ISSN 1063-7842, Technical Physics, 2017, Vol. 62, No. 3, pp. 365–373. © Pleiades Publishing, Ltd., 2017.
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ATOMIC AND MOLECULAR PHYSICS

Ion Velocity Distribution Function in Intrinsic Gas at Cryogenic Gas Temperatures

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Received June 7, 2016

Abstract—The influence of elastic scattering on the ion distribution function in the plasma of an intrinsic gas in weak fields has been considered. An analytical expression valid for cryogenic temperatures of atoms has been obtained. The reduced He^+–He, Ar^+–Ar mobilities as functions of the temperature of atoms in a range of 4–1000 K have been calculated in the approximation of the zero field taking into account elastic collisions; the calculated dependences well agree with the available experimental data. It has been demonstrated that elastic collisions play an important role in the formation of the ion distribution function at low temperatures. The results of measurement of the ion mobility in the limit of the zero field at low temperatures can be used to obtain data on the ratio of elastic scattering and resonance charge exchange cross sections.

DOI: 10.1134/S106378421703015X

INTRODUCTION

The influence of elastic collisions on ion drift in the plasma of intrinsic gas in a strong field was considered earlier [1, 2]. It was found out that elastic collisions weakly influence the ion drift along the field and considerably impact the drift characteristic across the electric field. This was explained by the fact that in strong fields when average ion energies are large, as compared to the temperature of atoms, the elastic scattering cross section turns out to be much smaller than the resonance charge exchange cross section, thus, this collision type insignificantly influences the ion mobility. At the same time, elastic collisions (although rather rare) transform part of the energy acquired by the ion in the electric field into the energy of ion motion in the plane orthogonal to the electric field. This has a decisive influence on the growth of the ion transverse diffusion coefficient with increasing field, which, in the absence of elastic collisions, is determined by the temperature of atoms. It is known that the elastic scattering cross section increases with reducing relative energy of colliding particles much faster than the resonance charge exchange cross section [3–5]. For small fields, when the ion energy is on the order of the average thermal energy of atoms, at low gas temperature, the cross sections can be on the same order of magnitude [5, 6]. Thus, the influence of elastic collisions in the case of weak fields on the ion distribution function can be quite substantial.

From a physical point of view, the influence of elastic collisions on the ion distribution function in an intrinsic gas at weak fields is quite clear. Indeed, under these conditions, the ion distribution function is close to Maxwellian. The presence of a weak electric field and resonance charge exchange results in the anisotropy of the ion distribution function (extended along the field), and the value of this perturbation depends on the ion energy according to a certain law [6, 7]. Elastic collisions implying energy transfer in all directions of ion motion obviously result in the isotropization of the ion distribution function, i.e., the reduction of perturbation of the Maxwellian distribution.

The ion drift in the intrinsic gas for small fields was theoretically studied by many authors [8–11], the results are given in detail in [11]. Most of these studies were based on various approximations of the Chapman–Enskog theory [12] for the drift velocity or mobility in the limit of the zero field.

The ion distribution function in an intrinsic gas at weak fields was studied in [6, 7]. In the first of these studies; however, elastic collisions were taken into account by estimating their influence at room temperature and, in the second one, they were not taken into account at all. In [7], the cross section of the resonance charge exchange was assumed to be constant.

It will be shown below that, for low temperatures of atoms taking into account elastic collisions is important when calculating the ion distribution function in an intrinsic gas.

Let us consider the problem of finding the ion distribution function in a plasma of an intrinsic gas in a weak field taking into account the dependence of the resonance charge exchange and elastic scattering cross sections on the relative energies of the ion and the atom.

BASIC RELATIONS

To solve the formulated problem, it is necessary to choose the model of the differential scattering cross section at the ion collision with the intrinsic atom taking into account the resonance charge exchange and elastic scattering. It was indicated in a number of papers that "isotropic in the center-of-mass system, elastic scattering cross section" well describes this situation [3, 5, 13]. Then, we take the function $\sigma_\theta(\varepsilon, \theta)$ as the differential scattering cross section as follows:

$$\sigma_\theta(\varepsilon, \theta) = \frac{1}{2\pi \sin\theta}\sigma_b(\varepsilon)\delta(\theta - \pi) + \frac{\sigma_i(\varepsilon)}{4\pi}, \tag{1}$$

where ε, θ are the energy of relative motion in the center of mass and the scattering angle in this system, respectively; $\sigma_b(\varepsilon)$ is the so called backscattering cross section; and $\sigma_i(\varepsilon)$ is the isotropic scattering cross section. Note that the cross section $\sigma_b(\varepsilon)$ is not the resonance charge exchange cross section. Let us find $\sigma_b(\varepsilon)$ and $\sigma_i(\varepsilon)$ using the experimental and (or) quantum mechanically calculated diffusion cross section (or the momentum transfer cross section) $\sigma_m(\varepsilon)$ and the energy transfer cross section (or the viscosity cross section) $\sigma_v(\varepsilon)$,

$$\begin{gathered}\sigma_m(\varepsilon) = 2\pi\int_0^\pi \sigma_\theta(\varepsilon, \theta)(1 - \cos\theta)\sin\theta d\theta = 2\sigma_b(\varepsilon) + \sigma_i(\varepsilon), \\ \sigma_v(\varepsilon) = 2\pi\int_0^\pi \sigma_\theta(\varepsilon, \theta)(1 - \cos^2\theta)\sin\theta d\theta = \frac{2}{3}\sigma_i(\varepsilon).\end{gathered} \tag{2}$$

Taking into account that with high accuracy [11]

$$\sigma_m \cong 2\sigma_c, \tag{3}$$

where σ_c is the resonance charge exchange cross section, we obtain from (2) that

$$\sigma_i = 1.5\sigma_v, \quad \sigma_b = \sigma_c - 0.5\sigma_i. \tag{4}$$

Thus, the backscattering cross section σ_b is equal to the resonance charge exchange cross section for $\sigma_i = 0$.

