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FABRICATION, TREATMENT, AND TESTING OF MATERIALS AND STRUCTURES

The Temperature Dependence of the Conductivity Peak Values in the Single and the Double Quantum Well Nanostructures *n*-InGaAs/GaAs after IR-illumination[1]

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Abstract—The dependences of the longitudinal and Hall resistances on a magnetic field in *n*-InGaAs/GaAs heterostructures with a single and double quantum wells after infrared illumination are measured in the range of magnetic fields $B = 0–16$ T and temperatures $T = 0.05–4.2$ K. Analysis of the experimental results was carried out on a base of two-parameter scaling hypothesis for the integer quantum Hall effect. The value of the second (irrelevant) critical exponent of the theory of two-parameter scaling was estimated.

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1. INTRODUCTION

The phenomenon of the integral quantum Hall effect (QHE), detected by Von Klitzing et al. [1], is closely associated with the problem of electron localization in a two-dimensional (2D) system in a quantizing magnetic field B. Laughlin [2] and Halperin [3] showed that, for the QHE to exist, narrow bands of delocalized states must be present close to the middle of each Landau subband, provided that all the other states are localized. On the other hand, earlier, in a seminal paper Abrahams et al. [4], on a base of the one-parameter scaling theory for a conductance G of a system, came to the conclusion that for 2D disordered systems at $B = 0$ quantum diffusion should be absent, i.e., there are no delocalized states in 2D systems in a presence of even a small degree of disorder. The supposition of Laughlin [2] and Halperin [3] thus appeared to be in an apparent contradiction with the results of the scaling theory with a single parameter σ (for 2D systems the concepts of conductance G and conductivity σ coincide).

Pruisken [5] was the first to express the idea that, in a quantizing magnetic field, it is necessary to consider renormalization of both the dissipative component σ_{xx} and the Hall component σ_{xy} of the conductivity tensor as the macroscopic size L of the system varies. To explain the QHE, Pruisken [5–7] and also Khmel'nitski [8] proposed the hypothesis of two-parameter scaling, which results in the existence of both localized and delocalized states (close to the middle of the Landau subbands) in the spectrum of a disordered 2D system in a quantizing magnetic field.

Historically, it happened that the vast number of works on the scaling in QHE regime are devoted to a study of only one aspect of the two-parameter theory, specifically, to an investigation of the QHE plateau-plateau transitions as the quantum phase transitions with the evaluation of the first critical index relevant to a divergence of localization length [9].

In our previous works [10, 11] we have also investigated the temperature dependences of the width of plateau-plateau QHE transitions in *n*-InGaAs/GaAs nanostructures before and after IR-illumination with an emphasis on a search of the scaling patterns.

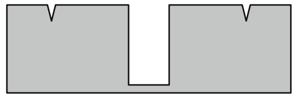
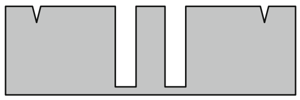
In a present paper we have analyzed the temperature dependence of the longitudinal conductivity peak values and attempted to estimate a second ("irrelevant") critical index of the two-parameter scaling QHE theory for *n*-InGaAs/GaAs heterostructures after IR-illumination.

2. CHARACTERISTICS OF THE SAMPLES

2D-structures of $GaAs/In_xGa_{1-x}As/GaAs$ with single (SQW) and double (DQW) quantum wells were grown by the method of organometallic vapor phase epitaxy on a semi-insulating GaAs substrate in the Scientific-Research Institute of Physics and Technology of the Nizhny Novgorod University by the B.N. Zvonkov group, and are studied here. The heterostructures are a sequence of epitaxial layers, forming one or two quantum wells, $In_xGa_{1-x}As$.

[1] The article is published in the original.

Table 1. Technological parameters of $GaAs/In_xGa_{1-x}As/GaAs$ structures

Sample	L_s, Å	d_w, Å	d_b, Å	x	Profile
SQW	190	100	0	0.2	
DOW	190	2 × 50	100	0.2	

d_w – well width, d_b – barrier width, L_s – spacer width [12].

The structures are symmetrically δ-doped by Si in the barriers at a distance of 19 nm from the heteroboundaries. Technological parameters and structural profiles of the samples are shown in Table 1.

