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Dynamic Characteristics of Double-Barrier Nanostructures with Asymmetric Barriers of Finite Height and Widths in a Strong ac Electric Field

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Abstract—The theory of the interaction of a monoenergetic flow of injected electrons with a strong high-frequency ac electric field in resonant-tunneling diode (RTD) structures with asymmetric barriers of finite height and width is generalized. In the quasi-classical approximation, electron wavefunctions and tunneling functions in the quantum well and barriers are found. Analytical expressions for polarization currents in RTDs are derived in both the general case and in a number of limiting cases. It is shown that the polarization currents and radiation power in RTDs with asymmetric barriers strongly depend on the ratio of the probabilities of electron tunneling through the emitter and collector barriers. In the quantum mode, when $\delta = \varepsilon - \varepsilon_r = \hbar\omega \gg \Gamma$ (ε is the energy of electrons injected in the RTD, $\hbar$ is Planck's constant, ω is the ac field frequency, ε_r and Γ are the energy and width of the resonance level, respectively), the active polarization current in a field of $E \approx 2.8\hbar\omega/ea$ (e is the electron charge and a is the quantum-well width) reaches a maximum equal in magnitude to 84% of the direct resonant current, if the probability of electron tunneling through the emitter barrier is much higher than that through the collector barrier. The radiation-generation power at frequencies of $\omega = 10^{12}$–10^{13} s^{-1} can reach 10^5–10^6 W/cm^2 in this case.

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1. INTRODUCTION

The development of the physical principles of amplification, generation, and detection of terahertz electromagnetic radiation is one of the urgent problems determining the possibility of the advancement of solid-state electronics toward ultrahigh performance. Developments in solid-state devices based on resonant tunneling effects, characterized by extremely low inertia of the internal electronic processes, seem most promising in this direction. Based on resonant-tunneling diodes (RTDs), the highest results in terms of performance in solid-state electronics are achieved [1, 2].

A large number of works [3–10] are devoted to the theoretical study of the interaction of electrons with an ac electric field in RTD structures. We have indicated only the most recent works in this field, directly related to the subject of the present study. We will not dwell on previous studies in this field, which are listed and analyzed in [3–10].

In [6–10], the theory of the interaction of electrons with an ac electric field in RTD structures with δ-functional barriers (infinitely large barrier height V and infinitesimal barrier width b while retaining the constant product $Vb = \alpha$) was constructed. In [6–10], it was shown that the amplification and generation of an ac electric field in a wide frequency range, including terahertz frequencies, are possible in such structures.

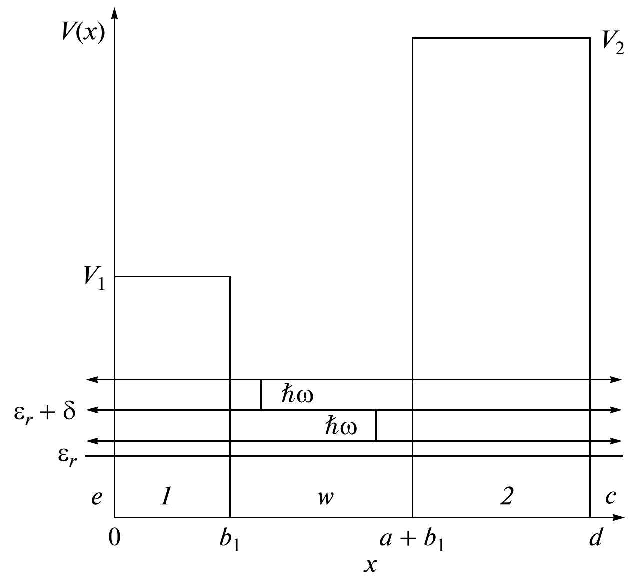
The objective of this study is to construct an analytical theory of the interaction of electrons with a strong ac electric field in RTD structures with asymmetric barriers of finite height and width.