Let us consider the steady-state ion velocity distribution under the following conditions:

(i) atoms of the gas move according to the Maxwell law;

(ii) ion motion takes place in the intrinsic gas with a low degree of ionization;

(iii) the dominant processes that form the ion velocity distribution function are the resonance charge exchange and elastic collisions of ions with atoms;

(iv) the plasma is homogeneous;

(v) the average ion energy is close to the average thermal energy of atoms.

In the steady-state plasma under the above assumptions the Boltzmann equation has the form

$$\frac{eE}{m}\nabla_v(f_i) = S_i, \tag{5}$$

where e, m are the ion charge and mass; $\mathbf{E}$ is the electric field strength; f_i is the ion velocity distribution function; and S_i is the collision integral, which can be split into two terms S_{ci} and S_{ei}. The first term corresponds to backscattering (in the center of mass system) and the second term corresponds to isotropic (in the center of mass system) scattering of the ion on the intrinsic atom. Let us consider first S_{ci}. Taking into account that the ion produced in the charge exchange process has the velocity of the atom, let us determine the collision integral as follows [2, 6]:

$$\begin{aligned}S_{ci}(\mathbf{v}_i) = n_a\Big\{&f_a(\mathbf{v}_i)\int \sigma_b v_r f_i(\mathbf{v}'_i)d\mathbf{v}'_i \\ &- f_i(\mathbf{v}_i)\int \sigma_b v_r f_a(\mathbf{v}'_a)d\mathbf{v}'_a\Big\} \equiv S_{1ci} - S_{2ci},\end{aligned} \tag{6}$$

where $\mathbf{v}_i$ is the ion velocity, $\mathbf{v}_a$ is the atom velocity, $f_a(\mathbf{v}_a)$ is the Maxwellian atom velocity distribution function (normalized to unity), and v_r is the absolute value of the relative velocity of the ion and the atom before the collision; the ion distribution function is normalized to concentration. It is known that the resonance charge exchange cross section σ_c in the energy range up to several electronvolt weakly depends on the relative energy of the ion and the atom. It was shown in [1, 2] that the ion distribution function with constant σ_c can be used to obtain the solution to the Boltzmann equation taking into account this dependence. Therefore, first, let us solve the formulated problem for constant resonance charge exchange cross section and then formulate the rules for taking into account the dependence σ_c on the relative energy of colliding particles.

It was demonstrated in [1, 14] that to notation the expressions for $S_{1ci}(\mathbf{v}_i)$, $S_{2ci}(\mathbf{v}_i)$ have the form

$$\begin{gathered}S_{1ci}(\mathbf{v}_i) = n_a f_a(\mathbf{v}_i)\int \sigma_b v_r f_i(\mathbf{v}'_i)d\mathbf{v}'_i, \\ S_{2ci}(\mathbf{v}_i) = n_a\sigma_b f_i(\mathbf{v}_i)w_{0e}(v_i); \\ w_{0e}(v_i) = \left(v_i + \frac{1}{2\beta v_i}\right)\mathrm{erf}(\sqrt{\beta}v_i) + \frac{\exp(-\beta v_i^2)}{\sqrt{\pi\beta}}.\end{gathered} \tag{7}$$

Here, $\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int\limits_0^x \exp(-t^2)dt$, $\beta = \frac{1}{2kT_a}$ $w_{0e}(v_i)$ are the relative velocities of the ion and the atom averaged over the atom distribution function, k, T_a are the Boltzmann constant and the temperature of atoms, respectively.

Let us consider the collision integral S_{ei} corresponding to isotropic in the center of mass system scattering of the ion on the intrinsic atom. Collisions in which the ion is deflected to small angles due to the interaction of the ion charge and the induced electric moment of the atom which take place at large impact parameters weakly influence the ion distribution function, since they slightly change the velocity [15], therefore, they will not be taken into account. Moreover, for small ion energies, polarization capture can take place [11]. In this case, the corresponding differential cross section is probably close to isotropic in the center of mass system.

Then, taking into account that, before the collision, the atom moves with the arbitrary velocity $\mathbf{v}_a$, the probability density $g_e(\mathbf{v}'_i \to \mathbf{v}_i; \mathbf{v}_a)$ that the ion with the velocity $\mathbf{v}'_i$ acquires the velocity $\mathbf{v}_i$ in collision with the ion can be written as [14]

$$g_e(\mathbf{v}'_i \to \mathbf{v}_i; \mathbf{v}_a) = \frac{v_r(v'_i)}{2\pi}\delta\left(\boldsymbol{\xi}\mathbf{v}_a - \frac{\xi^2}{2} - \frac{v_i^2 - v_i'^2}{2}\right), \tag{8}$$

where $\boldsymbol{\xi} = \mathbf{v}_i - \mathbf{v}'_i$; $\xi = |\mathbf{v}_i - \mathbf{v}'_i|$.

