

Quadrupole Radiation of an Uncharged Droplet That Oscillates in the Presence of Uniform Electric Field

A. I. Grigor'ev^{a,*}, S. O. Shiryayeva^a, and N. Yu. Kolbneva^a

^aDemidov State University, Yaroslavl, 150000 Russia

*e-mail: grig@uniyar.ac.ru

Received April 10, 2018; revised September 17, 2018; accepted September 17, 2018

Abstract—Capillary oscillations of an uncharged spheroid droplet of nonviscous conducting incompressible liquid in the presence of uniform electric field are analyzed in the first order of smallness with respect to the ratio of the oscillation amplitude to a linear size of the droplet and in the second order of smallness with respect to the squared ratio of the linear size to the radiation wavelength. It is shown that the electric quadrupole moment of the droplet is time-dependent due to surface oscillations, which lead to the emission of quadrupole electromagnetic waves. A mathematical model of the quadrupole electromagnetic radiation of an uncharged droplet that oscillates in the presence of electrostatic field is constructed, and the radiation intensity and frequency are estimated.

DOI: 10.1134/S1063784219030137

INTRODUCTION

Emission of electromagnetic waves has been demonstrated for an oscillating charged droplet of a perfectly conducting incompressible perfect liquid [1]. Indeed, intrinsic charge of such a droplet is distributed over the surface, charge particles exhibit accelerated motion in the course of oscillations, and, hence, electromagnetic radiation is generated in accordance with the classical physical principles (see, for example, [2]). The calculation method of [1] is reduced to derivation of a dispersion relation for an oscillating droplet of perfect liquid for the wave zone of the electric field of the droplet that has complex solutions. The imaginary part of the frequency corresponds to damping that is impossible for perfect liquid. Therefore, the emission of accelerated charges serves as the source of the energy loss.

In accordance with the electromagnetic theory [3], the radiation of a system of accelerated charges consists of a sum of dipole, quadrupole, and magnetic dipole contributions. Electromagnetic radiation is divided into multipole components using small parameter δ , which represents squared ratio of a linear size of the droplet to the radiation wavelength. The radiation intensities decrease in a row: dipole, quadrupole, and magnetic dipole components. The calculations of an oscillating charged droplet in the first order of smallness with respect to parameter ϵ [1] (ϵ is the ratio of the oscillation amplitude to a linear size of the droplet) yield only quadrupole component. The dipole radiation of an oscillating charged droplet is obtained in the calculations of the second order of

smallness with respect to parameter ϵ . However, the intensity of the dipole radiation appears to be greater than the intensity of the quadrupole radiation by 14–15 orders of magnitude.

Different results are obtained for an uncharged droplet of perfectly conducting incompressible nonviscous liquid in the presence of uniform electrostatic field: dipole radiation is observed in the first order of smallness with respect to parameters δ and ϵ [5], and quadrupole radiation is observed in the second order of smallness with respect to parameter δ and the first order with respect to parameter ϵ .

In this work, we calculate the intensity of quadrupole radiation of an uncharged droplet of perfect incompressible conducting liquid that oscillates in the presence of uniform electrostatic field.

PHYSICAL FORMULATION OF THE PROBLEM

We consider quadrupole electromagnetic radiation generated by an uncharged droplet of perfect incompressible liquid that oscillates in the presence of uniform electrostatic field \mathbf{E}_0 . The mass density of the liquid is ρ , and the coefficient of surface tension is σ . The droplet is located in vacuum, and its volume is the volume of a sphere with radius R . Capillary wave motion is always excited on the surface of the droplet due to thermal motion of water molecules. The amplitude of such motion is relatively small (about $\propto \sqrt{kT/\sigma}$, where k is the Boltzmann constant and T is the absolute temperature [6]). At temperatures of about room tempera-

ture such an amplitude is less than one angstrom for all liquids.

On the surface of a freely falling droplet that is affected by external forces (e.g., a force related to the air flow), the amplitudes of single modes of the oscillating droplet may amount to several tens of percents of the radius [7, 8]. In this case, the capillary oscillations of the surface of the droplet that is charged by induced charge lead to emission of electromagnetic waves.

In accordance with the general theory of electromagnetic waves, the intensity of the quadrupole radiation of a system of accelerated charges is calculated as [3]

$$I = \frac{1}{180c^5} \sum_{\alpha,\beta=1}^3 \left(\frac{\partial^3}{\partial t^3} D_{\alpha\beta} \right)^2, \quad (1)$$

where tensor of quadrupole moment $D_{\alpha\beta}$ is given by

$$D_{\alpha\beta}(t) = \iiint_V \gamma(\mathbf{r}, t) (3x_\alpha(t)x_\beta(t) - \delta_{\alpha\beta}r^2(t)) dV. \quad (2)$$

Here, $\gamma(\mathbf{r}, t)$ is the volume charge density, $\delta_{\alpha\beta}$ is the Kronecker delta, $x_\alpha(t)$ and $x_\beta(t)$ are the coordinates of radius vector $\mathbf{r}(t)$ in Cartesian coordinates (x, y, z) for a point that is located in droplet volume V .

