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SOLID STATE

Origin of Elastic–Plastic Deformation Invariant

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Abstract—We consider the autowave mechanism of evolution of a localized plastic deformation of crystalline solids of different origins. It is found that localization of the plastic flow is determined by the relation between elastic and plastic phenomena in deforming materials. It is shown that the main parameter of deformation processes is the elastic–plastic deformation invariant, viz., a dimensionless quantity connecting quantitatively the parameters of elastic waves and self-sustained waves (autowaves) of localized plastic deformation. The correctness of this statement is verified for metals, alkali-halide crystals, and rocks. The physical origin of the invariant is explained on the basis of thermodynamic considerations.

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INTRODUCTION

Plastic deformation of solids is a complex physical phenomenon the evolution of which depends on the crystal lattice and crystal structure defects. The process of plastic flow is usually described by the dependence $\sigma(\varepsilon)$ of the deforming stress on the strain. It is important that at all its stages from the beginning of plastic flow (yield stress) to fracture (ultimate strength), this process is accompanied by plastic strain localization [1].

The model of evolution of localized plastic strain proposed in [1] presumes that self-organization of the defect structure [2] occurs in the form of autowaves of a localized plastic flow [1, 3], which appear in the deforming medium as a result of interaction of elastic waves and relaxation events of breaking of elastic stress concentrators. Each relaxation event contributes to the overall plastic deformation and generates new stress concentrators. The autowave pattern of localized plasticity distribution regularly changes in accordance with the stage nature of the $\sigma(\varepsilon)$ curve so that the deformation process can be treated as a natural evolution of localized plasticity autowaves [1].

Therefore, autowaves (localized plasticity) and wave (elastic) deformation phenomena coexist in a plastically deforming medium. The former are characterized by wavelength λ of the localized plasticity autowave and velocity V_{aw} of its propagation, while the latter are determined by the interplanar distance χ in the crystal lattice of the tested material and the velocity V_t of propagation of transverse elastic waves.

It was noted in earlier experiments with a number of metals [1, 4, 5] that dimensionless ratio $\lambda V_{aw}/\chi V_t$ formed by these four characteristics is the same for all cases of straining of different metals at the stages of linear strain-hardening when $\sigma \sim \varepsilon$. This suggested that ratio $\lambda V_{aw}/\chi V_t$ is invariant in general. This study aims at the verification of this regularity not only for metals, but also for other materials with a linear law hardening. It is also important to clarify the origin of this relation using general thermodynamic concepts.

1. EXPERIMENTAL DATA

The quantitative data on the localized plasticity patterns were estimated experimentally for linear stages of the processes at which the deforming stresses and strains are connected by a linear relation. In these cases, a phase localized plasticity autowave corresponding to the condition of the constancy of phase $2\pi(t/T - x/\lambda) = \text{const}$ is observed, where T is the period of oscillations in the wave, x is the coordinate, and t is the running time. The localized plasticity pattern formed in such cases is stable and can be observed relatively easily [1].

For estimating ratio $\lambda V_{aw}/\chi V_t$ characterizing various materials, the range of metals under investigation was extended. In addition, we studied the localization of plastic deformation in alkali-halide crystals (KCl, NaCl, and LiF) and rocks such as sandstone (SiO_2) and marble ($CaCO_2$). The method for observing the patterns of a localized plastic flow in deforming materials, which was based on speckle photography, was described in detail earlier [1] and will not be discussed here. For illustrating the potentialities of this method, Fig. 1 shows a typical pattern of local plasticity distribution for sequential stages of easy slip and linear

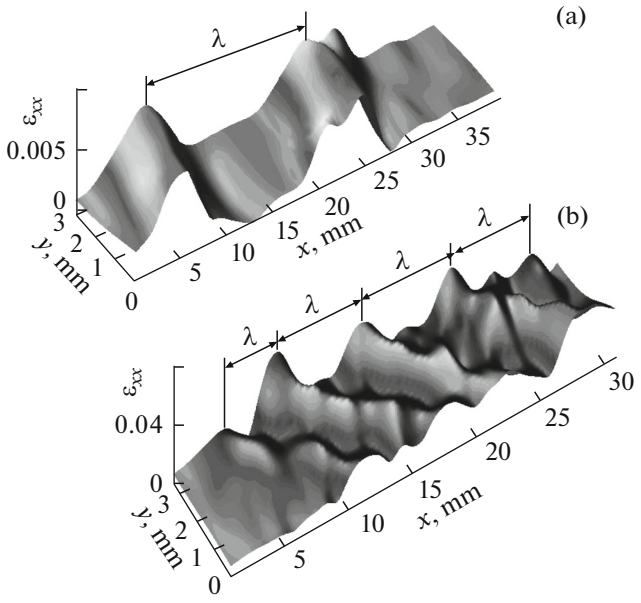

Fig. 1. Strain distribution over a sample of Fe–12 wt %Mn alloy (a) at the easy slip state and (b) at the linear strain-hardening stage.