For S_{ei}, we have

$$\begin{aligned} S_{ei}(\mathbf{v}_i; \mathbf{v}_a) &= n_a\int \sigma_i(v_r(v'_i))v_r(v'_i)g_e(\mathbf{v}'_i \to \mathbf{v}_i; \mathbf{v}_a) \\ &\times f_a(\mathbf{v}_a)f_i(\mathbf{v}'_i)d\mathbf{v}'_i - n_a\int \sigma_i(v_r(v_i))v_r(v_i) \\ &\times g_e(\mathbf{v}_i \to \mathbf{v}'_i; \mathbf{v}_a)f_a(\mathbf{v}_a)f_i(\mathbf{v}_i)d\mathbf{v}'_i. \end{aligned} \tag{8a}$$

Integrating (8a) over $\mathbf{v}_a$ and taking into account the weak dependence of the cross section $\sigma_i(v_r(v'_i))$ on the velocity, as compared to the Maxwellian function, we obtain [14]

$$\begin{aligned} \bar{S}_{ei}(\mathbf{v}_i) &= n_a\int \sigma_i(w_{0e}(v_i))w_e(\mathbf{v}'_i \to \mathbf{v}_i)f_i(\mathbf{v}'_i)d\mathbf{v}'_i \\ &- n_a\sigma_i(w_{0e}(v_i))w_{0e}(v_i)f_i(\mathbf{v}_i), \end{aligned} \tag{8b}$$

where

$$w_e(\mathbf{v}'_i \to \mathbf{v}_i) = \frac{\beta}{\pi^{1.5}}\frac{\exp\left[-\beta\left(\frac{v_i^2 - v_i'^2}{2\xi} + \frac{\xi}{2}\right)^2\right]}{\xi}. \tag{9}$$

Introducing the dimensionless velocity $x = v_i\sqrt{\beta}$, $\beta = m/2kT_a$, we write Eq. (5), integrated over $\mathbf{v}_a$ in the form

$$\begin{aligned} &\varepsilon_0\mu\frac{\partial f_i}{\partial x} + 2\tilde{w}_{oe}(x)\left[1 + \frac{\sigma_i(w_{0e}(x))}{2\sigma_c}\right]f_i \\ &= 2\left[1 - \frac{\sigma_i(w_{0e}(x))}{2\sigma_c}\right]f_a(x)\int x_r f_i(\mathbf{x}')d\mathbf{x}' \\ &+ \frac{2}{\sigma_c}\int \sigma_i(\tilde{w}_{0e}(\mathbf{x}'))\tilde{w}_e(\mathbf{x}' \to \mathbf{x})f_i(\mathbf{x}')d\mathbf{x}', \end{aligned} \tag{10}$$

where $\varepsilon_0 = \frac{eE}{n_a\sigma_c kT_a} \ll 1$, $\mu = \cos\theta$, θ is the angle between the ion velocity and the electric field vector, $\tilde{w}_{oe}(x) = \sqrt{\beta}w_{0e}(x/\sqrt{\beta})$, $\tilde{w}_e(\mathbf{x}' \to \mathbf{x}) = \beta^{1.5}w_e(\mathbf{x}'/\sqrt{\beta} \to \mathbf{x}/\sqrt{\beta})$. It was shown in [1, 2] that, if the charge exchange cross section is approximated by the formula [12]

$$\sigma_c(E_c) = \sigma_{0c}[1 + a\ln(E_c(\mathrm{eV}))]^2, \tag{11}$$

where E_c is the energy of relative motion of the ion and the atom, taking into account the weak dependence of the charge exchange cross section on the relative velocity of the ion and the atom is reduced with sufficient accuracy to the replacement of the cross section σ_c by $\sigma_{0c}k(x)$, where

$$k(x) = \left[1 + a\ln\left(\frac{x_0^2}{w_{0e}(x)^2}\right)\right]^2, \tag{11a}$$

and the dimensionless velocity x_0 corresponds to the energy of relative motion of the ion and the atom equal to 1 eV. Finally, we have

$$\begin{aligned} &\varepsilon_0\mu\frac{\partial f_i}{\partial x} + 2\tilde{w}_{oe}(x)[k(x) + 0.5\sigma k_e(x)]f_i \\ &= 2[k(x) - 0.5\sigma k_e(x)]f_a(x)\int x_r f_i(\mathbf{x}')d\mathbf{x}' \\ &+ 2\sigma\int k_e(x')\tilde{w}_e(\mathbf{x}' \to \mathbf{x})f_i(\mathbf{x}')d\mathbf{x}', \end{aligned} \tag{11b}$$

where the parameter ε_0 is determined for an energy of 1 eV, $k_e(x) = \frac{\sigma_i(\tilde{w}_{oe}(x))}{\sigma_i(\tilde{w}_{oe}(x_0))}$, $\sigma = \frac{\sigma_i(\tilde{w}_{oe}(x_0))}{\sigma_{0c}}$ is the ratio of elastic scattering and resonance charge exchange cross sections for the energy of relative motion in the center of mass system equal to 1 eV.