The charge carrier concentration was varied by infrared illumination both of the SQW and DQW samples. The electro-physical parameters of the investigated samples are shown in Table 2.

Note the sharp increase in both the concentration and the mobility of charge carriers in the samples after exposure to infrared light due to an effect of positive persistent photoconductivity (see [13] and references therein). In a present paper the magnetotransport data only for illuminated samples, SQW (b) and DQW (b), as the most homogeneous, are analyzed.

3. EXPERIMENTAL RESULTS

Measurements of the longitudinal and Hall resistivity tensor components $\rho_{xx}(B, T)$ and $\rho_{xy}(B, T)$ were carried out in magnetic fields $B \leq 16$ T in the temperature range $T = (0.05–4.2)$ K. Figure 1 shows the dependences of resistances $R_{xx}(B)$ and $R_{xy}(B)$ for the samples with single and double quantum wells $In_xGa_{1-x}As/GaAs$ after maximum illumination at $T = 0.05$ K.

It is clear from Fig. 1 that after the maximum IR-illumination heterostructures with single and double quantum wells have similar pictures of the magnetore-sistance dependencies in the QHE regime ($B > 3T$) with similar concentrations of charge carriers.

We are going to examine a temperature dependence of conductivity peak values in the QHE regime for investigated systems. The maximum (peak) value of $\sigma_{xx}(B)$ is reached when the Fermi level coincides with the energy E_c of the delocalized states at the center of each Landau subband. In order to facilitate direct comparison with the QHE scaling theory, we have converted, for a given T, the measured $\rho_{xx}(B)$ and dependences $\rho_{xy}(B)$ into $\sigma_{xx}(B)$ and $\sigma_{xy}(B)$ curves. Onwards, we separately provide data for SQW and DQW systems.

3.1. n-InGaAs/GaAs Structure with Single Quantum Well

Figure 2 shows the dependences of the longitudinal σ_{xx} and Hall σ_{xy} conductivity on magnetic field in the QHE regime for IR-illuminated InGaAs/GaAs SQW sample at different temperatures $T \leq 4.2$ K.

In the Fig. 2, the peak 0^- corresponds to the transition 1 → 2 between the QHE plateaus (spin-split sublevel Landau) and peak $1^{\pm}$ corresponds to the transition 2 → 4 (spindegenrate Landau sublevel). At T, 1K we see a tendency to spin splitting of the peak $1^{\pm}$ with forming a plateau with $i = 3$.

In our previous work [11] we have found that the temperature dependence of the width Δ for the QHE plateau-plateau transition 1 → 2 in an illuminated SQW sample is well described by the power-law scaling function $\Delta(T) \sim (T/T_0)^{\kappa}$ with critical exponent $\kappa = 0.25 \pm 0.02$ (He3 insertion) and $\kappa = 0.70 \pm 0.12$ (He4 insertion). The crossover temperature from one temperature regime to the other was found to be $T_{cross} \approx 2$ K. This behavior was interpreted [14] as a transition from a quasiclassical percolation through a

Table 2. Electron concentration, carrier mobility μ and conductivity at $B = 0$ in the structures depending on the impact of IR radiation

Sample	T, K	n, $m^{-2} \times 10^{15}$	μ, $m^2/(V\,s)$	σ, $\Omega^{-1} \times 10^{-4}$
SQW(a)	1.8	2.1	1.2	4
SQW(b)	0.4(He3)	3.8	3.0	18
	1.8(He4)	3.7	2.9	17
DQW(a)	1.7	2.1	1.2	4
DQW(b)	0.05(He3–He4)	4.7	2.7	20
	1.6(He4)	4.9	2.8	22

(a) Dark sample; (b) illuminated sample. Concentrations shown are determined by the method of the Shubnikov–de Haas oscillations. The second column shows the temperatures, at which the parameters of the samples were determined, and in parentheses—the method for temperature control for the illuminated samples: liquid 4He, liquid 3He, and dilution refrigeration by 3He–4He [12].