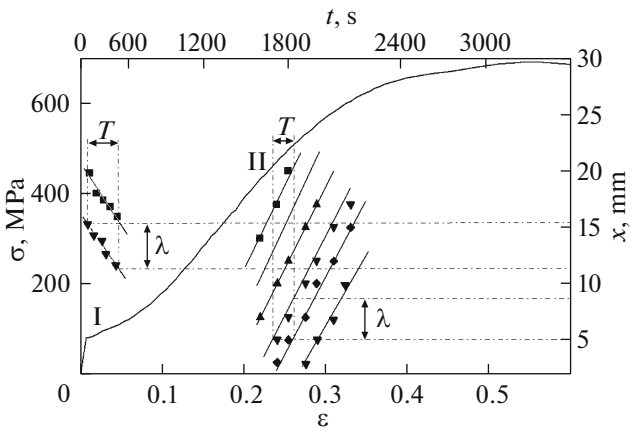

Fig. 2. *X–t* diagram for Garfield steel single crystal, plotted for the case shown in Fig. 1.

strain-hardening during tension with a constant rate for a single crystal sample of alloy Fe–12 wt %Mn (γ–Fe).

The quantitative characteristics λ and V_{aw} required for analyzing the data on the evolution of localized plasticity were determined from the processing of the so-called *X–t* diagrams proposed in [1] for such purposes and shown in Fig. 2. It can be seen from the figure that the values of autowave wavelength λ and period *T* can be determined from the vertical and horizontal sections of families of the *X*(*t*) curves. Characteristics λ and $V_{aw} = \lambda/T$ of localized plasticity autowaves were determined for linear strain-hardening of metals, easily slip in metal single crystals, compression of rock samples, and the phase-transformation straining of the NiTi single crystal.

Let us analyze the data obtained in these experiments. The values of products λV_{aw} for 18 tested metals are given in Table 1. It can be seen that the results differ insignificantly and the mean value of the product of these quantities is $\langle \lambda V_{aw} \rangle_{lwh} = (2.52 \pm 0.36) \times 10^{-7}$ m^2/s.

We managed to supplement these data with the results of analogous processing of localized plasticity patterns observed at the easy slip stages in Cu, Ni, α-Fe, γ-Fe, Zn, and Sn single crystals for which the proportionality $\sigma \sim \varepsilon$ also holds and a phase autowave is observed. For this stage, we have $\langle \lambda V_{aw} \rangle_{eg} \sim (2.95 \pm 1.05) \times 10^{-7}$ m^2/s (see Table 1).

The stage of linear strain-hardening and corresponding localized plasticity phase autowaves were observed for compressed samples of alkali-halide crystals and rocks [6, 7]. The results of these experiments given in Table 2 lead to $\langle \lambda V_{aw} \rangle_{ahc} = (3.44 \pm 0.49) \times 10^{-7}$ m^2/s and $\langle \lambda V_{aw} \rangle_{rock} = (1.44 \pm 0.34) \times 10^{-7}$ m^2/s.

In the case of staining due to the slip of individual dislocations, the process is usually characterized by dislocation path length *l* and dislocation velocity V_{disl};