External electrostatic field leads to perturbation of the equilibrium spherical shape of the droplet. In the linear approximation with respect to stationary deformation, we assume spheroid shape [9–13] with squared eccentricity e^2

$$e^2 = \frac{9E_0^2 R}{16\pi\sigma}.$$

The calculations are performed in spherical coordinates (r, θ, φ) with the origin at the center of mass of the droplet using dimensionless variables: $\mathbf{R} = \rho = \sigma = 1$. The remaining parameters are represented in fractions of characteristic values:

$$[E_0] = R^{-1/2}\sigma^{1/2}, [t] = R^{3/2}\rho^{1/2}\sigma^{-1/2}, \\ [V] = R^{-1/2}\rho^{-1/2}\sigma^{1/2}, [r] = R, \text{ and } [P] = R^{-1}\sigma.$$

We mathematically formulate the problem of the quadrupole radiation of a spheroid uncharged droplet of nonviscous incompressible conducting liquid that oscillates in the presence of external electrostatic field.

MATHEMATICAL FORMULATION OF THE PROBLEM

We assume that, at initial moment $t = 0$, equilibrium spheroid droplet $r(\theta)$ is perturbed and virtual axisymmetric perturbation $\xi(\theta, t)$ with fixed amplitude ε is significantly less than the radius of the droplet. Based on the smallness and axisymmetric character of the initial perturbation, we assume that the droplet shape is axisymmetric at any time moment. We also assume that the equation that describes the surface of the droplet in the spherical coordinates with the origin

at the center of mass is written in terms of dimensionless variables as

$$r(\theta, t) = r(\theta) + \xi(\theta, t), \quad |\xi| \ll 1.$$

We consider potential motion of the liquid and assume that the field of velocities for the liquid in the droplet $\mathbf{V}(\mathbf{r}, t) = \nabla\psi(\mathbf{r}, t)$ is fully determined by velocity potential $\psi(\mathbf{r}, t)$ [14]. The amplitude of the velocity field for the liquid has the order of smallness that is identical to the order smallness of the oscillation amplitudes on the surface of the droplet: $\psi(\mathbf{r}, t) \sim \xi(\theta, t) \sim \varepsilon$. In the vicinity of the droplet, the electric field is described using electric potential $\Phi(\mathbf{r}, t)$.

The mathematical problem of the motion of liquid in the uncharged droplet that oscillates in the presence of external electrostatic field is written as [1, 15]

$$\Delta\psi(\mathbf{r}, t) = 0, \quad \Delta\Phi(\mathbf{r}, t) = 0, \\ r \rightarrow 0: \psi(\mathbf{r}, t) \rightarrow 0, \quad r \rightarrow \infty: \Phi(\mathbf{r}, t) \rightarrow -E_0 r \cos\theta, \\ r = r(\theta) + \xi(\theta, t): \\ \frac{\partial\xi(\theta, t)}{\partial t} = \frac{\partial\psi(\mathbf{r}, t)}{\partial r} - \frac{1}{r^2} \frac{\partial\psi(\mathbf{r}, t)}{\partial\theta} \left(\frac{\partial r(\theta)}{\partial\theta} + \frac{\partial\xi(\theta, t)}{\partial\theta} \right), \quad (3) \\ \Phi(\mathbf{r}, t) = \Phi_s(t),$$

$$t = 0: \xi(\theta) = \varepsilon \sum_{j \in \Xi} h_j P_j(\mu), \quad \sum_{j \in \Xi} h_j = 1, \\ \varepsilon \ll 1, \quad \frac{\partial\xi(\theta)}{\partial t} = 0,$$

where h_j are the coefficients that determine the partial contribution of the j th oscillation mode to the total initial perturbation, Ξ is the set of numbers of the initially excited oscillation modes, $P_j(\mu)$ is the Legendre polynomial of the j th order, j is the integer, and $\mu \equiv \cos\theta$.