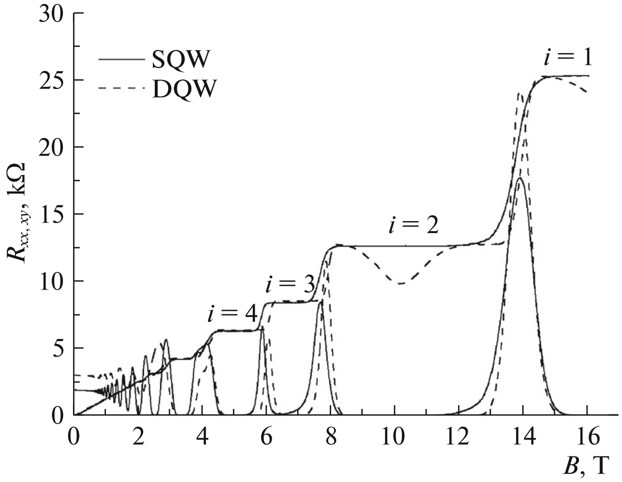


Fig. 1. The dependences $R_{xx}(B)$ and $R_{xy}(B)$ at $T = 0.05$ K for $In_xGa_{1-x}As/GaAs$ samples with single and double quantum wells after maximum illumination.

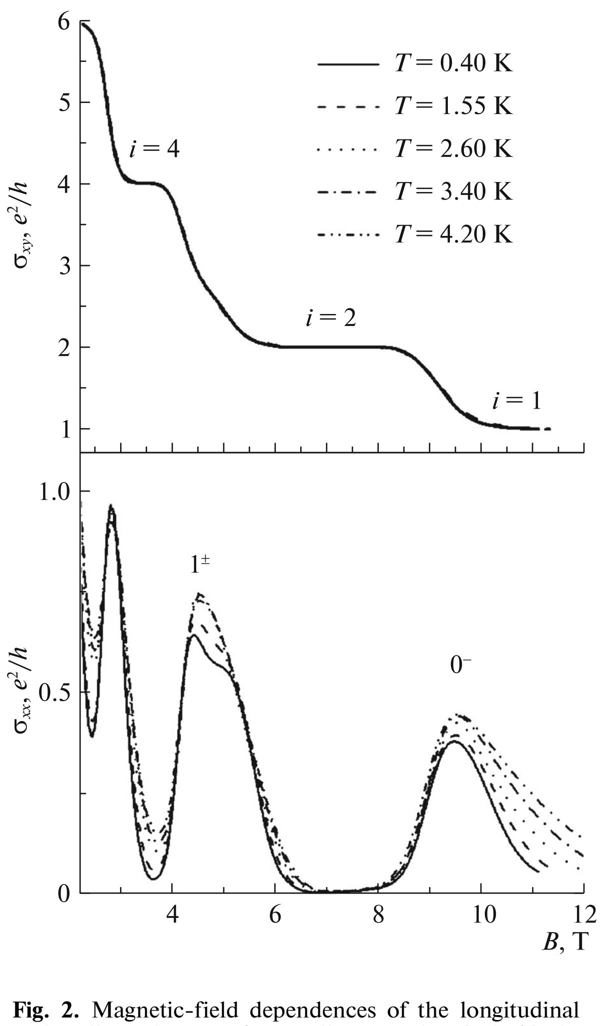


Fig. 2. Magnetic-field dependences of the longitudinal and Hall conductivity for IR-illuminated InGaAs/GaAs sample with a single quantum well at $T = (0.4–4.2)$ K.

potential barrier at different saddle points for $T > T_{cross}$ to a quantum tunneling process for $T < T_{cross}$.

We have analyzed magnetic-field and temperature dependences of the longitudinal conductivity for IR-illuminated SQW sample just for the peak 0^+ for which the system manifests a scaling behavior of $\Delta(T)$. Figure 3 is a plot of the temperature dependence of the maximum (peak) values σ_{xx}^{peak} of the conductivity σ_{xx} in the region of the 1 ... 2 QHE plateau-plateau transition (0^- peak) for IR-illuminated SQW sample InGaAs/GaAs.

It can be seen that at $T < 4.2$ K the amplitude of the peak, σ_{xx}^{peak}, goes down as the temperature decreases, while simultaneously the peak is narrowed (see the inset in Fig. 3). Just such a behavior of $\sigma_{xx}^{peak}(T)$ indicates a transition to the genuine scaling of the conductances as it was pointed out in the first experimental work on localization and scaling in the quantum Hall regime [15].

