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Magnetic Minibands in Superlattices Based on the Semi-Dirac Crystals

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Abstract—Semi-Dirac crystals with a weak periodic modulation of the energy gap has been considered. The formation of minibands in such crystals has been studied when the latter are placed in a quantizing magnetic field. The width of the formed magnetic minibands has been shown to depend not only on the magnetic field intensity, but also on the band gap of the initial sample in contrast to gap graphene. The effect of this feature on the magneto conductivity of studied material has been investigated.

Keywords: superlattice, magnetic minibands, Weiss oscillations
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Recently, in the field of physics of low-dimensional structures, researchers have paid special attention to the study of electronic properties of 2D crystals belonging to the group of so-called Dirac [1, 2] materials. Transport effects in such materials are more robust to temperature decay compared to 2D electron gas with a standard parabolic spectrum [3, 4]. An example of such effects are oscillations of the conductivity of a crystal with a superlattice (SL) in a magnetic field when the strength of the latter changes (Weiss oscillations). In [3, 4], Weiss oscillations have been studied for ideal graphene, in which the SL is formed due to an electrostatic potential periodic along the spatial axis. However, such a potential due to the semimetallic type of graphene conductivity will lead to redistribution of free charge carriers across the graphene surface and, consequently, to its distortion. This circumstance creates the necessity to search for alternative ways to create in Dirac crystals additional SL potentials [5–7]. One of the ways is to form a spatially modulated energy gap in the zone structure of graphene [5]. In [6, 7], so-called semi-Dirac crystals have been proposed. Such materials have been obtained relatively recently and represent 2D structures whose effective mass tensor of charge carriers is significantly anisotropic. In one direction, the electrons have a relativistic-type dispersion, and in the transverse direction—a quadratic dispersion [8, 9]. An example of such a material is phosphorene, whose conductivity has a strong directional dependence [8].

Let us associate the xy plane with the semi-Dirac crystal and place it in a uniform magnetic field whose intensity vector $\mathbf{H}$ is perpendicular to xy. Let us write the model Hamiltonian for charge carriers with magnetic field in the form [10]

$$\hat{H}_{\mathrm{SD}} = \upsilon_{\mathrm{F}} p_x \hat{\sigma}_x + \left(\frac{1}{2m}\left(p_y + \frac{\hbar x}{\lambda^2}\right)^2 + \Delta\right)\hat{\sigma}_y, \qquad (1)$$

where $\hat{\sigma}_{x,y,z}$—Pauli matrices, m—effective mass of the carrier in the direction Oy, Δ—half-width of the energy slit ($\Delta > 0$), $\lambda = \sqrt{c\hbar/eH}$. The energy eigenvalue equation $\hat{H}_{\mathrm{SD}}\psi_n = \varepsilon_n\psi_n$ is solved under the condition that the terms containing the small dimensionless parameters $\hbar^2/m\Delta\lambda^2$ and $\hbar/m\upsilon_{\mathrm{F}}\lambda \ll 1$ as multipliers can be neglected. This is justified if the magnetic field strengths are such that the period of the SLd is not much larger than the minimum value of the cyclotron radius ($\lambda_{\min}/d > 0.1$). In addition, at the standard value of the SL period $d = 10^{-5}$ cm, the parameters m and Δ should satisfy the inequalities $m \gg 10^{-29}$ g, $\Delta \gg 5$ meV, which is quite consistent with the real [8, 11] materials. In result, the problem can be reduced to the harmonic oscillator problem and found for energy:

$$\varepsilon_n = \sqrt{\Delta^2 + \frac{\hbar^2\upsilon_{\mathrm{F}}^2}{\Lambda^2}(2n+1) - \left(\frac{\hbar^2\upsilon_{\mathrm{F}}^2}{2\Delta\Lambda^4}\right)^2}. \qquad (2)$$