Since for weak electric fields, the ion distribution function is weakly anisotropic, we use the method of spherical harmonics [14] to solve Eq. (11b). The only complication is that this equation represents a singularly perturbed integro-differential equation with the small parameter ε_0 at the highest derivative [16, 17]. The solution to this equation is obtained in the Appendix in the following form:

$$f_i(x,\mu) = f_a(x) + 3\varepsilon_0\mu F_{11}(x), \tag{12}$$

where $F_{11}(x)$ is defined by formulas (A5).

For mobility in the limit of the zero field, we have (see (A5))

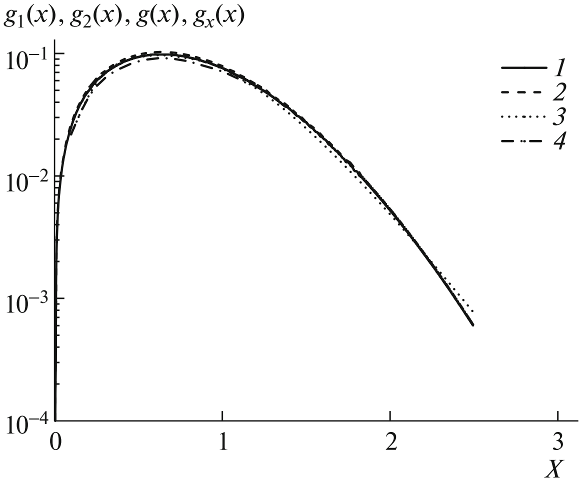

Fig. 1. Perturbations of the Maxwellian ion distribution function without account of elastic collisions in a weak field for constant resonance charge exchange cross section: (*1*) $g_1(x)$ is the first approximation, (*2*) $g_2(x)$ is the second approximation, (*3*) $g(x)$ is the approximate solution from [6], and (*4*) $g_x(x)$ is the solution from [7].

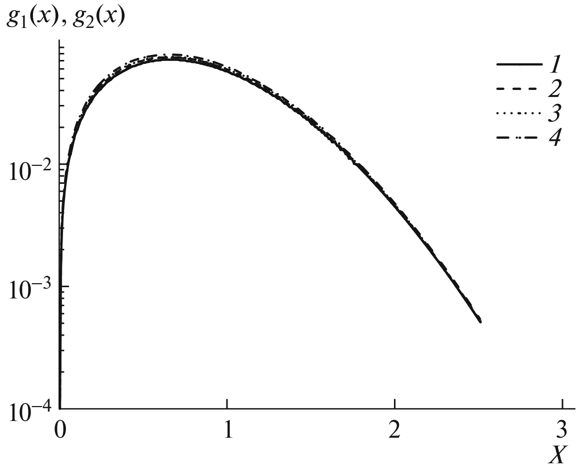

Fig. 2. Perturbations of the Maxwellian ion distribution function without account of elastic collisions in a weak field: (*1*) $g_1(x)$ is the first approximation for the He^+–He case, (*2*) $g_2(x)$ is the second approximation for the He^+–He case, (*3*) $g_1(x)$ is the first approximation for the Xe^+–Xe case, (*4*) $g_1(x)$ is the first approximation for the Xe^+–Xe case. Parameters of cross sections of the resonance charge exchange are taken from [18].

$$K[\mathrm{cm}^2/\mathrm{V\,s}] = \frac{6.98\times 10^4}{\sigma_{0c}(10^{-15}\,\mathrm{cm}^{-2})\sqrt{T_a(K)m}}\int\limits_0^\infty x^3 F_{11}(x)dx, \tag{13}$$

where m is the ion mass in a.m.u.

DISCUSSION OF RESULTS

It was already mentioned above that the ion drift in the intrinsic gas in weak fields was studied by many authors. Let us compare our results with some of the previously published data. It was obtained in [6, 12] that, without taking into account elastic collisions, the drift ion velocity is given by the following expressions, respectively:

$$\frac{0.341E}{N\sqrt{T_a m}\sigma\left(2.12\sqrt{\frac{2T_a}{m}}\right)} \quad \text{and} \quad \frac{0.332E}{N\sqrt{T_a m}\sigma\left(2.24\sqrt{\frac{2T_a}{m}}\right)}.$$

A similar relation obtained using $F_{11}^{(2)}(x)$ yields the numerical coefficient in this formula equal to 0.343, which weakly depends on the temperature of atoms and the parameter a in (11a).

Figure 1 shows the results of comparison of the correction to the Maxwellian ion distribution function $g_i(x) = 2\pi F_{11}^{(i)}(x)_{\sigma = 0}$, $i = 1, 2$, calculated using the obtained relations in the assumption of constant charge exchange cross section ($a = 0$) and absence of elastic collisions, and the similar functions $g(x)$, $g_x(x)$ calculated in [6] and [7], respectively. It can be seen that, first, these functions are close and second, the first approximation $g_1(x)$ hardly differs from the second one $g_2(x)$ (see Appendix). Thus, it can be stated that the obtained results for the ion distribution function in the intrinsic gas and weak fields agree well in particular cases with the data obtained by other authors.

Figure 2 shows the calculated ion distribution functions $g_1(x)$ and $g_2(x)$ without account of elastic collisions for He^+–He and Xe^+–Xe. Formula (11) with the data from [18] was used for calculation of the resonance charge exchange cross section. Similar to the previous case, it can be seen that the second approximation practically coincides with the first one. Moreover, it follows from these results that the form of $g(x)$ weakly depends on the charge exchange cross section.