For closing of the above system of equations, we use general physical principles and formulate additional natural conditions for constancy of the volume, immobility of the center of mass, and zero charge of the droplet:

$$\int_V r^2 dr \sin\theta d\theta d\varphi = \frac{4}{3}\pi, \quad \int_V r r^2 dr \sin\theta d\theta d\varphi = 0, \\ V = [0 \leq r \leq r(\theta) + \xi(\theta, t), 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi], \\ \frac{1}{4\pi} \oint_S (\mathbf{n}(\mathbf{r}, t), \nabla\Phi) dS = 0, \quad (4)$$

$$S = [r = r(\theta) + \xi(\theta, t), 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi].$$

Here, we use the following notation: $\infty_s(t)$, electrostatic potential that is constant over the surface; $P = P_0 - \frac{\partial\psi}{\partial t}$, hydrodynamic pressure; P_0 , constant internal pressure in the equilibrium state; $P_E = (\nabla\Phi)^2/8\pi$, electric field pressure; $P_\sigma = \text{divn}(\mathbf{r}, t)$, capillary pressure; $\mathbf{n}(\mathbf{r}, t)$, unit vector that is orthogonal to the perturbed surface [16] given by

$$\mathbf{n}(\mathbf{r}, t) = \frac{\nabla(r - r(\theta, t))}{|\nabla(r - r(\theta, t))|_{r=r(\theta, t)}}. \quad (5)$$

The desired quantities are expanded in terms of the smallness of dimensionless oscillation amplitude ε [17]:

$$\begin{aligned} \xi(\theta, t) &= \xi^{(1)}(\theta, t) + O(\varepsilon^2), & \psi_j(\mathbf{r}, t) &= \psi_j^{(1)}(\mathbf{r}, t) + O(\varepsilon^2), \\ P(\mathbf{r}, t) &= P^{(0)}(\mathbf{r}) + P^{(1)}(\mathbf{r}, t) + O(\varepsilon^2), \\ P_\sigma(\mathbf{r}, t) &= P_\sigma^{(0)}(\mathbf{r}) + P_\sigma^{(1)}(\mathbf{r}, t) + O(\varepsilon^2), \\ P_E(\mathbf{r}, t) &= P_E^{(0)}(\mathbf{r}) + P_E^{(1)}(\mathbf{r}, t) + O(\varepsilon^2), \\ \Phi(\mathbf{r}, t) &= \Phi^{(0)}(\mathbf{r}) + \Phi^{(1)}(\mathbf{r}, t) + O(\varepsilon^2), \end{aligned} \quad (6)$$

where $\Phi^{(0)}(r, \theta)$ is the electric potential in the vicinity of the equilibrium position of the uncharged spheroid in the presence of external electric field and $\Phi^{(1)}(r, \theta, t)$ is the term of the first order of smallness added to the electric potential due to perturbation of the surface. The superscript shows the order of smallness with respect to parameter ε .

Substituting expansions (6) in expressions (3) and (4), we select problems of the zero and first order with respect to parameter ε .

Potential $\Phi^{(0)}(r, \theta)$ in the linear approximation with respect to e^2 can be obtained using a transition from the known expression of [18] for the electric potential of a prolate conducting ellipsoid in the presence of a uniform external field in the spheroidal coordinates to the spherical coordinates or with the aid of a direct solution of the electrostatic problem of the zero order of smallness with respect to parameter ε in the spherical coordinates using the perturbation method:

$$\begin{aligned} \Phi^{(0)}(r, \theta) &= E_0 r P_1(\mu) \left(\frac{1}{r^3} - 1 \right) + \frac{2}{5} \frac{1}{r^2} e^2 \\ &\times E_0 \left(P_1(\mu) + \frac{3}{2} \frac{1}{r^2} P_3(\mu) \right). \end{aligned}$$

Accurate to the terms of the first order with respect to small parameters ε and e^2 , the shape of the perturbed surface $r(\theta, t)$ is described using the equation

$$r(\theta, t) = 1 + e^2 h(\theta) + \varepsilon \sum_{j \in \Xi} M_j(t) P_j(\mu), \quad (7)$$

where amplitude coefficients $M_j(t)$ are represented as

$$\begin{aligned} M_j(t) &= h_j \cos(\omega_j t) + e^2 \left(h_{j-2} \frac{j(j-1)(3j^2 - 3j + 2)}{4(2j-1)(2j-3)(3j-4)} \right. \\ &\times (\cos(\omega_{j-2} t) - \cos(\omega_j t)) \\ &- h_{j+2} \frac{(j+1)(3j^3 + 27j^2 + 44j + 16)}{4(2j+3)(2j+5)(3j+2)} \\ &\left. \times (\cos(\omega_{j+2} t) - \cos(\omega_j t)) \right) \quad (j \in \Xi). \end{aligned}$$

In the absence of the external electric field, the second term of about e^2 vanishes and perturbation ampli-

tude $M_j(t)$ is determined by the term with frequency ω_j : $M_j(t) = h_j \cos(\omega_j t)$.