The following notations are introduced here: $\Lambda = \lambda/\kappa$, $\kappa^4 = \Delta/m\upsilon_{\mathrm{F}}^2$. The corresponding eigenspinors have the form

$$\psi_{n,p_y} = \frac{1}{\sqrt{2\Lambda}}\left(-i\Phi_n\left(\frac{x - x_+}{\Lambda}\right)\Phi_n\left(\frac{x - x_-}{\Lambda}\right)\right)^T, \qquad (3)$$

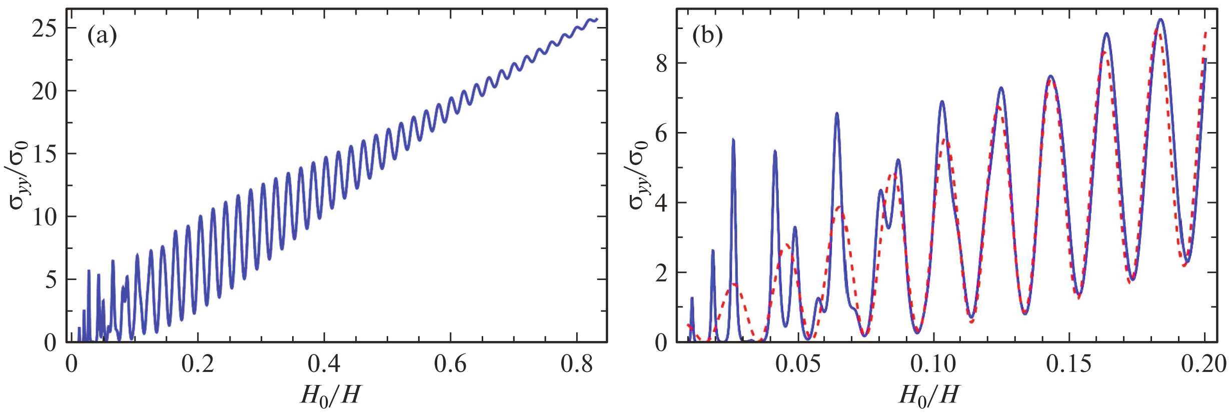

Fig. 1. The dependence of the magnetic conductivity of a semi-Dirac crystal on the inverse magnetic field strength: (a) calculation by formula (7); (b) comparison of the graph (solid line) plotted by formula (7) and the graph (dashed line) plotted by approximate formula (8). $d = 10^{-5}$ cm, $n_0 = 2 \times 10^{11}$ cm^{-2}, $T = 4$ K, H_0 corresponds to a magnetic induction of 0.06 T.

where $\Phi_n(\xi)$—harmonic oscillator functions, $x_\pm$—cyclotron centers equal to

$$x_\pm = -\frac{p_y\lambda^2}{\hbar} \mp \frac{\hbar\upsilon_F}{2\Delta}. \tag{4}$$

From (2) and (3) it can be seen that, in contrast to graphene [3, 4], the cyclotron radius for the semi-Dirac electron, equal to Λ, depends on the parameter Δ, and the position of its Larmor center is determined by the pseudo spin.

We now consider that the energy slit has a spatial periodic modulation:

$$\Delta_g = \Delta - \Delta_0 \cos(2\pi x/d),$$

and $\Delta_0 \ll \Delta$. As is known, the SL potential leads to the removal of degeneracy by p_y for Landau levels whose broadening forms magnetic mini-zones. Let us substitute $\Delta \to \Delta_g$ in the Hamiltonian (1). The calculations performed in the first strand of perturbation theory lead to the following expression for the law of dispersion in the minizone:

$$\varepsilon_{n,p_y} = \varepsilon_n - \Delta_0 g_n \cos\left(\frac{2\pi p_y\lambda^2}{\hbar d}\right). \tag{5}$$

Here, ε_n—the n-energy of the Landau go level in the absence of slit modulation, equal to (2),

$$g_n = e^{-\frac{\alpha^2+\beta^2}{4}} L_n\left(\frac{\alpha^2+\beta^2}{2}\right), \tag{6}$$

$L_n(\xi)$—Laguerre polynomials, $\alpha = 2\pi\Lambda/d$, $\beta = \hbar\upsilon_F/\Delta\Lambda$. As can be seen from formula (6), the parameter Δ is contained in the function argument g_n. Consequently, for SLs based on a semi-Dirac crystal (in contrast to the Dirac [3, 4]), the width of the magnetic mini-zone, equal to $2\Delta_0 g_n$, depends, among other things, on the width of the band gap 2Δ.