2. SOLUTION OF THE SCHRÖDINGER EQUATION FOR ELECTRONS IN RTD STRUCTURES WITH ASYMMETRIC BARRIERS OF FINITE HEIGHT AND WIDTH IN A STRONG ac ELECTRIC FIELD

Let us consider a one-dimensional RTD structure with two asymmetric barriers of finite height and width (see the figure). An electron flux proportional to q^2, with energy ε differing slightly from the energy of the resonance level ε_r ($\varepsilon - \varepsilon_r = \delta \ll \varepsilon_r$) is directed to the diode from the left ($x < 0$). The RTD region is under an ac electric field $E(t)$ with the frequency ω and potential

$$U(x, t) = U(x)\cos\omega t,$$

$$U(x) = \begin{cases} 0, & x < 0 \\ -eEx, & 0 < x < d \\ -eEd, & x > d. \end{cases} \quad (1)$$



Double-barrier resonant-tunneling structure with the resonance level ε_r.

The electron wavefunction satisfies the Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m(x)}\frac{\partial^2\Psi}{\partial x^2} + V(x)\Psi + U(x,t)\Psi, \quad (2)$$

where (see the figure)

$$m(x) = m(x<0,\, b_1<x<a+b_1,\, x>d),$$
$$m(x) = m_b(0<x<b_1,\, a+b_1<x<d). \quad (3)$$

We will solve Eq. (2) in the quasi-classical approximation in a relatively strong electric field. The conditions of the applicability of such an approximation are defined by the inequalities

$$\varepsilon_r \gg \Gamma, \hbar\omega; \quad \varepsilon_r \gg eEa, \quad (4)$$

where ε_r and Γ are the energy and half-width of the resonance level. The behavior of the potential $V(x)$ is shown in the figure.

We present the solutions to Eq. (2) in the emitter ($x<0$), in the first barrier ($0<x<b_1$), in the quantum well ($b_1<x<a+b_1$), in the second barrier ($a+b_1<x<d$), and in the collector ($x>d$), respectively, in the form [9]

$$\Psi_e(x,t) = q\exp(-i\omega_0 t + ikx) + \exp[-i\omega_0 t - ikx + iS_e(x,t)], \quad (5)$$

$$\Psi_1(x,t) = \exp[-i\omega_0 t - k_1 x + iS_1(v_1,x,t)] + \exp[-i\omega_0 t + k_1 x + iS_1(-v_1,x,t)], \quad (6)$$

$$\Psi_W(x,t) = \exp[-i\omega_0 t + ik(x-b_1) + iS_W(v,x,t)] + \exp[-i\omega_0 t - ik(x-b_1) + iS_W(-v,x,t)], \quad (7)$$

$$\Psi_2(x,t) = \exp[-i\omega_0 t - k_2(x-a-b_1) + iS_2(v_2,x,t)] + \exp[-i\omega_0 t + k_2(x-a-b_1) + iS_2(-v_2,x,t)], \quad (8)$$

$$\Psi_c(x,t) = \exp[-i\omega_0 t + ik(x-d) + iS_c(x,t)], \quad (9)$$

where

$$\omega_0 = \varepsilon/\hbar, \quad k = \sqrt{2m\varepsilon}/\hbar, \quad k_n = \sqrt{2m_b(V_n-\varepsilon)}/\hbar,$$
$$v = \hbar k/m, \quad v_n = i\hbar k_n/m_b, \quad (n=1,2). \quad (10)$$

Substituting functions (5)–(9) into Eq. (2) for the x ranges shown in the figure, we obtain the equations for the functions $S_e(x,t)$, $S_c(x,t)$, $S_n(\pm v_n,x,t)$, $S_w(\pm v,x,t)$,

$$\frac{\partial S_e}{\partial t} - v\frac{\partial S_e}{\partial x} = 0, \quad \frac{\partial S_c}{\partial t} + v\frac{\partial S_c}{\partial x} = 0, \quad (11)$$

$$\frac{\partial S_n(\pm v_n,x,t)}{\partial t} \pm v_n\frac{\partial S_n(\pm v_n,x,t)}{\partial x} = \frac{eEx}{\hbar}\cos\omega t, \quad (12)$$

$$\frac{\partial S_w(\pm v,x,t)}{\partial t} \pm v\frac{\partial S_w(\pm v,x,t)}{\partial x} = \frac{eEx}{\hbar}\cos\omega t. \quad (13)$$

Equations (11)–(13) are only valid if, after substituting wavefunctions (5)–(9) into the Schrödinger equation (2), the terms containing the second derivatives (quasi-classical approximation) and the squared first derivatives (provided that the second inequality in (4) is satisfied) of the functions $S_e(x,t)$, $S_c(x,t)$, $S_n(\pm v_n,x,t)$, $S_w(\pm v,x,t)$ with respect to the coordinate x can be neglected in them. The solutions to Eqs. (11) are the arbitrary functions

$$S_e\left(-\frac{x}{v}-t\right), \quad S_c\left(\frac{x-d}{v}-t\right) \quad (14)$$

of the parenthetical arguments.