The term added to the electric potential in the vicinity of the perturbed spheroid is written as

$$\begin{aligned} \Phi^{(1)}(\mathbf{r}, t) &= E_0 \varepsilon \left(\frac{6}{35} \frac{e^2 M_3(t)}{r} + \left(\sum_{j \in \Xi} [3(\mu_{j-1}^+ M_{j-1}(t) \right. \right. \\ &+ \mu_{j+1}^- M_{j+1}(t)) + e^2 (M_{j-3}(t) l_1(j) + M_{j-1}(t) l_2(j) \\ &\left. \left. + M_{j+1}(t) l_3(j) + M_{j+3}(t) l_4(j))] r^{-(n+1)} \right) \right) P_j(\mu). \end{aligned}$$

SURFACE CHARGE DENSITY

Surface charge density $\mathbf{v}(\mathbf{r}, t)$ that is distributed over the perturbed surface of the axisymmetric surface of a perfectly conducting droplet is determined using the known formula

$$\begin{aligned} \mathbf{v}(\mathbf{r}, t) &= -\frac{1}{4\pi} (\mathbf{n}(\mathbf{r}, t), \nabla \Phi(\mathbf{r}, t))_{r=r(\theta, t)} \\ &= -\frac{1}{4\pi} (\mathbf{n}(\mathbf{r}, t), \nabla \Phi^{(0)}(\mathbf{r}) + \nabla \Phi^{(1)}(\mathbf{r}, t))_{r=r(\theta, t)}, \end{aligned} \quad (8)$$

where $r(\theta, t)$ is given by expression (7).

We use expression (8) for the components of the electric potential and the expression for the normal vector orthogonal to the perturbed surface of the spheroid in the presence of the external field

$$\begin{aligned} \mathbf{n}(\mathbf{r}, t) &= \left(1 - \frac{1}{3} e^2 \frac{\partial P_2(\mu)}{\partial \theta} \frac{\partial \xi(\theta, t)}{\partial \theta} \right) \mathbf{e}_r - \left(\frac{\partial \xi(\theta, t)}{\partial \theta} \right. \\ &\left. + \frac{1}{3} e^2 \left((1 - \xi(\theta, t)) \frac{\partial P_2(\mu)}{\partial \theta} - P_2(\mu) \frac{\partial \xi(\theta, t)}{\partial \theta} \right) \right) \mathbf{e}_\theta \end{aligned} \quad (9)$$

to obtain the surface charge density on the perturbed droplet $r(\theta, t)$ accurate to the terms on the order of $\sim E_0 e^2 \varepsilon$ and $\sim E_0^3 \varepsilon$:

$$\begin{aligned} \mathbf{v}(\mathbf{r}, t) &= \frac{1}{4\pi} \left(3E_0 \left(P_1(\mu) + \frac{2}{5} e^2 P_3(\mu) \right) \right. \\ &- \left(\frac{\partial \Phi^{(1)}}{\partial r} + E_0 \xi(\theta, t) \left(6P_1(\mu) + \frac{2}{5} e^2 (P_1(\mu) + 15P_3(\mu)) \right) \right. \\ &\left. \left. + \frac{1}{3} e^2 \left(\frac{\partial^2 \Phi^{(1)}}{\partial r^2} P_2(\mu) - \frac{\partial \Phi^{(1)}}{\partial \theta} \frac{\partial P_2(\mu)}{\partial \theta} \right) \right) \right)_{r=1}. \end{aligned}$$

Here, functions $\xi(\theta, t)$ and $\Phi^{(1)}(\mathbf{r}, t)$ are the functions of the first order of smallness with respect to parameter ε . In expression (9), \mathbf{e}_r and \mathbf{e}_θ are the unit vectors of the spherical coordinate system.

Taking into account function $\xi(\theta, t)$ from expression (7) and the solution for additional electric potential $\Phi^{(1)}(\mathbf{r}, t)$, we represent the surface charge density as an expansion in terms of the Legendre polynomials:

$$\begin{aligned} \mathbf{v}(\mathbf{r}, t) = & \frac{3}{4\pi} E_0 \left(P_1(\mu) + \frac{2}{5} e^2 P_3(\mu) + \varepsilon \sum_{j \in \Xi} (M_{j-1}(t) k_1 \right. \\ & + M_{j+1}(t) k_2 - e^2 \left(\frac{2}{35} M_3(t) \delta_{j,0} + M_{j-3}(t) k_3 + M_{j-1}(t) k_4 \right. \\ & \left. \left. + M_{j+1}(t) k_5 + M_{j+3}(t) k_6 \right) \right) P_j(\mu) \quad (j \in \Xi). \end{aligned} \quad (10)$$

Numerical factors k_1 – k_6 that depend on only subscript j are presented in the Appendix.