According to the theory of two-parameter scaling the dependence of the quantity σ_{xx}^{peak} on the linear size L of a 2D system is determined by the so named "irrelevant" critical exponent $y_\sigma < 0$ [9, 16]:

$$(\sigma_{xx}^{peak} - \sigma_{xx}^{c}) \sim L^{y_\sigma} \sim T^{\mu_\sigma}, \quad (1)$$

where σ_{xx}^{c} is the limiting (for $L \to \infty$ or $T \to 0$) value of a "metallic" conductivity on delocalized states at the center of a Landau level at $E_F = E_c$, $\mu_\sigma = -py_\sigma/2$ and the exponent p depends on the inelastic-scattering mechanism. The second part of the expression (1) is derived using the concept of the Thouless length $L_{in} \sim T^{-p/2}$, the length of inelastic scattering of electrons, which, at finite temperature, plays the role of the effective size of the sample [17].

As is evident from the expression (1), a contribution to σ_{xx} determined by the critical exponent y_σ is actually "irrelevant" (absent) in an infinite sample at $T = 0$, but for a real sample at finite temperatures this contribution is quite observable (an effect of finite size scaling [9]).

By matching the expression

$$\frac{h}{e^2}\sigma_{xx}^{peak} = \frac{h}{e^2}\sigma_{xx}^{c} + \left(\frac{T}{T_1}\right)^{\mu_\sigma}, \quad (2)$$

with T_1 as phenomenological parameter and $\mu_\sigma = -py_\sigma/2$, y_σ being the leading irrelevant exponent, to the experimental data (see Fig. 3) it is possible to estimate the limiting conductivity σ_{xx}^{c} and parameters μ_σ and T_1 for the 0^- peak in SQW sample. The best fit gives $\sigma_{xx}^{c} = (0.37 \pm 0.03)e^2/h$, $T_1 = 28.3 \pm 0.4$ and $\mu_\sigma = 1.2 \pm 0.2$.

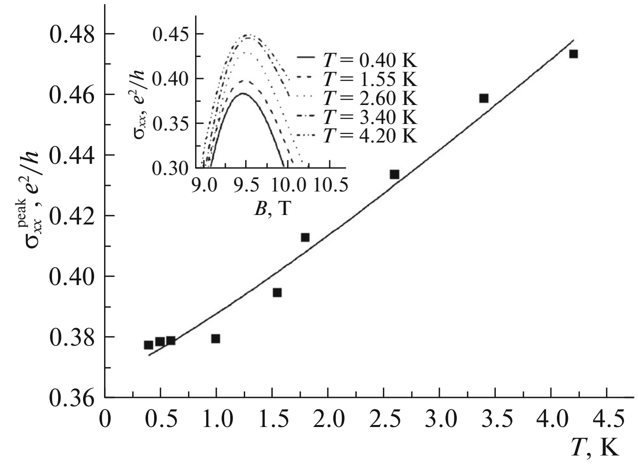


Fig. 3. The maximum conductivity, $\sigma_{xx}^{\text{peak}}$, for the peak as a function of temperature for IR-illuminated InGaAs/GaAs SQW sample. The inset shows the magnetic-field dependences of the longitudinal conductivity for the 0^- peak.