The solutions to Eqs. (12) and (13) can be presented in the form [9]

$$S_W(\pm v,x,t) = S_{UW}(\pm v,x,t) + S_W\left(\pm\frac{x-b_1}{v}-t\right), \quad (15)$$

$$S_1(\pm v_1,x,t) = S_{U1}(\pm v_1,x,t) + S_1\left(\pm\frac{x}{v_1}-t\right), \quad (16)$$

$$S_2(\pm v_2,x,t) = S_{U2}(\pm v_2,x,t) + S_2\left(\pm\frac{x-a-b_1}{v_2}-t\right), \quad (17)$$

where

$$S_{UW}(\pm v, x, t) = \mp\int_{b_1}^{x} dx' \frac{U(x')}{\hbar v}\cos\left[\omega\left(\pm\frac{x-x'}{v}-t\right)\right] \approx \mp S_{UW}(v, x)\cos\omega t; \quad (18)$$

$$S_{UW}(v, x) = \int_{b_1}^{x} dx' \frac{U(x')}{\hbar v},$$

$$iS_{U1}(\pm v_1, x, t) = \mp\int_{0}^{x} dx' \frac{U(x')}{\hbar|v_1|}\cos\left[\omega\left(\pm\frac{x-x'}{v_1}-t\right)\right] \approx \mp S_{U1}(|v_1|, x)\cos\omega t; \quad (19)$$

$$S_{U1}(|v_1|, x) = \int_{0}^{x} dx' \frac{U(x')}{\hbar|v_1|},$$

$$iS_{U2}(\pm v_2, x, t) = \mp\int_{a+b_1}^{x} dx' \frac{U(x')}{\hbar|v_2|}\cos\left[\omega\left(\pm\frac{x-x'}{v_2}-t\right)\right] \approx \mp S_{U2}(|v_2|, x)\cos\omega t; \quad (20)$$

$$S_{U2}(|v_1|, x) = \int_{a+b_1}^{x} dx' \frac{U(x')}{\hbar|v_2|}$$

are particular solutions to the inhomogeneous equations (12) and (13), and the functions $S_W\left(\pm\frac{x-b_1}{v}-t\right)$, $S_1\left(\pm\frac{x}{v_1}-t\right)$, and $S_2\left(\pm\frac{x-a-b_1}{v_2}-t\right)$ are general solutions (arbitrary functions of the parenthetical arguments) of the homogeneous equations (12) and (13) (without the right part).

Taking into account Eqs. (14)–(20), wavefunctions (5)–(9) are written in the form

$$\Psi_e(x, t) = q\exp(-i\omega_0 t + ikx) + \exp(-i\omega_0 t - ikx)f_e\left(-\frac{x}{v}-t\right), \quad (21)$$

$$\Psi_1(x, t) = \exp[-i\omega_0 t - k_1 x + iS_{U1}(v_1, x, t)]f_{k_1}\left(\frac{x}{v_1}-t\right) + \exp[-i\omega_0 t + k_1 x + iS_{U1}(-v_1, x, t)]f_{-k_1}\left(-\frac{x}{v_1}-t\right), \quad (22)$$

$$\Psi_W(x, t) = \exp[-i\omega_0 t + ik(x-b_1) + iS_{UW}(v, x, t)] \times f_k\left(\frac{x-b_1}{v}-t\right) + \exp[-i\omega_0 t - ik(x-b_1) + iS_{UW}(-v, x, t)]f_{-k}\left(-\frac{x-b_1}{v}-t\right), \quad (23)$$

$$\Psi_2(x, t) = \exp[-i\omega_0 t - k_2(x-a-b_1) + iS_{U2}(v_2, x, t)] \times f_{k_2}\left(\frac{x-a-b_1}{v_2}-t\right) + \exp[-i\omega_0 t + k_2(x-a-b_1) + iS_{U2}(-v_2, x, t)]f_{-k_2}\left(-\frac{x-a-b_1}{v_2}-t\right), \quad (24)$$