CALCULATION OF THE QUADRUPOLE MOMENTS

To obtain the intensity of the quadrupole electromagnetic radiation of the conducting droplet, we calculate tensor of quadrupole moment $D_{\alpha\beta}(t)$ that enters expression (1). Substituting squared radius vector $r^2 = x^2 + y^2 + z^2$ in formula (2), we obtain a general expression for the tensor of quadrupole moment in the Cartesian coordinates [3, 18]:

$$\begin{aligned} D_{\alpha\beta}(t) = & \int_V \mathbf{v}(\mathbf{r}, t) \delta(r - r(\mathbf{r}, t)) (3x_\alpha x_\beta - r^2 \delta_{\alpha\beta}) dV, \\ V = & [-r \leq x \leq r; -\sqrt{r^2 - x^2} \leq y \leq \sqrt{r^2 - x^2}; \\ & -\sqrt{r^2 - x^2 - y^2} \leq z \leq \sqrt{r^2 - x^2 - y^2}]. \end{aligned}$$

Here, $r = r(\mathbf{r}, t)$ is the equation for the surface of the droplet, $\mathbf{v}(\mathbf{r}, t)$ is the surface charge density, \mathbf{r} is the radius vector of a point on the surface of the droplet, $\delta(r - r(\mathbf{r}, t))$ is the Dirac delta function [19], and $\delta_{\alpha\beta}$ is the Kronecker delta.

For further calculation of the quadrupole moments, we use the approach of [20]. The expression for components of quadrupole moments is represented in the spherical coordinates (r, θ, φ) with the origin at the center of the droplet. The presence of delta function in the expressions for the components of the tensor of quadrupole moment makes it possible to transform integration over the volume into integration over the surface of the droplet that is perturbed due to capillary wave motion. Thus, we obtain

$$\begin{aligned} D_{11}(t) = & \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t) ((2 - 3 \sin^2 \varphi) \sin^2 \theta \\ & - \cos^2 \theta) \sin \theta d\theta d\varphi, \\ D_{22}(t) = & \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t) ((3 \sin^2 \varphi - 1) \sin^2 \theta \\ & - \cos^2 \theta) \sin \theta d\theta d\varphi, \end{aligned}$$

$$D_{33}(t) = \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t) (3 \cos^2 \theta - 1) \sin \theta d\theta d\varphi,$$

$$D_{12}(t) = D_{21}(t) = 3 \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t) \quad (11)$$

$$\times \cos \varphi \sin \varphi \sin^3 \theta d\theta d\varphi,$$

$$D_{13}(t) = D_{31}(t) = 3 \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t)$$

$$\times \cos \varphi \cos \theta \sin^2 \theta d\theta d\varphi,$$

$$D_{23}(t) = D_{32}(t) = 3 \int_S \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t)$$

$$\times \sin \varphi \cos \theta \sin^2 \theta d\theta d\varphi,$$

$$S = [r = r(\theta, t); 0 \leq \theta \leq \pi; 0 \leq \varphi \leq 2\pi].$$

Here, quantities $r(\theta, t)$, $v(\theta, t)$, and $\mathbf{n}(\mathbf{r}, t)$ are given by expressions (7), (10), and (9), respectively.

The droplet is axisymmetric, so that equation of the perturbed surface $r(\theta, t)$ is independent on the φ coordinate. After integration of expressions (11) with respect to angle φ , we obtain

$$\begin{aligned} D_{11}(t) = D_{22}(t) = & -\frac{1}{2} D_{33}(t) = \pi \int_0^\pi \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} r^4(\theta, t) \\ & \times (1 - 3 \cos^2 \theta) \sin \theta d\theta = -2\pi \int_0^\pi \frac{v(\theta, t)}{(\mathbf{n}(\mathbf{r}, t), \mathbf{e}_r)} \\ & \times r^4(\theta, t) P^2(\mu) \sin \theta d\theta, \end{aligned} \quad (12)$$

$$D_{12} = D_{21} = D_{13} = D_{31} = D_{23} = D_{32} = 0.$$

We substitute surface charge density (10) and normal vector (9) on perturbed surface of the droplet $r(\theta, t)$ in nonzero tensor components to obtain the corresponding analytical expressions in terms of amplitude coefficients $M_j(t)$:

$$\begin{aligned} D_{11}(t) = D_{22}(t) = & -\frac{1}{2} D_{33}(t) = -\frac{3}{2} E_0 \\ & \times \int_0^\pi \left(\cos \theta \left(1 + e^2 \left(3 \cos \theta - \frac{19}{15} \right) \right) \right. \\ & + \varepsilon \sum_{j \in \Xi} (M_{j-1}(t) p_1 + M_{j+1}(t) p_2 \\ & + e^2 \left(\frac{2}{35} M_3(t) \delta_{j,0} + M_{j-3}(t) p_3 + M_{j-1}(t) p_4 \right. \\ & \left. \left. + M_{j+1}(t) p_5 + M_{j+3}(t) p_6 \right) \right) P_j(\cos \theta) \left. \right) P_2(\mu) \sin \theta d\theta. \end{aligned} \quad (13)$$

Numerical coefficients p_1 – p_6 , which depend only on subscript j , can be found in the Appendix.