We also calculated the dependences of reduced zero field mobility $K_0 = \frac{P[\mathrm{Torr}]}{760}\frac{273.16}{T_a[\mathrm{K}]}$ K for He^+ and Ar^+ ions in the intrinsic gases on the gas temperature in a range from cryogenic temperatures to 1000 K. These gases were chosen because there are reliable data on elastic scattering cross sections in a wide range of relative energies of the atom and the ion [5, 19, 20]. The results for He^+ and Ar^+ are shown in Figs. 3 and 4, respectively. Figure 3 shows the experimental data from [21–27] and the results of calculations from [28, 29]. It was demonstrated in [21] that the dependence of reduced mobility for He^+ in He on the gas temperature can be obtained from the dependence of this quantity on the parameter E/N if the effective temperature T_{eff} is used as the gas temperature; this effective temperature for He should be calculated using the formula

$$T_{\mathrm{eff}} = T_a + 2.9\times10^{-4} m_{\mathrm{He}}\left(K_0\frac{E}{N}\right)^2, \qquad (14)$$

where T_{eff}, T_a are taken in kelvins, m_{He} is the mass of He atom in a.m.u., the reduced mobility K_0 is expressed in cm^2/V s, and E/N is expressed in Td. The experimental data from [21] on He$^+$ mobility in He as a function of E/N is recalculated using formula (14) are also shown in Fig. 3. Here, the results of our calculations of the resonance charge exchange cross section according to (11) with the parameters taken from [18] and the correction for deviation of the trajectories of colliding particles from straight lines at small relative energies of the ion and the atom in the form of the factor $\left[1 + \frac{V(R_0)}{\varepsilon}\right]$, where $R_0 = \sqrt{\frac{2\sigma_c}{\pi}}$ [30] and the parameter $\sigma = 0.31$ are also shown. This correction is necessary, since the calculations included cryogenic temperatures for which this correction may play a substantial role. Moreover, it was taken into account that, for small energies of relative motion, the resonance charge exchange cross section becomes equal to one-half of the polarization capture cross section [30]. The energy transfer cross section given in [29] is well approximated by the formula

$$\sigma_v(\varepsilon) = \frac{\sigma_m(\varepsilon)}{1.5(1+\varepsilon^{1.1})}, \quad \sigma_m(\varepsilon) = 2\sigma_c(\varepsilon). \qquad (15)$$

It follows from these relations and (4) that $\sigma = 1$. However, the value of this parameter found from the condition of best matching with the experimental data is $\sigma = 0.31$. This can be explained as follows: experimental data [20] indicate that the differential cross section of elastic scattering for He$^+$ ion on He atom for energies on the order of 1 eV has a pronounced maximum at small scattering angles. This circumstance increases the integral elastic scattering cross section, but hardly influences the ion mobility, since it corresponds to scattering with small ion momentum and energy variation. Thus, if the differential cross section of ion scattering on an intrinsic atom is approximate according to (1), i.e., without taking into account the maximum at small scattering angles, the integral scattering cross section should be taken to be smaller than the real one. This fact explains why the ratio of elastic scattering and charge exchange cross sections at 1 eV found from the best match between calculations and the experiment is smaller than that obtained from quantum mechanical calculations [29].

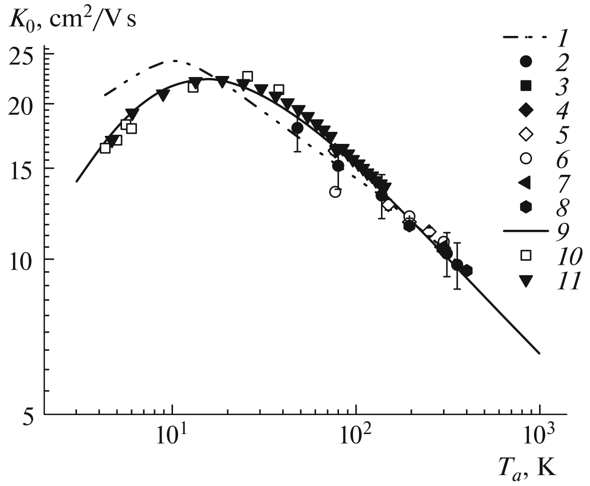

Fig. 3. Ion He$^+$ mobility K_0 in He as a function of temperature of atoms: (*1*) calculation [28], (*2*) experiment [21], (*3*) experiment [22], (*4*) calculation [29], (*5*) experiment [23], (*6*) experiment [24], (*7*) experiment [25], (*8*) experiment [26], (*9*) calculation using the obtained formulas, He scattering cross sections from [18, 29], and $\sigma = 0.31$, (*10*) recalculated data [29] using T_{eff} (see formula (14)), and (*11*) recalculated data [21] using T_{eff} (see formula (14)).

