{(ka)^2}{2e}\left(\left(1 + \frac{1}{e^2}\right)F_2 - \frac{1}{e}\right)\gamma_1 + \frac{2iV_*}{3} \\ &+ \frac{(ka)^2}{e}\sum_{m=2}\frac{(ikbe)^m}{m!(m+2)}\left(\left(2 - (m-1)\frac{1-e^2}{e^2}\right)F_{m+2}\right. \\ &\left. + (m-1)\frac{1-e^2}{e^4}F_m\right)\gamma_{m+1}, \end{aligned} \quad (8)$$

where $\gamma_m = \int_0^1 d\xi\xi^m f$,

$$F_2 = \frac{1}{2}\ln\frac{1+e}{1-e},$$

$$F_{m+2} = \frac{1}{m}\left(\frac{1}{e^{m-1}} + (m-1)\frac{1-e^2}{e^2}F_m\right), \quad m \ge 1.$$

Here, $e^2 = 1 - (a/b)^2$ and $V_* = k^3a^2b\gamma_2$.

In the electrostatic limit $kb \ll 1$ ($\omega \ll c/b$), the complex amplitude at the plasmoid center

$$E_c \approx \frac{E_0}{1 + \frac{i\sigma_*}{1 - i\vartheta}\frac{1-e^2}{e^2}\left(\frac{1}{2e}\ln\frac{1+e}{1-e} - 1\right)} \quad (9)$$

is independent of the distribution $f(\xi, p_j)$ within the used model. Note that formula (9) has the same form as in the case of a uniform ($f = 1$) plasma ellipsoid of revolution (see, for example, [6]).

The most important characteristic of the plasmoid forming as a result of development of a microwave streamer is absorbed power W_j (Joule heat),

$$W_f = \frac{\int d\mathbf{R}\sigma_e|E|^2}{2(1 + \vartheta^2)}, \quad (10)$$

which can be estimated, according to the aforesaid, using the following approximation:

$$W_j \approx W_0\frac{|d_*|^2}{\pi\sigma_*(g, h)V_*}, \quad (11)$$

where $W_0 = \lambda^2 c\varepsilon_0 E_0^2/2$. The main energy contribution is made in the quasi-stationary stage of plasmoid evolution under conditions of ionization–recombination quasi-equilibrium [5]. This stage begins when the electrostatic field of uncompensated space charges cannot significantly pull the streamer further along vector $\mathbf{E}_0$.

We estimate the integral characteristics of the plasma dipole formed in air as a result of discharge [1, 3]: P = 75 Torr, λ = 2.3 cm, and $|E_0| = (1.1–1.2)E_{br}$ (E_{br} is the breakdown value).

Before the quasi-stationary stage, the luminous region was in the volume, which (according to the photographs in the **kE** and **kH** planes) can be approximated by an ellipsoid of revolution with semiaxes $b \approx 0.28\lambda$ and $a \approx b/7$. For these sizes,

$$\begin{aligned} \Psi \approx &-\frac{\ln 2b/a - 1}{(b/a)^2} + \frac{(ka)^2}{2}(2\ln 2b/a - 1)\gamma_1 \\ &+ \frac{2iV_*}{3} - \frac{(k^2ab)^2}{8}\gamma_3. \end{aligned} \quad (12)$$

The luminescence intensity was maximum at the center. The normalized dipole-moment amplitude of this plasmoid was found as a result of processing the experimental data: about $|d_*| \approx 0.6$.

For estimations, we used the model distribution of the electronic conductivity:

$$f(\xi, p) = 1 - \xi^{2p}, \quad p \ge 1, \quad (13)$$

which depends on only one parameter p (Fig. 1); variation of this parameter allows one to change the degree of filling η of volume V with plasma,

$$\beta = 3\gamma_2 = \left(1 + \frac{1.5}{p}\right)^{-1} \quad (14)$$

in the range $0.4 \le \eta < 1$.

The calculation results obtained based on formulas (7)–(9) and (11) are shown in Fig. 2 in the form of dependences of the values of electron concentration $\bar{N}_e$ and conductivity $\bar{\sigma}_e$ averaged over the volume V, external-field attenuation coefficient $|E_c|/E_0$ in the plasmoid, and absorbed power W_j on

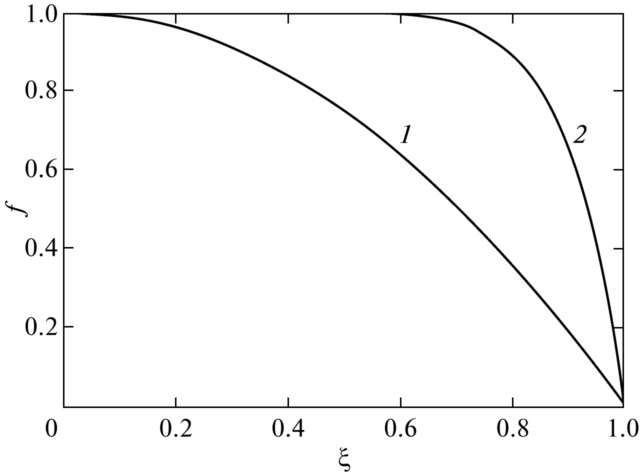

Fig. 1. Model distributions of electronic conductivity at different values of parameter p: p = (*1*) 1 and (*2*) 5.

the degree of filling η. It can be seen that the integral plasmoid characteristics under consideration change by about 30%,

$$\bar{N}_e \approx 3.5 \times (1 \pm 0.14) \times 10^{14}\ \mathrm{cm}^{-3},$$

$$\bar{\sigma}_e \approx 2.3 \times (1 \pm 0.13)\ \Omega^{-1}\ \mathrm{m}^{-1},$$

$$|E_c|/E_0 \approx 0.55 \times (1 \pm 0.14),$$

$$W_j \approx 15.8 \times (1 \pm 0.14)\ \mathrm{kW},$$

when uncertainty parameter p is varied in a wide range ($1 \le p < 500$); i.e., they are low-sensitive to deformation of electronic-conductivity profile (13).