Table 1. Comparison of quantities χV_t and λV_{aw} for metals

$\times 10^7$ m^2/s	Linear strain hardening stage												
	Cu	Zn	Al	Zr	Ti	V	Nb	α-Fe	γ-Fe	Ni	Co	Mo	
λV_{aw}	3.6	3.7	7.9	3.7	2.5	2.8	1.8	2.55	2.2	2.1	3.0	1.2	
χV_t	4.8	11.9	7.5	11.9	7.9	6.2	5.3	4.7	6.5	6.0	6.0	7.4	
λV_{aw}	0.75	0.3	1.1	0.3	0.3	0.45	0.33	0.54	0.34	0.35	0.5	0.2	
$\times 10^7$ m^2/s	Linear strain hardening stage							Easy slip stage					
	Sn	Mg	Cd	In	Pb	Ta	Hf	α-Fe	γ-Fe	Cu	Zn	Ni	Sn
λV_{aw}	2.4	9.9	0.9	2.6	3.2	1.1	1.0	7.4	2.9	1.9	1.0	1.3	3.3
χV_t	5.3	15.8	3.5	2.2	2.0	4.7	4.2	6.5	6.0	4.7	5.0	6.0	4.9
$\lambda V_{aw}/\chi V_t$	0.65	0.63	0.2	1.2	1.6	0.2	0.24	1.1	0.49	0.4	0.2	0.2	0.67

these parameters were determined from analysis of the available data on the mobility of individual dislocations in single crystals [8–12] in the quasi-viscous flow regime, in which $V_{disl} \sim \sigma$ [13]. In such conditions, the characteristic dislocation path lengths lie in the interval 10^{-5} m $\le l \le 10^{-4}$ m and the velocities of dislocations belong to the range 10^{-3} m/s $\le V_{disl} \le 10^{-2}$ m/s. The product of this quantities was estimated using the relation $lV_{disl} = V_{disl}^2\tau$, where τ is the duration of the load pulse acting during the crystal loading. The results of calculations of products lV_{disl} in these cases are given in Table 3. It can be seen that $\langle lV\rangle_{disl} = (3.2 \pm 0.35) \times 10^{-7}$ m^2/s in this case.

The experimental estimation of the parameters of a localized plasticity autowave for plastic deformation of TiNi intermetallide single crystal of the equiatomic composition (strain-induced phase transformation [14]) resulted in the value $\langle\lambda V_{aw}\rangle_{pt} \approx 0.85 \times 10^{-7}$ m^2/s.

Comparing pairwise the above data by calculating the Student t-test [15], we can conclude that the resultant values differ insignificantly (i.e., belong to the same general population). This leads to

$$\langle\lambda V_{aw}\rangle_{lwh} \approx \langle lV\rangle_{disl} \approx \langle\cdot V_{aw}\rangle_{pt} \approx \langle\cdot V_{aw}\rangle_{ahc} \approx \langle\cdot V_{aw}\rangle_{rock}. \quad (1)$$

Elastic processes in the tested materials were characterized by interplanar distances χ in the crystal lattice [16] and velocities V_t of propagation of transverse elastic waves [17]. As follows from Tables 1–3, we have $\langle\chi V_t\rangle_{el} \approx (5.8 \pm 0.3) \times 10^{-7}$ m^2/s for all tested materials.

Normalizing expressions (1) to the corresponding products χV_t, we obtain dimensionless relations

$$\frac{\langle\lambda V_{aw}\rangle_{lwh}}{\langle\lambda V_t\rangle_{el}} = \hat{Z}_{lwh}, \quad \frac{\langle lV\rangle_{disl}}{\langle\chi V_t\rangle_{el}} = \hat{Z}_{disl},$$

$$\frac{\langle\lambda V_{aw}\rangle_{eg}}{\langle\lambda V_t\rangle_{el}} = \hat{Z}_{eg}, \quad \frac{\langle\lambda V_{aw}\rangle_{pt}}{\langle\chi V_t\rangle_{el}} = \hat{Z}_{pt},$$

$$\frac{\langle\lambda V_{aw}\rangle_{ahc}}{\langle\lambda V_t\rangle_{el}} = \hat{Z}_{ahc}, \quad \text{and} \quad \frac{\langle\lambda V_{aw}\rangle_{rock}}{\langle\chi V_t\rangle_{el}} = \hat{Z}_{rock}.$$

Numerical estimates of quantities $\hat{Z}$ show that

$$\hat{Z}_{lwh} \approx \hat{Z}_{disl} \approx \hat{Z}_{eg} \approx \hat{Z}_{pt} \approx \hat{Z}_{ahc} \approx \hat{Z}_{rock} \approx 1/2, \quad (2)$$

which gives

$$\left\langle\frac{\lambda V_{aw}}{\chi V_t}\right\rangle = \hat{Z} \approx \frac{1}{2}. \quad (3)$$

This relation, which is known as the elastic–plastic deformation invariant, holds in all cases of plastic deformation considered above. It can be treated as a universal characteristic of plastic deformation processes. The data considered here are represented graphically in Fig. 3.