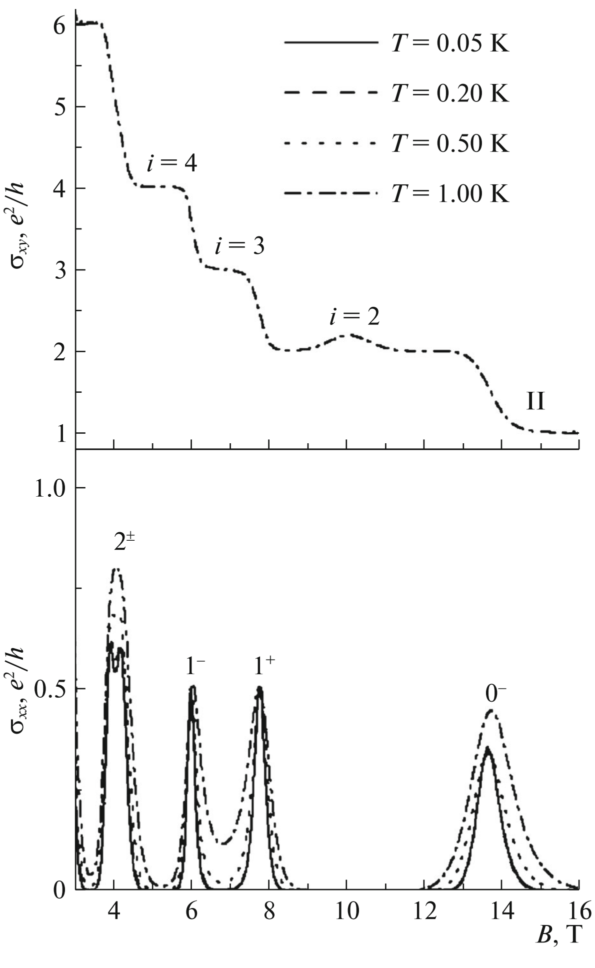


Fig. 4. Magnetic-field dependences of the longitudinal and Hall conductivity for IR-illuminated InGaAs/GaAs DQW sample at $T = (0.05–1.0)$ K.

The obtained value of σ_{xx}^c is in quite good accordance with theoretical calculations ($\sigma_{xx}^c \approx 0.5e^2/h$ for the lowest Landau level [9]) and for $1 < p < 2$ $1.2 \pm 0.2 < |y_\sigma| < 2.4 \pm 0.2$.

These results are similar to our results in heterostructures p-Ge/Ge$_{1-x}$Si$_x$ [16] where for peaks 0^- and 1^+ the values $\sigma_{xx}^c \approx 0.3e_2$ and $\mu_\sigma = -py_\sigma/2 \cong 1$ were obtained.

3.2. n-InGaAs/GaAs Structure with Double Quantum Well

Figure 4 demonstrates the function $\sigma_{xx}(B)$ and $\sigma_{xy}(B)$ at low temperatures ($T \le 1$K) in the QHE regime for DQW n-InGaAs/GaAs system with maximum concentration and mobility after IR-illumination.

The unique results for the longitudinal conductivity σ_{xx} have been obtained in this sample. It is remarkable that, for spin-split peaks 1^+ and 1^- (corresponding to the transitions $2 \to 3$ and $3 \to 4$ between the QHE plateaus) for $T \le 0.05$ K we have $\sigma_{xx}^c = (0.5 \pm 0.05)e^2/h$ for the maximum (critical) values of σ_{xx}: $\sigma_{xx}^c \equiv \sigma_{xx}(B_c)$. This observation is in perfect agreement with the results of numerical simulation for various impurity potential models: $\sigma_{xx}^c = (0.5 \pm 0.05)e^2/h$ [9] and is evidenced of rather high quality of DQW n-InGaAs/GaAs system after IR-illumination.

Let us note, that most researchers report the critical values of the peak amplitudes of σ_{xx} in the QHE regime, σ_{xx}^c ($T \to 0$), which (40–80)% less than the theoretically predicted value of $0.5e^2/h$ (see, for example, the surveys [18, 19]). This discrepancy, just as the observed deviations of the temperature dependence of the bandwidth of QHE transitions, $\Delta(T)$, from the scaling behavior, is usually attributed to the insufficient homogeneity of the samples under test [20, 21].

Figure 4 also shows that the spin splitting of the peak $2^{\pm}$ becomes more and more pronounced as temperature decreases (which corresponds to the formation of a QHE plateau with $i = 5$); for each of these peaks, $\sigma_{xx}^c \to 0.6e^2/h$ as $T \to 0.05$ K. On the other hand, in ultraquantum magnetic fields, for the peak 0^-, we have $\sigma_{xx}^c \cong 0.35e^2/h$, which, just as in many other experimental studies [18, 19, 22], is notably less than the theoretical value $0.5e^2/h$.