$$\Psi_c(x, t) = \exp[-i\omega_0 t + ik(x-d)]F\left(\frac{x-d}{v}-t\right), \quad (25)$$

where $S_{UW}(\pm v, x, t)$ and $S_{Un}(\pm v_n, x, t)$ are defined by expressions (18)–(20), and $f_e\left(-\frac{x}{v}-t\right)$, $f_{\pm k_1}\left(\pm\frac{x}{v_1}-t\right)$, $f_{\pm k}\left(\pm\frac{x-b_1}{v}-t\right)$, $f_{\pm k_2}\left(\pm\frac{x-a-b_1}{v_2}-t\right)$, $F\left(\frac{x-d}{v}-t\right)$ are arbitrary tunneling functions whose interrelation is determined from boundary conditions. Using Eqs. (14)–(20), it is easy to verify that Eqs. (11)–(13), their solutions (14)–(20), hence, the form of wavefunctions (21)–(25), are valid upon satisfaction of the inequalities $2ka$, $2k_n b_n \gg 1$, and inequalities (4).

Joining the wavefunctions $\Psi(x, t)$ (see (21)–(25)) and the products $\Psi'(x, t)/m(x)$ at the barrier boundaries, beginning from the boundary of the right (collector) barrier and finishing at the left boundary of the left (emitter) barrier (as a result, electron fluxes at the barrier boundaries are joined), we obtain the equation for the single tunneling function $F\left(\pm\frac{a}{v}\pm\frac{b_1}{v_1}\pm\frac{b_2}{v_2}\right)$ depending on eight biased arguments:

$$\left\{\left[2+i\left(\frac{1}{\xi_1}-\xi_1\right)\right]\left[2+i\left(\frac{1}{\xi_2}-\xi_2\right)\right]\exp[k_1b_1+k_2b_2 + (S_1+S_2)\cos\omega t]F\left(-\frac{a}{v}-\frac{b_1}{v_1}-\frac{b_2}{v_2}-t\right)\right.$$

$$+\left[2+i\left(\frac{1}{\xi_1}-\xi_1\right)\right]\left[2-i\left(\frac{1}{\xi_2}-\xi_2\right)\right]\exp[k_1b_1-k_2b_2 + (S_1-S_2)\cos\omega t]F\left(-\frac{a}{v}-\frac{b_1}{v_1}+\frac{b_2}{v_2}-t\right)$$

$$+\left[2-i\left(\frac{1}{\xi_1}-\xi_1\right)\right]\left[2-i\left(\frac{1}{\xi_2}-\xi_2\right)\right] \times \exp[-k_1b_1-k_2b_2-(S_1+S_2)\cos\omega t] \times F\left(-\frac{a}{v}+\frac{b_1}{v_1}+\frac{b_2}{v_2}-t\right)+\left[2-i\left(\frac{1}{\xi_1}-\xi_1\right)\right]$$

$$\times\left[2+i\left(\frac{1}{\xi_2}-\xi_2\right)\right]\exp[k_2b_2-k_1b_1$$

$$+ (S_2 - S_1)\cos\omega t]F\left(-\frac{a}{v} + \frac{b_1}{v_1} - \frac{b_2}{v_2} - t\right)\Bigg\} \quad (26)$$

$$\times \exp(-ika + iS_W\cos\omega t) + \Bigg\{\exp[k_1b_1 + k_2b_2$$

$$+ (S_1 + S_2)\cos\omega t]F\left(\frac{a}{v} - \frac{b_1}{v_1} - \frac{b_2}{v_2} - t\right)$$

$$- \exp[k_1b_1 - k_2b_2 + (S_1 - S_2)\cos\omega t]$$

$$\times F\left(\frac{a}{v} - \frac{b_1}{v_1} + \frac{b_2}{v_2} - t\right)$$

$$- \exp[k_2b_2 - k_1b_1 + (S_2 - S_1)\cos\omega t]$$

$$\times F\left(\frac{a}{v} + \frac{b_1}{v_1} - \frac{b_2}{v_2} - t\right)$$

$$+ \exp[-k_1b_1 - k_2b_2 - (S_1 + S_2)\cos\omega t]$$

$$\times F\left(\frac{a}{v} + \frac{b_1}{v_1} + \frac{b_2}{v_2} - t\right)\Bigg\}\left(\xi_1 + \frac{1}{\xi_1}\right)\left(\xi_2 + \frac{1}{\xi_2}\right)$$