In the frames of the constant relaxation time approximation τ, the magnetic conductivity of the semi-Dirac crystal in the direction Oy is equal to

$$\sigma_{yy} = \frac{\pi\sigma_0 m\upsilon_F^2\lambda^2}{4kTd^2}\sum_{n=0}^{\infty} g_n^2 \cosh^{-2}\left(\frac{\varepsilon_n - \varepsilon_F}{2kT}\right), \tag{7}$$

where $\sigma_0 = e^2\tau\Delta_0^2/\hbar^2 m\upsilon_F^2$, ε_F—Fermi energy, T—temperature. Figure 1a shows a plot of the dependence of conductivity (7) on the inverse magnetic field strength plotted for surface concentration $n_0 = 2 \times 10^{11}$ cm^{-2}, $\Delta = 0.1$ eV and $T = 4$ K. In the case of low temperatures ($kT \ll \varepsilon_F$) and weak magnetic fields such that a large number of Landau levels ($n \ll 1$) appear below the Fermi level, the following formula is valid:

$$\sigma_{yy} = \frac{\sigma_0 m\upsilon_F^2}{\varepsilon_0\kappa\gamma w}\frac{\lambda^3}{d^3}\left(1 + Q_T\left(\frac{2\pi kT}{\varepsilon_0\kappa}\frac{w\lambda}{\gamma d}\right)\sin\left(\frac{2\varepsilon_F}{\varepsilon_0\kappa}\frac{\gamma w\lambda}{d}\right)\right), \tag{8}$$

where

$$\varepsilon_0 = \hbar\upsilon_F/d, \quad w = \sqrt{\alpha^2+\beta^2},$$
$$\gamma = \sqrt{1-(1+\beta^2)\Delta^2/\varepsilon_F^2}, \quad Q_T(\xi) = \xi\sinh^{-1}\xi.$$

A comparison of the Weiss oscillations constructed by the formulas (7) and (8) is shown in Fig. 1b, from which we can see the asymptotic convergence of the plots with decreasing magnetic field strength. According (8), for semidirac crystals, the periodicity of the conductivity oscillations in terms of the magnitude H^{-1} is preserved only under the condition $\beta \ll \alpha$. In this case, the Weiss oscillation period is equal to $\delta(H^{-1}) = ed\Delta^{1/2}/2c\varepsilon_F\gamma m^{1/2}$.

In conclusion, we will point out the difference in the structures of the eigenstates of the electrons of the semidirac crystal and the slit modification of graphene. As can be seen from (3) and (4), the magnitude of the cyclotron radius for the semi-Dirac electron depends on the half-width of the band gap A, and its Larmor center is determined by the pseudospin. As a consequence, the width of the magnetic mini-zone, according to (6), also depends on the parameter Δ. Such a feature is absent in graphene SL [3, 4]. This leads to a more complex dependence of the magnetic mini-zone width on the magnetic field strength, which in turn is reflected in the character of the periodicity of the Weiss oscillations in the inverse magnetic field. The latter can be considered periodic by inverse intensity H^{-1} only for relatively weak fields, whose band is given by the inequality $\beta \ll \alpha$. The period of the Weiss oscillations at this $\delta(H^{-1}) \propto \sqrt{\Delta/m}$. This fact makes it possible to use the Weiss oscillations as a basis for the experimental method of measuring the parameters of the zone structure of semi-Dirac crystals.

CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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