In our work [10] a real scaling behavior of the width, $\Delta(T) \sim (T/T^0)^\kappa$, for the QHE plateau-plateau transitions $2 \to 3$ ($\kappa = 0.22 \pm 0.01$) and $3 \to 4$ ($\kappa = 0.21 \pm 0.01$) has been observed in DQW heterostructure after IR-illumination. The value $\kappa = 0.21$ is in consent with the theoretical results of [23–25] and with an estimate of [26] that take into consideration

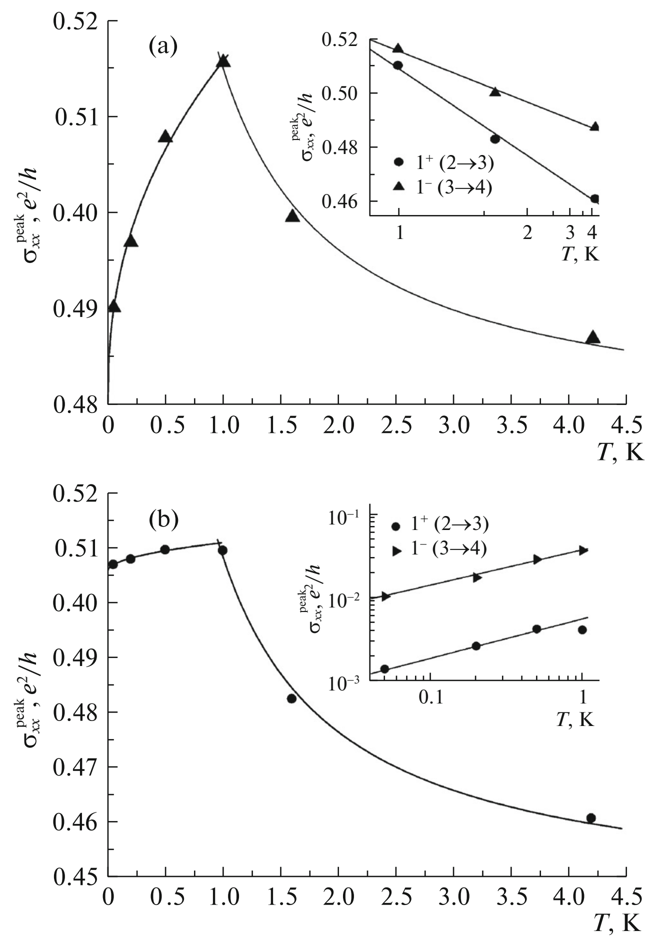


Fig. 5. (a) The maximum conductivity, $\sigma_{xx}^{\text{peak}}$, as a function of temperature for 1^- peak (transition $3 \to 4$) in IR-illuminated DQW InGaAs/GaAs sample. The inset shows $\sigma_{xx}^{\text{peak}}(T)$ at $T > 1.0$ K as a function of $1/T$ for the 1^+ and 1^- peaks. (b) The maximum conductivity, $\sigma_{xx}^{\text{peak}}$, for the 1^+ peak (transition $2 \to 3$) as a function of temperature for IR-illuminated InGaAs/GaAs DQW sample. The inset shows $\sigma_{xx}^{\text{peak}}(T)$ at $T < 1.0$ K in log-log scale for the 1^+ and 1^- peaks.

the short-range electron-electron interaction potential.

Temperature dependences of the maximum (peak) values, $\sigma_{xx}^{\text{peak}}$, of the conductivity σ_{xx} in the regions of the $2 \to 3$ (1^+ peak) and $3 \to 4$ (1^- peak) QHE plateau-plateau transition for IR-illuminated DQW sample are shown, respectively, on Figs. 5a, 5b.

To analyze the temperature dependence of the conductivity, it is convenient to start from the expression [27, 28]:

$$\sigma_{xx}(T) = -\int dE \frac{\partial f(E - E_F)}{\partial E} \sigma_{xx}(E), \tag{3}$$

where $f(E - E_F)$ is the Fermi-Dirac distribution function, and $\sigma_{xx}(E)$ is the partial contribution to the dissipative conductivity of the states with energy E. Since only delocalized states in the energy interval $|E - E_c| \le \Delta$ contribute to the conductivity, we can write the partial conductivity as

$$\sigma_{xx}(E) = \sigma_{xx}^{\text{peak}} \frac{\Delta^2}{(E - E_c)^2 + \Delta^2}. \tag{4}$$

When $E_F = E_c$ we find from Eqs. (3) and (4) that

$$\sigma_{xx}(T) \approx \sigma_{xx}^{\text{peak}} \frac{\Delta}{kT} \quad (kT > \Delta), \tag{5}$$

$$\sigma_{xx}(T) = \sigma_{xx}^{\text{peak}} \quad (kT < \Delta). \tag{6}$$

The quantity $\sigma_x^{\text{peak}} \equiv \sigma(E_c)$ in Eqs. (4)–(6) at zero temperature depends only on the linear size L of a 2D system (see Eq. (1)).