Table 2. Comparison of quantities χV_t and λV_{aw} for alkali-halide crystals [6] and rocks [7]

$\times 10^7$ m^2/s	KCl	NaCl	LiF	Marble	Sandstone
λV_{aw}	3.0	3.1	4.3	1.75	0.6
χV_t	7.0	7.5	8.8	3.7	1.5
$\lambda V_{aw}/\chi V_t$	0.43	0.4	0.5	0.5	0.4

Table 3. Comparison of quantities χV_t and lV_{disl} determined by measuring of path lengths of individual dislocations

$\times 10^7$ m^2/s	NaCl [8]	LiF [9]	CsI [10]	KCl [11]	Zn [12]
lV_{dist}	4.1	4.1	1.9	4.1	1.8
χV_t	7.3	8.6	4.0	6.8	4.0
$lV_{dist}/\chi V_t$	0.56	0.47	0.47	0.6	0.45

2. ON THE ORIGIN OF ELASTIC–PLASTIC INVARIANT

Let us consider some factors concerning the origin of invariant (3), which plays an important role in the evolution of localized plasticity because it connects the characteristics of elastic (χ and V_t) and plastic (λ and V_{aw}) deformation. It was shown earlier [1] that the localization of plastic deformation is a consequence of self-organization of a nonlinear active deforming medium with structure defects [2]. A general feature of self-organization processes in the open thermodynamic system such as a deforming medium is the decrease in its entropy in such a process [18]. This condition holds indeed in the case of generation of autowaves of a localized plastic flow [19]. For this reason, the use of the entropy factor for clarifying the physical nature of plastic deformation localization processes is quite justified.

In the localized plasticity evolution, the processes of generation and relaxation of elastic stress concentrators with the generation of dislocations compete [1, 4, 5]. In this case, the spatiotemporal distributions of fields of stressed $\sigma(x, y, t)$ and plastic strains $\varepsilon(x, y, t)$ are transformed in a correlated way so that in accordance with the elastic–plastic invariant, velocities V_t and V_{aw} control the kinetics of the transformation processes of the corresponding fields, while length χ and wavelength λ specify the spatial scales of the redistribution processes.

For this reason, it is convenient to write Eq. (3) for the invariant in the form

$$\frac{\lambda/\chi}{V_t/V_{aw}} = \frac{p_{scale}}{p_{kin}} = \hat{Z}, \quad (4)$$

where ratios $\lambda/\chi = p_{scale} > 1$ and $V_t/V_{aw} = p_{kin} > 1$ have the meaning of the scale and kinetic thermodynamic probabilities, respectively [18]. Scale thermodynamic probability p_{scale} is interpreted as the number of possi-

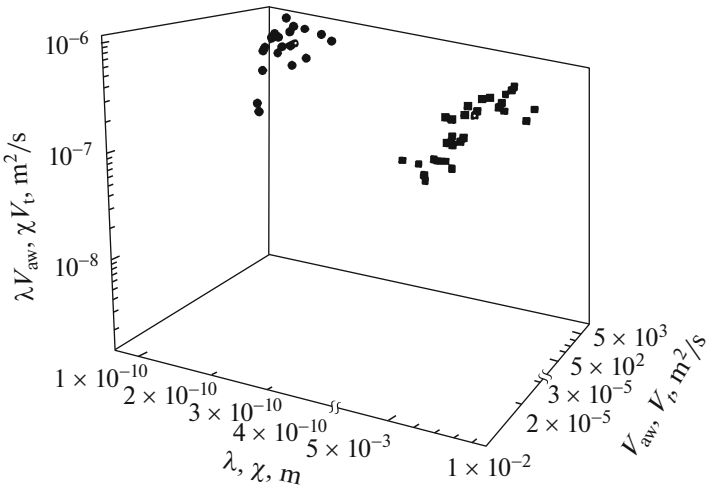

Fig. 3. Comparison of products (■) and (●) appearing in the expression for the elastic–plastic invariant.

ble sites of generation of an autowave of the localized plastic flow in a deforming medium; i.e., it determines the significant difference in the spatial scales of elastic and plastic deformation processes. As it regards kinetic thermodynamic probability p_{kin}, it determines the choice of the autowave velocity from the interval of its possible values $0 \le V_{aw} \le V_t$ by the deforming system.