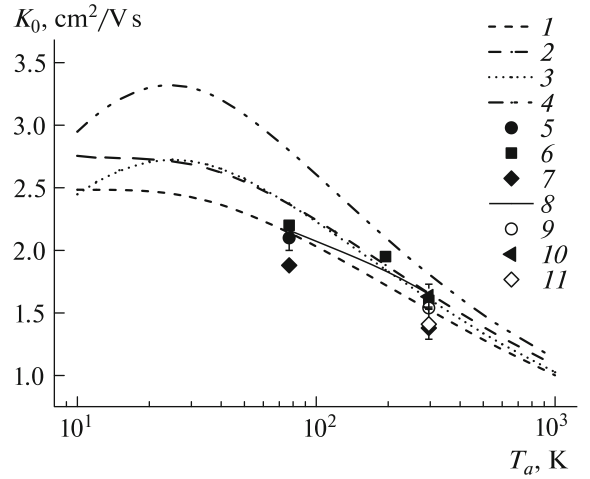

Fig. 4. Ion Ar$^+$ mobility in Ar as a function of temperature of atoms: (*1*) calculation using the obtained formulas, charge exchange and elastic scattering cross sections from [31], and $\sigma = 0.488$, (*2*) same as (*1*) but for $\sigma = 0$, (*3*) calculation using the obtained formulas, charge exchange cross section from [18], elastic scattering cross section from [31], and $\sigma = 0.514$, (*4*) same as (*3*) but for $\sigma = 0$, (*5*) experiment [32], (*6*) experiment [24], (*7*) experiment [33], (*8*) calculation [36], (*9*) experiment [34], (*10*) experiment [11], and (*11*) experiment [35].

Figure 4 shows the data on reduced zero field mobility for Ar$^+$ in Ar calculated for different charge exchange cross sections. The calculations were performed for the resonance charge exchange cross section taken from [18] and the elastic cross section taken from [30] (Fig. 4, curve *3*). In the other variant, the resonance charge exchange cross section was determined from relation (3) with the momentum transfer cross section σ_m and the elastic scattering cross section taken from [31] (Fig. 4, curve *1*). In both cases, the parameter σ was not varied, but calculated using the known cross sections [18, 31], it was equal to $\sigma = 0.514$, 0.488, respectively. This figure also shows the results of calculations using the same resonance charge exchange cross sections but without account of

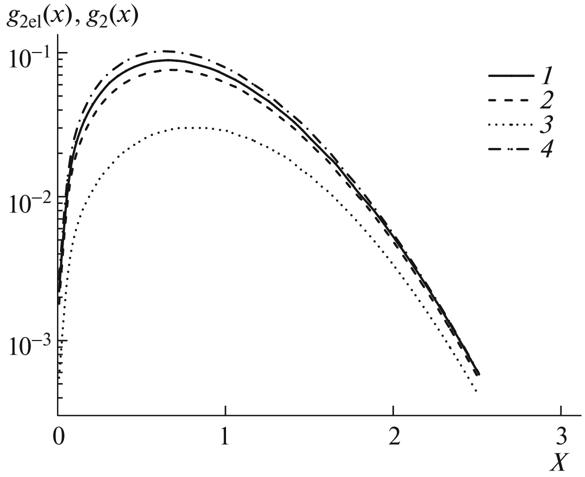

Fig. 5. Influence of elastic collisions on the form of the ion distribution function perturbation in a weak field for the He^+–He case, different atomic temperature, and $\sigma = 0.31$: (*1*) $g_{el}(x)$ is the second approximation taking into account elastic collisions $T_a = 1000$ K, (*2*) $g_{el}(x)$ is the second approximation taking into account elastic collisions $T_a = 76$ K, (*3*) $g_{el}(x)$ is the second approximation taking into account elastic collisions $T_a = 10$ K, (*4*) $g_2(x)$ is the second approximation without account of elastic collisions. Cross sections of both resonance charge exchange and elastic scattering are taken from [18, 29].

elastic scattering. It can be seen from these data that, since the resonance charge exchange cross section given in [18] is larger than that calculated using the data from [31] on σ_m, curve *3* is noticeably higher than curve *1*. Moreover, in the low temperature region (for $T_a < 30$ K), the calculated mobilities depend on the temperature of atoms in a different way. This is probably also connected with the different temperature dependence of the charge exchange cross sections given in [18] and calculated using the diffusion cross section from [31] at low relative energies of the ion and the atom. Concerning the agreement between calculations and experimental data, on the whole, calculations according to [31] better agree with experiment. However, it should be noted that the data on mobility of Ar^+ ions in Ar differ noticeably in [11, 24, 32–36].

It should also be emphasized that it can be seen from the curves in Fig. 4 that neglecting elastic scattering results in a considerable error in calculating the mobility at temperatures below 300 K, and this error increases with decreasing temperature of the atoms. It is clear that the reason for this is the sharper growth in the elastic scattering cross section with decreasing relative energy of the ion and the atom (which at the zero field is determined by the temperature of atoms) compared to the resonance charge exchange cross section. It should also be noted that, while for better agreement of the calculated He^+ mobility in He with the experiment, we had to take the parameter σ smaller than the ratio of the applied elastic cross section and the cross section of resonance charge exchange (for the reason explained above), for argon this was not required, and the parameter σ was taken from the cross section ratio. The reason for this difference is that, as follows from a comparison of the data on differential cross sections of elastic Ar^+ ion scattering on Ar atoms [3] and He^+ ion on He atoms [20], for an energy of relative motion of the ion and the atom of about 1 eV (for which the parameter σ is calculated) in the first case, the cross section is close to isotropic in the center of mass system, while in the second case there is a noticeable maximum at small scattering angles. As was already mentioned above, the latter circumstance reduces the contribution of isotropic scattering of He^+ ion, which results in the requirement to reduce the parameter σ, since elastic scattering at small angles does not influence the ion mobility.