Thus, two regions can be distinguished in the temperature dependence of the peak amplitude of $\sigma_{xx}(T)$. When $kT > \Delta$ a conventional thermal smearing of the Fermi step kT is the main reason for the temperature dependence of σ_{xx}. In this region the width of the peak is *growing* ($\sim kT$) and the amplitude, $\sigma_{xx}^{\text{peak}}$, *decreases* in process of *T increasing*.

In the low-temperature region, $kT \ll \Delta$, the scaling regime, in which the temperature dependence $\sigma_{xx}(T)$ (*if any*) is completely determined by the Thouless length $L_{\text{in}}(T)$, may be realized. In this regime both the σ_{xx} peak width, Δ, and its amplitude, $\sigma_{xx}^{\text{peak}}$, are *reduced* in process of *T decreasing* (see Eq. (1)).

The maximum of $\sigma_{xx}^{\text{peak}}$(T) dependence should be achieved at $kT \cong \Delta$.

Experimentally, the temperature dependences of the maximum conductivities, $\sigma_{xx}^{\text{peak}}$, both for the 1^- peak (Fig. 5a) and for the 1^+ peak (Fig. 5b) at temperatures $T = (1.0–4.2)$ K may be described by the expression (see Eq. (5)):

$$\frac{h}{e^2}\sigma_{xx}^{\text{peak}} = a + \frac{T_2}{T}, \tag{7}$$

with $a = 0.44 \pm 0, 01$, $T_2 = (0.06 \pm 0.02)$ K and $a = 0.48 \pm 0.005$, $T_2 = 0.04 \pm 0.005$, respectively. Thus, it obviously is the region of thermal broadening of the peak width produced by the Fermi-Dirac distribution function (see inset Fig. 5a).

As it may be seen from the Figs. 5a, 5b the transition to the scaling regime of Eq. (1), in which the amplitude of the peaks 1^- and 1^+ for IR-illuminated DQW sample begins to decrease with decreasing temperature, occurs at $T \cong 1$ K. As $T \to 0$ the temperature dependences of the maximum conductivities, $\sigma_{xx}^{\text{peak}}$, at $T = (0.05–0.5)$ K are well described by Eq. (2) with $\sigma_{xx}^c = (0.49 \pm 0.05)e^2/h$, $T_1^{\mu_\sigma} = (27.8 \pm 0.5)$ K for the

1^- peak (Fig. 5a) and with $\sigma_{xx}^c = (0.5 \pm 0.05)e^2/h$, $T_1^{\mu_\sigma} = (188.2 \pm 0.5)$ K for the 1^+ peak (Fig. 5b) and $\mu_\sigma = 0.48 \pm 0.02$ for 1^+ peak and $\mu_\sigma = 0.42 \pm 0.02$ for 1^- peak (see inset on Fig. 5b).

4. DISCUSSION

Let us compare the results obtained by us with the data of theoretical and experimental estimations by the other authors. Chalker and Eastmond pioneered the theoretical analysis in terms of irrelevant scaling fields in the context of the QHE (see Sec. VIII.C of [9]). They obtained the irrelevant scaling index $y_\sigma = -0.38 \pm 0.02$, in agreement with the results of the subsequent work by Huckestein [29]. In [29] finite-size corrections to scaling laws in the centers of Landau levels are studied systematically by numerical calculations. At the center of the lowest and the second lowest Landau level the irrelevant scaling index was found to be $y_\sigma = -0.38 \pm 0.04$, i.e. for $1 < p < 2$ should be $\mu_\sigma \cong (0.17–0.42)$.