We take into account orthogonality of the Legendre polynomials, substitute explicit expression for $M_j(t)$, and make a transition to dimensional variables:

$$\begin{aligned}
D_{11}(t) &= D_{22}(t) = -\frac{1}{2} D_{33}(t) \\
&= -E_0 R^4 \varepsilon \left(\frac{9}{7} h_3 \cos(\omega_3 t) + e^2 \left(\frac{52}{245} h_3 \cos(\omega_3 t) \right. \right. \\
&\quad \left. \left. + \frac{2}{847} h_5 (236 \cos(\omega_3 t) + 149 \cos(\omega_5 t)) \right) \right), \\
\omega_j &= \left(\frac{\sigma}{\rho R^3} j \left((j-1)(j+2) \right. \right. \\
&\quad \left. \left. - e^2 \frac{(2j^5 + 23j^4 + 21j^3 - 17j^2 - 7j^2 - 2)}{(2j-1)(2j+1)(2j+3)} \right) \right)^{1/2}.
\end{aligned}$$

Note that the quadrupole moment of the uncharged droplet in the presence of the external field differs from zero only for the perturbed surface of the droplet (the quadrupole moment of an equilibrium uncharged spheroid in the presence of external electrostatic field is zero) and is determined by the third and fifth oscillation modes in the calculations accurate to the squared eccentricity.

We determine the maximum sum of squares of the third derivative with respect to time of the tensor components of the quadrupole moment:

$$\begin{aligned}
\sum_{\alpha, \beta=1}^3 \left(\frac{\partial^3}{\partial t^3} D_{\alpha\beta}(t) \right)_{\max}^2 &= 6E_0^2 R^8 \varepsilon^2 \left(\frac{9}{7} h_3 \omega_3^3 \right. \\
&\quad \left. + e^2 \left(\frac{52}{245} h_3 \omega_3^3 + \frac{2}{847} h_5 (236 \omega_3^3 + 149 \omega_5^3) \right) \right)^2. \quad (14)
\end{aligned}$$

Substituting expression (14) in formula (1), we find the intensity of the quadrupole electromagnetic radiation of the uncharged droplet that oscillates in the presence of electrostatic field:

$$\begin{aligned}
I &= \frac{1}{30c^5} E_0^2 R^8 \omega^2 \left(\frac{9}{7} h_3 \omega_3^3 + e^2 \left(\frac{52}{245} h_3 \omega_3^3 \right. \right. \\
&\quad \left. \left. + \frac{2}{847} h_5 (236 \omega_3^3 + 149 \omega_5^3) \right) \right)^2. \quad (15)
\end{aligned}$$

Based on expression (15), we numerically estimate the quadrupole moment of the quadrupole noise radio radiation of convective clouds.

First, note that a possible source of electromagnetic radiation is related to the oscillations of low modes of small droplets the concentration of which in the cloud is about 10^3 cm^{-3} [21]. Characteristic sizes of such droplets range from 3 to 30 μm . Maximum concentration in the cloud corresponds to size interval of 3–7 μm . The surface oscillations of the droplets are primarily determined by microphysical intracloud processes: coagulation with smaller particles, changes of the aggregate state, splitting into smaller droplets due to collisions or electrostatic instability, and electric and hydrodynamic interaction with the neighboring droplets. Experiments under natural conditions show that the oscillation amplitude of cloud droplets may amount to several tens of percents of the droplet

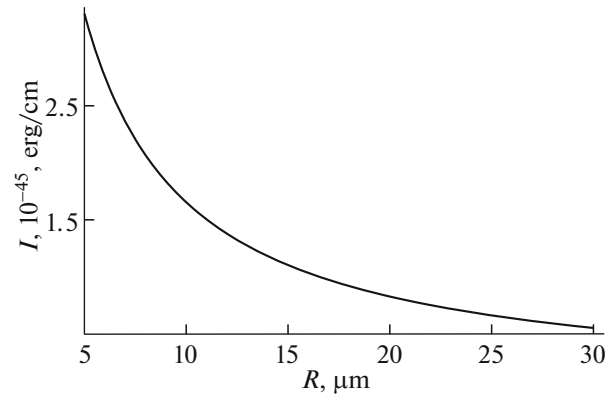


Fig. 1. Plot of the intensity of the quadrupole electromagnetic radiation of a single uncharged droplet that oscillates in the presence of electrostatic field vs. radius of an equal-size spherical droplet for $\varepsilon = 0.1$, $\sigma = 73 \text{ dyn/cm}$, $\rho = 1 \text{ g/cm}^3$, and $E_0 = 50 \text{ V/cm}$ (about $2 \times 10^4 E_{0\text{cr}}$ and $7 \times 10^4 E_{0\text{cr}}$ at $R = 3$ and $30 \mu\text{m}$, respectively).

size [7, 8]. Thus, we estimate the radiation intensity on the assumption that the ratio of the oscillation amplitude to the droplet radius $\xi(\theta, t)/R = \varepsilon$ is 0.1.