Figure 5 shows the results of calculation of the function $g_2(x)$ without account of elastic collisions for He^+–He and the function $g_{2el}(x)$ calculated taking into account elastic collisions for He^+–He at different gas temperatures $T_a = 10, 76, 1000$ K and $\sigma = 0.31$. As expected, taking into account elastic collisions reduces the correction of the ion distribution function to the Maxwellian function caused by the presence of the electric field and the process of resonance charge exchange, since it first results in the isotropization of the ion distribution function and, second, in the additional energy exchange between atoms and ions. Moreover, unlike the function $g_2(x)$ (calculated taking into account resonance charge exchange only), which weakly depends on the gas temperature, the influence on the ion distribution function $g_{2el}(x)$ of elastic collisions increases with reducing temperature. In the considered He^+–He case at 10 K, $g_{2el}(x)$ decreases by a factor of approximately three compared to the correction calculated without taking into account elastic collisions. This is probably determined by the fact that, at low temperatures, the ratio of elastic scattering and cross sections of resonance charge exchange increases noticeably.

Thus, it can be concluded that the ion mobility in the intrinsic gas at low temperatures is sensitive to the ratio of elastic scattering and resonance charge exchange cross sections, and the obtained formulas can be used to find the energy dependence of this ratio from experimental data.

CONCLUSIONS

Let us summarize the main results of the study:

(i) The expression for the ion distribution function in weak fields in an intrinsic gas was obtained. This expression takes into account elastic ion collisions with atoms, along with resonance charge exchange. It was demonstrated that elastic collisions result in a smaller deviation of the ion distribution function from the Maxwellian one.

(ii) The calculations of the drift velocity and mobility using the obtained formulas neglecting elastic collisions agree well with the theoretical calculations made by other authors.

(iii) The temperature dependences of zero field He^+ and Ar^+ ion mobilities in the intrinsic gases were calculated for a gas temperature range of 4–1000 K. The calculated dependences well describe the available experimental data. Unlike the case of strong and moderate fields, neglecting elastic ion collisions with atoms results in the considerable overestimation of the calculated mobility, especially at cryogenic temperatures.

(iv) The results of a comparison of the calculated and experimental mobilities at low temperatures can be used to reconstruct the ratios of the cross sections of elastic ion scattering on atom and resonance charge exchange.

APPENDIX

Equation (11b) represents a singularly perturbed integro-differential equation with the small parameter ε_0 at the highest derivative [16]. The proof of closeness of its approximate solution obtained by expansion over the small parameter requires a special examination. Therefore, to find this solution, we use the well-known multiscale method [17], which can be applied to solve singularly perturbed equations. Introducing the variables $x_1 = x$, $\tau = x/\varepsilon_0$ instead of x, we obtain the integro-differential equation regular with respect to the small parameter ε_0, and its solution can be sought in the form of an expansion into a power series over this small parameter. Taking into account that $\frac{\partial}{\partial x} = \frac{\partial}{\partial x_1} + \varepsilon_0^{-1}\frac{\partial}{\partial \tau}$, for the first two terms of this expansion, we obtain

$$f_i = F_0(x_1,\tau,\mu) + \varepsilon_0 F_1(x_1,\tau,\mu),$$

$$\mu\frac{\partial F_0}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_0$$
$$= 2[k(x) - 0.5\sigma k_e(x)]f_a(x_1)\int x_r F_0(\mathbf{x}_1', \tau)d\mathbf{x}_1'$$
$$+2\sigma\int k_e(x')\tilde{w}_e(\mathbf{x}_1' \to \mathbf{x}_1)F_0(\mathbf{x}_1', \tau)d\mathbf{x}_1', \qquad \text{(A1)}$$

$$\mu\frac{\partial F_0}{\partial x_1} + \mu\frac{\partial F_1}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_1$$
$$= 2[k(x) - 0.5\sigma k_e(x)]f_a(x_1)\int x_r F_1(\mathbf{x}_1', \tau)d\mathbf{x}_1'$$
$$+2\sigma\int k_e(x')\tilde{w}_e(\mathbf{x}_1' \to \mathbf{x}_1)F_1(\mathbf{x}_1', \tau)d\mathbf{x}'.$$

To solve system (A1), we take into account that, for weak fields, the ion distribution function is close to isotropic and apply the well-known P_1 method [14],

$$F_i = \frac{1}{4\pi}(F_{i0} + 3\mu F_{i1}), \quad i = 0,1.$$

Substituting this expansion into (A1), integrating over the angles, then multiplying by μ, and integrating over the angles again, we obtain the system of equations for the functions $F_{ik}(x_1, \tau)$, i, $k = 0$, 1,

$$\frac{\partial F_{01}}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_{00}$$
$$= 2[k(x) - 0.5\sigma k_e(x)]\int \alpha_{0r}(x_1', x_1)F_{00}(x_1', \tau)dx_1'$$
$$+2\sigma\int k_e(x')\alpha_0(x_1' \to x_1)F_{00}(x_1', \tau)dx_1',$$

$$\frac{\partial F_{01}}{\partial x_1} + \frac{\partial F_{11}}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_{10}$$
$$= 2[k(x) - 0.5\sigma k_e(x)]f_a(x_1)\int \alpha_{0r}(x_1', x_1)F_{10}(x_1', \tau)dx_1'$$
$$+2\sigma\int k_e(x')\alpha_0(x_1' \to x_1)F_{10}(x_1', \tau)dx_1',$$