In view of the above arguments, Eq. (4) leads to the relation

$$\ln \hat{Z} = \ln p_{scale} - \ln p_{kin}, \tag{5}$$

which can be transformed with the help of the Boltzmann formula to the equation for the entropy variation associated with the difference in the scales,

$$\Delta S_{scale} = k_B \ln \frac{\lambda}{\chi} = k_B \ln p_{scale} \tag{6}$$

and the difference in the velocities,

$$\Delta S_{kin} = k_B \ln \frac{V_t}{V_{aw}} = k_B \ln p_{kin}. \tag{7}$$

Finally, we obtain the following relation from Eqs. (5)–(7):

$$\begin{aligned} \Delta S &= k_B \ln p_{kin} + k_B \ln p_{scale} \\ &= -\Delta S_{kin} + \Delta S_{scale} = k_B \ln \frac{1}{2} < 0, \end{aligned} \tag{8}$$

which means that in the course of generation of a phase autowave, the entropy of the deforming system decreases by $-\Delta S = \Delta S_{scale} - \Delta S_{kin}$. The signs of the terms $\Delta S_{scale} > 0$ and $\Delta S_{kin} < 0$ in Eq. (8) emphasize the antagonism of the contributions of the scale and kinetic factors to the nature of localized plastic deformation. The contribution from the difference in scales, $\lambda/\chi = p_{scale}$, is dissipative because it is equivalent to the existence of the structure of the medium, and this factor is responsible for the emergence of dispersion and dissipation processes in general [20]. Conversely, the contribution from the difference in velocities $V_t/V_{aw} = p_{kin}$, which reduces the total entropy of the system, facilitates the self-organization of the medium.

Quantity $\Delta S < 0$ characterizes the total decrease in the entropy during the formation of a phase autowave of plastic flow localization, i.e., corresponds to self-organization of the deforming medium. Since

$$\hat{Z} = \exp(\Delta S/k_B) \approx \frac{1}{2}, \tag{9}$$

we have $\Delta S = k_B \ln(1/2) \approx -0.7 k_B$ per elementary relaxation event [21].

Let us consider the relation between elastic and plastic displacements for a small deviation of the deforming system from equilibrium. In the vicinity of the energy minimum corresponding to equilibrium, the velocities of displacements of the stress and strain fields in the deforming system during spatiotemporal transformations are linear in the gradients of plastic and elastic strains to within the first-order terms [22], i.e.,

$$\dot{u}_{pl}^{(p)} \approx D_{\varepsilon\varepsilon} \nabla \varepsilon_{pl}, \tag{10}$$

$$\dot{u}_{el}^{(p)} \approx D_{\sigma\sigma} \nabla \varepsilon_{el}, \tag{11}$$

respectively. In these relations, we assume that $\lambda V_{aw} \equiv D_{\varepsilon\varepsilon}$ and $\chi V_t \equiv D_{\sigma\sigma}$.

In view of the nonlinear relation between the strain and stress, which is described by the flow curve $\sigma(\varepsilon)$, we must also take into account the emergence of velocities additional to expressions (10) and (11):

$$\dot{u}_{pl}^{(ad)} \approx D_{\varepsilon\sigma} \nabla \varepsilon_{pl},$$

$$\dot{u}_{pl}^{(ad)} \approx D_{\sigma\varepsilon} \nabla \varepsilon_{el}.$$

We obtain a system of equations that can be written in the form

$$\begin{cases}\dot{u}_{pl} = D_{\varepsilon\varepsilon}\nabla\varepsilon + D_{\varepsilon\sigma}\nabla\varepsilon_{el},\\ \dot{u}_{el} = D_{\sigma\varepsilon}\nabla\varepsilon_{el} + D_{\sigma\sigma}\nabla\varepsilon.\end{cases}$$

The coefficients in this system form matrix $\begin{pmatrix} D_{\varepsilon\varepsilon} & D_{\varepsilon\sigma} \\ D_{\sigma\varepsilon} & D_{\sigma\sigma}\end{pmatrix}$ [22] whose nondiagonal components are identical in accordance with the Onsager principle of symmetry of kinetic coefficients [22, 23]; i.e., $D_{\varepsilon\sigma} = D_{\sigma\varepsilon}$ and, accordingly, $\lambda V_{aw} \approx \chi V_t$. As it regards diagonal coefficients $D_{\varepsilon\varepsilon}$ and $D_{\sigma\sigma}$, which are the coefficients of the autowave equations of the localized plastic flow [1, 4, 5], they are not necessarily identical; for example, it was shown in [1] that $D_{\varepsilon\varepsilon} \ll D_{\sigma\sigma}$.