In a recent paper Slevin and Ohtsuki [30] reported an estimate of the critical exponents at the quantum Hall transition and, in particular, tabulate some previous estimates of the irrelevant exponent. It could be seen that the estimate of Huckestein [29] is not consistent with the subsequent estimates by Wang et al. [31] ($y_\sigma \approx -0.52$ or $y_\sigma \approx -0.72$ for geometric or arithmetic average of the two-terminal conductance) and thus a more precise numerical estimate of the irrelevant exponent is highly desirable.

Pruisken and Burmistrov [26] have evaluated critical exponents at the Fermi liquid fixed point and the best estimate for the irrelevant one lie in the range $y_\sigma = -(0.34–0.42)$ in remarkable agreement with the exponent values known from numerical works [9, 29]. The other estimates of [26] for the Fermi liquid exponents are: $\mu_\sigma = -py_\sigma/2 = 0.26 \pm 0.05$ (for $p = 1.22–1.48$) and $\kappa = 0.29 \pm 0.04$.

In [32] experimental data on the plateau-insulator (PI) transition in the QHE regime for a low mobility InGaAs/InP heterostructure have been reported and both relevant, κ, and irrelevant (as a leading corrections to scaling), μ_σ exponents have been simultaneously extracted for the PI transition.

The experimental values $\kappa = 0.57 \pm 0.03$ and $\mu_\sigma = 2.5 \pm 0.5$ provided an accurate description of the transport data on the quantum critical phase at $T < 4$ K in [32], are in conflict with the theoreticalestimations in the Fermi liquid approximation [26].

Pruisken et al. [32] supposed that the new result obtained for the exponent κ (≈ 0.57) indicates that the quantum critical phenomenon belongs to a non-Fermi-liquid universality class on account of *long-range* Coulomb electron-electron (*e–e*) interaction. In [32] an idea also is expressed that the plateau-plateau and plateau-insulator transitions may correspond to the different universality classes of the quantum phase transitions in the QHE regime.

Note, that in [32] irrelevant exponent was determined from an analysis of the small deviations in the Hall resistivity from exact quantization at PI transition due to a leading corrections to scaling. While inverting the resistivity tensor into the conductivity tensor extended to include the corrections to scaling (see Eq. (17) of [32]) it is not difficult to show that the ways of defining index μ_σ by Pruisken et al. [32] and by us are equivalent.

However, the results obtained are dissimilar: $\kappa \approx 0.57$ and $\mu_\sigma \approx 2.5$ in [32] and $\kappa = (0.21–0.25)$ and $\mu_\sigma = (0.42–1.2)$ in our systems. On the other hand, the value $\kappa = 0.21$ is in accordance with the theoretical results that take into consideration the *short-range e–e* interaction [23, 24] (see a detailed discussion in our previous article [10]). Potentially, a different range of *e–e* interaction may be a reason of a difference in experimental values for the both critical exponents.

5. CONCLUSIONS

The magnetic field dependences of the longitudinal and Hall resistivity in quantum Hall regime for *n*-InGaAs/GaAs nanostructures with single and double quantum wells after IR-illumination have been studied over a wide range of magnetic fields $B \leq 16$ T and temperatures $T = (0.05–4.2)$ K. The data on the temperature dependences of the maximum (peak) values of dissipative conductivity, σ_{xx}, were analyzed using the assumptions of two-parameter scaling theory for the integer quantum Hall effect [9].

The value of the second (irrelevant) critical exponent, μ_σ, was estimated for the 0^- peak in a structure with a single quantum well and for the peaks 1^+ and 1^- in a structure with a double quantum well. In combination with the values of the first (relevant) critical exponent, κ, which we obtained previously [10, 11], the results are the following: $\kappa = 0.25 \pm 0.04$ (at $T < 2$ K); $\mu_\sigma \approx 1.2$ for peak in the SQW structure and $\kappa = 0.21 \pm 0.02$; $\mu_\sigma \approx 0.48$ for peak 1^+ and $\mu_\sigma \approx 0.42$ for 1^- in the DQW structure.

The experimentally obtained values of the irrelevant exponent for our structures turned out to be in a reasonably good (for SQW) and in a very good (for DQW) accordance with the currently available theoretical estimates, in contrast with the experimental results of Pruisken et al. for the PI transition in InGaAs/InP heterostracture [32].

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