$$\frac{1}{3}\frac{\partial F_{00}}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_{01}$$
$$= 2[k(x) - 0.5\sigma k_e(x)]f_a(x_1)\int \alpha_{1r}(x_1', x_1)F_{01}(x_1', \tau)dx_1'$$
$$+2\sigma\int k_e(x')\alpha_1(x_1' \to x_1)F_{01}(x_1', \tau)dx_1',$$

$$\frac{1}{3}\frac{\partial F_{00}}{\partial x_1} + \frac{1}{3}\frac{\partial F_{10}}{\partial \tau} + 2\tilde{w}_{oe}(x_1)[k(x_1) + 0.5\sigma k_e(x_1)]F_{11}$$
$$= 2[k(x) - 0.5\sigma k_e(x)]f_a(x_1)\int \alpha_{1r}(x_1', x_1)F_{11}(x_1', \tau)dx_1'$$
$$+2\sigma\int k_e(x')\alpha_1(x_1' \to x_1)F_{11}(x_1', \tau)dx_1',$$

where

$$\alpha_{0r}(x_1', x_1) = 2\pi\int_{-1}^{1}\sqrt{x_1'^2 + x_1^2 - 2x_1'x_1\mu}\,d\mu,$$

$$\alpha_{1r}(x_1', x_1) = 2\pi\int_{-1}^{1}\mu\sqrt{x_1'^2 + x_1^2 - 2x_1'x_1\mu}\,d\mu,$$

$$\alpha_0(x_1' \to x_1) = 2\pi\int_{-1}^{1}\tilde{w}_e(\mathbf{x}_1' \to \mathbf{x}_1)d\mu,$$

$$\alpha_1(x_1' \to x_1) = 2\pi\int_{-1}^{1}\mu\tilde{w}_e(\mathbf{x}_1' \to \mathbf{x}_1)d\mu.$$

The following expressions hold for $\alpha_1(x_1' \to x_1)$ [13]:

$$\alpha_1(x_1' \to x_1) = 2\frac{\exp(x_1'^2 - x_1^2)}{x_1'}\psi(x_1') \quad \text{for} \quad x_1 > x_1', \tag{A2}$$

$$\alpha_1(x_1' \to x_1) = 2\frac{x_1}{x_1'^2}\psi(x_1) \quad \text{for} \quad x_1 < x_1',$$

$$\psi(x) = \frac{2}{\sqrt{\pi}}\exp(-x^2) + \left(x - \frac{1}{x}\right)\mathrm{erf}(x),$$

and for $\alpha_{1r}(x_1', x_1)$,

$$\alpha_{1r}(x_1', x_1) = \frac{2\pi}{15\eta^2}\{(1+\eta^2-3\eta)(1+\eta^2+2\eta)^{1.5} - (1+\eta^2+3\eta)(1+\eta^2-2\eta)^{1.5}\},$$

$$\eta = \frac{x_1'}{x_1}.$$

Using the P_2 approximation [14], we can show that the following relations hold:

$$F_{01}, F_{10} \equiv 0, \quad F_{00}(x_1, \tau) = f_a(x_1).$$

Then, going back to the variable x, we obtain

$$\frac{1}{3}\frac{\partial F_{00}}{\partial x} + 2\tilde{w}_{oe}(x)[k(x) + 0.5\sigma k_e(x)]F_{11} = 2[k(x) - 0.5\sigma k_e(x)]f_a(x)\int \alpha_{1r}(x', x)F_{11}(x')dx' + 2\sigma\int k_e(x')\alpha_1(x' \to x)F_{11}(x')dx'. \tag{A3}$$

To solve Eq. (A3), let us first consider the case $\sigma = 0$. In this case, the convergence method used in [2] can be applied,

$$F_{11}^{(n)}(x)_{\sigma=0} = \frac{x\exp(-x^2)}{3\pi^{1.5}\tilde{w}_{oe}(x)k(x)} + 2f_a(x)\int \alpha_{1r}(x', x)F_{11}^{(n-1)}(x')_{\sigma=0}dx', \tag{A4}$$

$$F_{11}^{(0)}(x)_{\sigma=0} = \frac{x\exp(-x^2)}{3\pi^{1.5}\tilde{w}_{oe}(x)k(x)}.$$

The calculations show that the second approximation differs from the first one by about 1%. The solution for $\sigma \neq 0$ is obtained in a similar way. Then, using the second approximation for $F_{11}(x)_{\sigma = 0}$, we obtain

$$F_{11}^{(n)}(x) = \frac{F_{11}^{(2)}(x)_{\sigma=0}}{1 + 0.5\sigma\Omega(x)} + \frac{\sigma[1 - 0.5\sigma\Omega(x)]}{\tilde{w}_{oe}(x)[1 + 0.5\Omega(x)]} \times \int_{-1}^{1} k_e(x')\alpha_1(x_1' \to x_1)F_{11}^{(n-1)}(x)dx, \tag{A5}$$

$$F_{11}^{(0)}(x) = \frac{F_{11}^{(2)}(x)_{\sigma=0}}{1 + 0.5\sigma\Omega(x)}, \quad \Omega(x) = \frac{k_e(x)}{k(x)},$$

where $\alpha_1(x_1' \to x_1)$ is defined by relations (A2). Similar to the above said, the second approximation of this convergence method yields an almost exact result.

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Translated by E. Baldina