CONCLUSIONS

1. It has been shown that the relation

$$\left\langle \frac{\lambda V_{aw}}{\chi V_t} \right\rangle = \hat{Z} \approx \frac{1}{2}$$

holds for all investigated cases of plastic deformation with a linear strain-hardening law and can be treated as the elastic–plastic invariant.

2. Analysis of the invariant origin indicates the validity of the assumption concerning self-organization of a deforming medium in the course of straining.

3. The invariant formalizes the relation between elastic and plastic components of strain of materials and is important for solving problems of theoretical and applied plasticity.

4. The elastic–plastic invariant necessitates the allowance for the role played by elastic (lattice) characteristics of materials and is important in the development of models and mechanisms of evolution of a plastic flow.

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REFERENCES

1. L. B. Zuev, V. I. Danilov, and S. A. Barannikova, *The Physics of Macrolocalization of Plastic Flow* (Nauka, Novosibirsk, 2008).
2. A. Seeger and W. Frank, in *Non-Linear Phenomena in Material Science*, Ed. by L. P. Kubin and G. Martin (Trans Tech, New York, 1987), p. 125.
3. V. A. Davydov, N. V. Davydov, V. G. Morozov, M. N. Stolyarov, and T. Yamaguchi, Condens. Matter Phys. **7**, 565 (2004).
4. L. B. Zuev and S. A. Barannikova, Solid State Phenom. **172–174**, 1279 (2011).
5. L. B. Zuev, Phys. Wave Phenom. **20**, 166 (2012).
6. S. A. Barannikova, M. V. Nadezhkin, and L. B. Zuev, Tech. Phys. Lett. **37**, 750 (2011).
7. L. B. Zuev, S. A. Barannikova, M. V. Nadezhkin, and V. V. Gorbatenko, J. Min. World Express **2** (1), 31 (2013).
8. V. F. Kurilov, L. B. Zuev, V. E. Gromov, V. P. Sergeev, and G. I. Gershtein, Kristallografiya **22**, 653 (1977).
9. E. V. Darinskaya, A. A. Urusovskaya, V. N. Opekunov, G. A. Abramchuk, and V. F. Alekhin, Fiz. Tverd. Tela **20**, 1250 (1978).
10. E. V. Darinskaya and A. A. Urusovskaya, Fiz. Tverd. Tela **17**, 2421 (1975).
11. L. B. Zuev, V. E. Gromov, and O. I. Aleksankina, Kristallografiya **19**, 889 (1974).
12. L. B. Zuev, V. E. Gromov, V. F. Kurilov, and L. I. Gurevich, Dokl. Akad. Nauk SSSR **239**, 84 (1978).
13. V. I. Al'shits and V. L. Indenbom, *Dislocations in Solids*, Ed. by F. R. N. Nabarro (Elsevier, Amsterdam, 1986), p. 43.
14. K. Otsuka and K. Shimizu, Int. Met. Rev **31** (3), 93 (1986).
15. D. J. Hudson, *Lectures on Elementary Statistics and Probability* (CERN, Geneva, 1963).
16. L. I. Mirkin, *Handbook on X-ray Structure Analysis of Polycrystals* (GIFML, Moscow, 1961).
17. O. L. Anderson, in *Physical Acoustics*, Ed. by W. P. Mason (Academic, New York, 1965), Vol. 3B, p. 43.
18. Yu. L. Klimontovich, *Introduction to Open-System Physics* (Yanus-K, Moscow, 2002).
19. L. B. Zuev, Tech. Phys. Lett. **31**, 89 (2005).
20. A. Scott, *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, 2nd ed. (Oxford Univ. Press, 2003).
21. A. I. Slutsker, Phys. Solid State **47**, 801 (2005).
22. L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Nauka, Moscow, 1964).
23. Yu. B. Rumer and M. Sh. Ryvkin, *Thermodynamics, Statistical Physics, and Kinetics* (Novosib. Gos. Univ., Novosibirsk, 2000).

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