$$\times \exp(ika - iS_W\cos\omega t) = 16q,$$

where (see (18)–(20))

$$\xi_n = \frac{m_b}{m}\frac{k}{k_n} = \left(\frac{m_b}{m}\frac{\varepsilon}{V_n - \varepsilon}\right)^{1/2}, \quad (27)$$

$$S_W = S_{UW}(v, a + b_1) = \frac{1}{2}\frac{U(a)}{\hbar\omega}\omega\frac{a}{v}\left(1 + 2\frac{b_1}{a}\right),$$

$$S_1 = S_{U1}(|v_1|, b_1) = \frac{1}{2}\frac{U(b_1)}{\hbar\omega}\omega\frac{b_1}{|v_1|}, \quad (28)$$

$$S_2 = S_{U2}(|v_2|, d) = \frac{U(d)}{\hbar\omega}\frac{\omega b_2}{|v_2|}\left(1 - \frac{b_2}{2d}\right).$$

Relations (28) were obtained from (18)–(20) under the conditions

$$\frac{\omega a}{v} \ll 1, \quad \frac{\omega b_n}{|v_n|} \ll 1, \quad (29)$$

which are satisfied in cases of practical interest right up to frequencies of $\omega \approx 10^{13}$ s^{-1}. Under additional conditions

$$\frac{a}{v} \gg \frac{b_n}{v_n}, \quad k_nb_n = \text{const}, \quad S_W \gg S_n, \quad (30)$$

equation (26) takes the form

$$GF(z) + Re^{2ika}A(t)F(z + 2T) = Y(t), \quad (31)$$

where $z = -a/v - t$, $T = a/v$,

$$\begin{aligned} G - &4\cosh(k_1b_1)\cosh(k_2b_2) \\ &-\left(\frac{1}{\xi_1} - \xi_1\right)\left(\frac{1}{\xi_2} - \xi_2\right)\sinh(k_1b_1)\sinh(k_2b_2) \\ &+ 2i\left(\frac{1}{\xi_1} - \xi_1\right)\sinh(k_1b_1)\cosh(k_2b_2) \\ &+ 2i\left(\frac{1}{\xi_2} - \xi_2\right)\cosh(k_1b_1)\sinh(k_2b_2), \end{aligned} \quad (32)$$

$$R = \left(\frac{1}{\xi_1} + \xi_1\right)\left(\frac{1}{\xi_2} + \xi_2\right)\sinh(k_1b_1)\sinh(k_2b_2), \quad (33)$$

$$\begin{aligned} A(t) &= \exp(-2iS_W\cos\omega t), \\ Y(t) &= 4q\exp(ika - iS_W\cos\omega t). \end{aligned} \quad (34)$$

When joining wavefunctions (21)–(25) and products $\Psi'(x, t)/m(x)$, a number of relations were also obtained, which will be used in calculating the RTD currents, i.e.

$$\begin{aligned} f_{\pm k_1}(-t) &= \frac{1}{2}\exp(\pm k_1b_1) \\ &\times\left[\left(1 \mp i\frac{k}{k_1}\right)f_p(-t) + \left(1 \pm i\frac{k}{k_1}\right)f_{-p}(-t)\right], \end{aligned} \quad (35)$$

$$f_e(-t) = \frac{1}{2}\left[\left(1 - i\frac{k_1}{k}\right)f_{k_1}(-t) + \left(1 + i\frac{k_1}{k}\right)f_{-k_1}(-t)\right], \quad (36)$$

$$f_{\pm k_2}(-t) = \frac{1}{2}\left(1 \mp i\frac{k}{k_2}\right)\exp(\pm k_2b_2)F(-t), \quad (37)$$

$$q = \frac{1}{2}\left[\left(1 + i\frac{k_1}{k}\right)f_{k_1}(-t) + \left(1 - i\frac{k_1}{k}\right)f_{-k_1}(-t)\right], \quad (38)$$

$$\begin{aligned} f_k(-t) &= \frac{1}{4}\exp(-ika + iS_W\cos\omega t)F\left(-\frac{a}{v} - t\right) \\ &\times\left\{\left[2 + i\left(\frac{1}{\xi_2} - \xi_2\right)\right]\exp(k_2b_2)\right. \\ &\left. + \left[2 - i\left(\frac{1}{\xi_2} - \xi_2\right)\right]\exp(-k_2b_2)\right\}, \end{aligned} \quad (39)$$

$$\begin{aligned} f_{-k}(-t) &= \frac{1}{4}\exp(ika - iS_W\cos\omega t)F\left(\frac{a}{v} - t\right) \\ &\times\left[i\left(\frac{1}{\xi_2} + \xi_2\right)\exp(-k_2b_2) - i\left(\frac{1}{\xi_2} + \xi_2\right)\exp(k_2b_2)\right]. \end{aligned} \quad (40)$$