The electron concentration values estimated by the proposed method agree with the results obtained in nitrogen according to Stark broadening of the hydrogen spectral line H_β under similar experimental conditions:

$$\lambda = 2\ \mathrm{cm}, \quad P = 70\ \mathrm{Torr}, \quad 2a \approx 0.1\ \mathrm{cm},$$

$$N_e \approx 0.9 \times 10^{14}\ \mathrm{cm}^{-3}\ [7],$$

$$\lambda = 4\ \mathrm{cm}, \quad P = 50\ \mathrm{Torr}, \quad 2a \approx 0.1\ \mathrm{cm},$$

$$N_e \approx 2 \times 10^{14}\ \mathrm{cm}^{-3}\ [8].$$

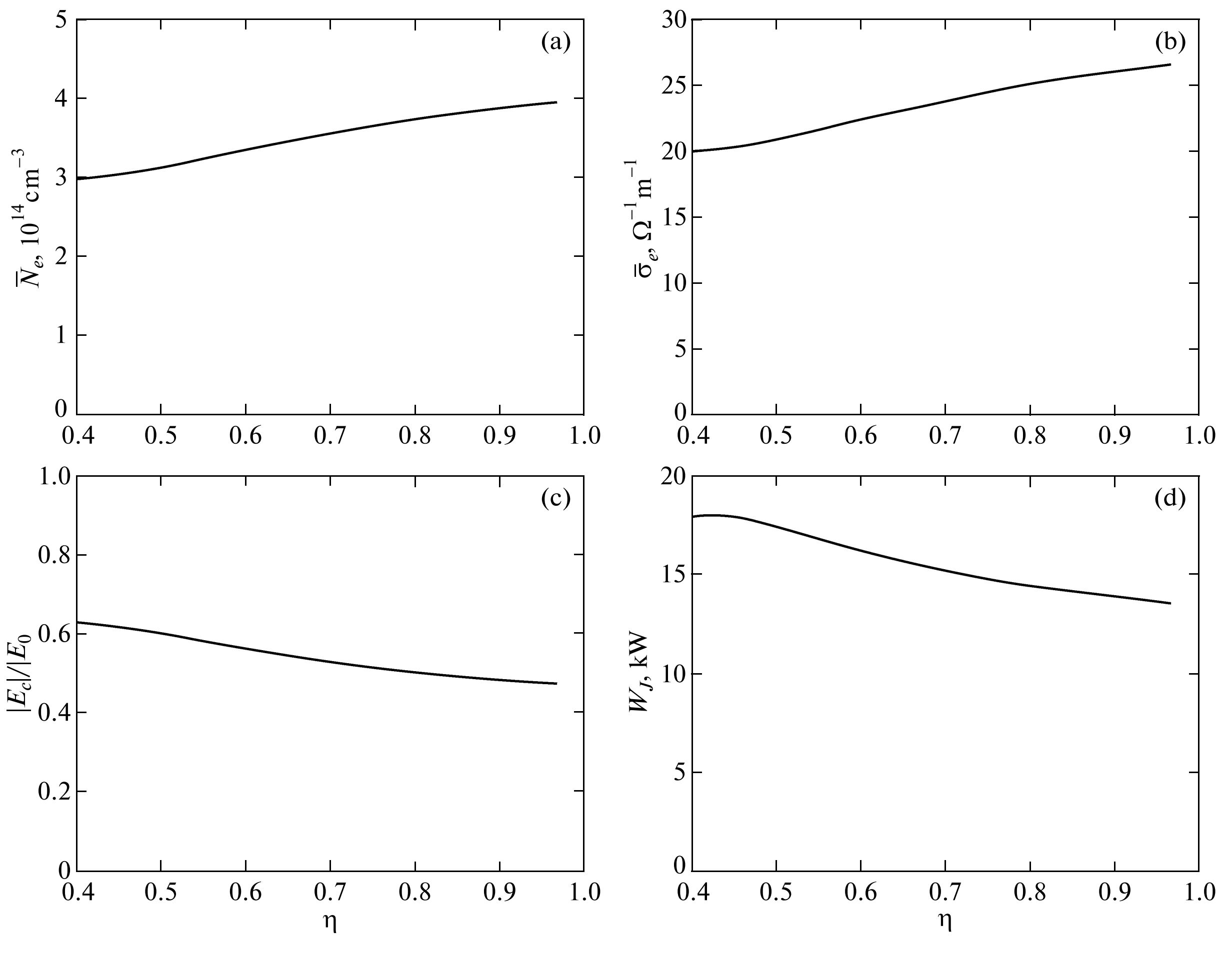

Fig. 2. Dependences of volume-averaged values of (a) electron concentration $\bar{N}_e$ and (b) conductivity $\bar{\sigma}_e$, (c) external-field attenuation coefficient $|E_c|/E_0$ in plasmoid, and (d) absorbed power W_j on degree of filling η.

To summarize, we report that the inverse problem of scattering a linearly polarized electromagnetic wave from a thin plasma dipole vibrator is solved. Development of this approach will also make it possible to analyze the scattering properties and integral characteristics of multiplasmoid structures formed upon ignition of a high-pressure microwave discharge.

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