We use expression (15) to calculate the quadrupole radiation of a single uncharged droplet that oscillates in the presence of the uniform electrostatic field. For the numerical estimations we use $h_3 = h_5 = 0.5$, $\varepsilon = 0.1$, $\sigma = 73 \text{ dyn/cm}$, $\rho = 1 \text{ g/cm}^3$, $R = 30 \mu\text{m}$, and $E_0 = 50 \text{ V/cm}$ [22] (about $2 \times 10^4 E_{0\text{cr}}$ and $\sim 7 \times 10^4 E_{0\text{cr}}$ at $R = 3$ and $30 \mu\text{m}$, respectively, where $E_{0\text{cr}}$ is the critical field). Then, expression (14) yields $I \sim 6 \times 10^{-46} \text{ erg/cm}$. For a droplet with a radius of $R = 8 \mu\text{m}$, we obtain an intensity of $I \sim 2 \times 10^{-45} \text{ erg/cm}$. For a radius of $R = 3 \mu\text{m}$, the intensity is $I \sim 6 \times 10^{-45} \text{ erg/cm}$ (Fig. 1).

The estimated results and Fig. 1 show that an increase in the droplet radius by an order of magnitude (from 3 to 30 μm) leads to a decrease in the intensity by an order of magnitude.

Figure 2 shows the characteristics of radiation versus the external electric field. It is seen that an increase in the electric field causes a rapid increase in the radiation intensity: an increase in the field by a factor of 3 leads to an increase in the radiation intensity by an order of magnitude in accordance with expression (15).

In comparison with the intensity of the dipole electromagnetic radiation of the uncharged droplet that oscillates in the presence of electrostatic field [5], the intensity of the quadrupole electromagnetic radiation is less by 14–15 orders of magnitude.

Figure 3 presents the dependences of the dimensionless axisymmetric component $D_{33}(t)$ of the quadrupole moment calculated with the aid of expression (13) for $h_3 = h_5 = 0.5$ (curve 1) and dimensionless projection along the symmetry axis of the dipole moment

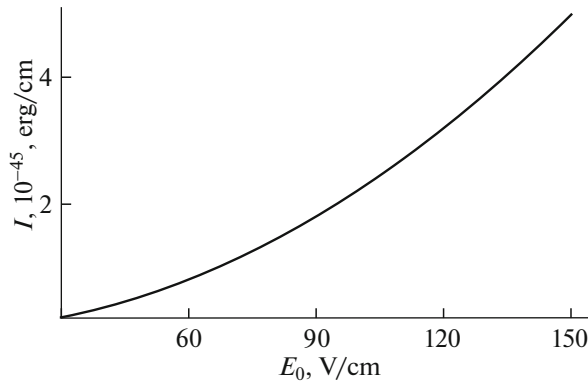


Fig. 2. Plot of the intensity of electromagnetic radiation of a single uncharged droplet that oscillates in the presence of electrostatic field vs. external electric field that is calculated for the parameters of Fig. 1 at $R = 30 \mu\text{m}$.

$d_3(t)$ (curve 2) on dimensionless time. The dipole moment is calculated with the aid of the expression

$$\mathbf{d}(t) = \frac{1}{2} \mathbf{e}_z E_0 \left[2 \left(1 + \frac{1}{3} e^2 \right) \left(1 + \frac{1}{15} e^2 \right) + H(t) \right],$$

$$\begin{aligned} H(t) = & 4\epsilon \sum_{j \in \Xi} h_j \left(1 + \frac{1}{3} e^2 \right) (G_1 \cos(\omega_j t) + e^2 (G_2 \cos(\omega_j t) \\ & + G_3 \cos(\omega_{j-2} t) - G_4 \cos(\omega_{j+2} t))) + \epsilon \sum_{j \in \Xi} h_j \left(1 + \frac{1}{15} e^2 \right) \\ & \times (G_5 \cos(\omega_j t) + e^2 (G_6 \cos(\omega_j t) + G_7 \cos(\omega_{j+2} t) \\ & + G_8 \cos(\omega_{j-2} t))) + \epsilon \sum_{j \in \Xi} h_j \left(1 + \frac{1}{15} e^2 \right) (G_9 \cos(\omega_j t) \\ & + e^2 (G_{10} \cos(\omega_j t) + G_{11} \cos(\omega_{j+2} t) + G_{12} \cos(\omega_{j-2} t))) \end{aligned}$$

by analogy with [5] for $j = 2$ and $h_2 = 1$ for the spheroid water droplet in the presence of the uniform electrostatic field $E_0 = 50 \text{ V/cm}$ using numerical coefficients G_i that depend only on the mode number (cumbersome expressions for the coefficients are not presented).

We plot dimensionless quantities, since quadrupole $D_{33}(t)$ and dipole $d_3(t)$ moments have different

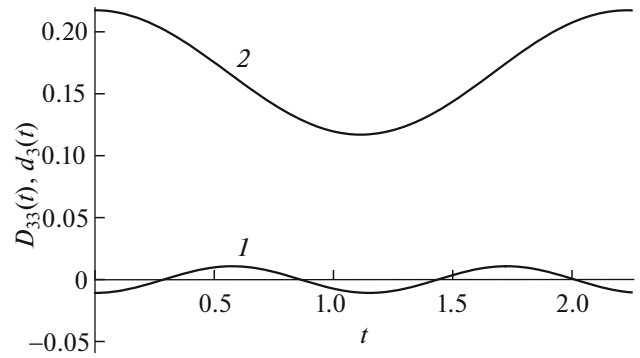


Fig. 3. Plots of the projections of the dimensionless electric dipole and quadrupole moments of an uncharged droplet that oscillates in the presence of the uniform electrostatic field vs. dimensionless time that are calculated for $\epsilon = 0.1$ and $E_0 = 50 \text{ V/cm}$: (1) quadrupole moment $D_{33}(t)$ calculated at $h_3 = h_5 = 0.5$ and (2) dipole moment $d_3(t)$ calculated at $j = 2$ and $h_2 = 1$.

dimensions but it is expedient to represent both quantities on a single plot for clearness.

It is seen that the frequency of variations in the quadrupole moment with time is approximately four times greater than the frequency of variations in the dipole moment. This circumstance can be used for experimental identification of the multipole components of electromagnetic radiation of a water droplet in the presence of external uniform electrostatic field.

CONCLUSIONS

We have estimated the intensity of the quadrupole electromagnetic radiation of an uncharged droplet that oscillates in the presence of electrostatic field in the first order of smallness with respect to the ratio of the oscillation amplitude to the linear size of the droplet and in the second order of smallness with respect to the ratio of the linear size of the droplet to the radiation wavelength. It has been shown that the intensity of the quadrupole radiation is less than the intensity of the dipole radiation of the same droplet by 14–15 orders of magnitude and the frequency of the quadrupole radiation is several times higher than the frequency of the dipole radiation.

APPENDIX

Coefficients k_1 – k_6 and p_1 – p_6 in expressions (10) and (13) are given by the following expressions:

$$\begin{aligned} k_1 &= \frac{j(j-1)}{(2j-1)}, & k_2 &= \frac{(j+1)(j-1)}{(2j+3)}, \\ k_3 &= \frac{j(j-1)(j-2)(2j^2+3j+7)}{2(2j-1)(2j-3)(2j-5)}, \\ k_4 &= \frac{j(100j^5+116j^4-165j^3-205j^2+75j-81)}{30(2j-1)^2(2j-3)(2j+3)}, \end{aligned}$$

$$k_5 = \frac{(j+1)(100j^5 + 765j^4 + 2159j^3 + 2591j^2 + 704j - 600)}{30(2j-1)(2j+3)^2(2j+5)},$$

$$k_6 = \frac{(j+1)(j+2)(j+3)(2j^2 + 11j + 21)}{2(2j+3)(2j+5)(2j+7)},$$

$$p_1 = \frac{j(j+3)}{(2j-1)}, \quad p_2 = \frac{(j+1)(j+3)}{(2j+3)},$$

$$p_3 = -\frac{j(j-1)(j-2)(9j-1)}{2(2j-1)(2j-3)(2j-5)},$$

$$p_4 = -\frac{j(40j^5 + 274j^4 - 265j^3 - 475j^2 + 375j - 9)}{30(2j-1)^2(2j-3)(2j+3)},$$

$$p_5 = \frac{(j+1)(135j^9 + 990j^8 + 2990j^7 + 4924j^6 + 6145j^5 + 8251j^4 + 7564j^3 + 1927j^2 - 886j - 360)}{30(2j-1)(2j+1)^2(2j+3)^2(2j+5)},$$

$$p_6 = -\frac{j(j+1)(j+2)(j+3)(4j^2 + 7j - 5)}{2(2j+3)(2j+5)(2j+7)}.$$

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Translated by A. Chikishev