Following the method used in [4, 5, 9], we present the function $F(z)$ in the form

$$F(z) = \sum_{n=-\infty}^{\infty} C_n \Phi_n(z), \quad \Phi_n(z) = \exp(-in\omega z)\Phi_0(z), \tag{41}$$

where $\Phi_0(z)$ satisfies the equation

$$\Phi_0(z) = A(z)\Phi_0(z+2T). \tag{42}$$

Substituting (41) into (31), then multiplying by $\omega/2\pi\exp(il\omega z)$, and integrating over z from 0 to $2\pi/\omega$, we find the expansion coefficients C_n

$$C_n = Y_n/\Delta_n, \quad \Delta_n = R + G\exp(-2ika + 2in\omega T), \quad Y_n = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} dz\frac{Y(z)}{\Phi_0(z)}\exp(-2ika + 2in\omega T + in\omega z). \tag{43}$$

Let us write the function $\Phi_0(z)$ in the form

$$\Phi_0(z) = \exp\left[i\sum_{m=-\infty}^{\infty} b_m \exp(im\omega z)\right]. \tag{44}$$

Substituting (44) into (42) and finding the coefficients b_m, we find

$$\Phi_0(z) = \exp\left[\sum_{n=-\infty}^{\infty}\exp(in\omega z)\frac{\omega}{2\pi} \times \int_0^{2\pi/\omega} dz'\frac{\exp(-in\omega z')\ln A(z')}{1-\exp(2in\omega T)}\right]. \tag{45}$$

From (34), we find (see (31))

$$\ln A(z') = -iS_W\{\exp[i\omega(z'+T)] + \exp[-i\omega(z'+T)]\}. \tag{46}$$

Substituting (46) into (45) and taking into account (28), (29), we obtain

$$\Phi_0(z) = \exp\left(i\frac{W}{\hbar\omega}\sin\omega z\right), \quad W = \frac{1}{2}U(a)\left(1+2\frac{b_1}{a}\right). \tag{47}$$

Substituting (34) and (47) into (43), we find Y_n with an accuracy up to the terms of the order of $(\omega T)^2$,

$$Y_n = 4qJ_n\left(\frac{W}{\hbar\omega}\right)\exp(-ika + 2in\omega T), \tag{48}$$

where $J_n\left(\frac{W}{\hbar\omega}\right)$ is the Bessel function.

Taking into account (41)–(48), we write the tunneling function $F(z)$ in the expanded form

$$F(z) = 4q\Phi_0(z)e^{-ika} \times \sum_{n=-\infty}^{\infty}\frac{J_n(W/\hbar\omega)}{\Delta_n}\exp(-in\omega z + 2in\omega T). \tag{49}$$

The tunneling function $F(z)$ has a sharp maximum under the condition that the imaginary part of the resonant determinants Δ_n is zero, i.e., provided that (see (32), (33), (43))

$$\tan(2k_r a - 2n\omega T_r) = \operatorname{Im}G(\varepsilon_r)/\operatorname{Re}G(\varepsilon_r) = L_r/D_r, \tag{50}$$

where

$$L_r = 2\left[\left(\frac{1}{\xi_1}-\xi_1\right)\coth k_2 b_2 + \left(\frac{1}{\xi_2}-\xi_2\right)\coth k_1 b_1\right]_{\varepsilon=\varepsilon_r}, \tag{51}$$

$$D_r = \left[4\coth k_1 b_1 \coth k_2 b_2 - \left(\frac{1}{\xi_1}-\xi_1\right)\left(\frac{1}{\xi_2}-\xi_2\right)\right]_{\varepsilon=\varepsilon_r}. \tag{52}$$

For symmetric barriers, instead of (50), we obtain

$$\tan k_r a = \frac{2\xi_r}{\xi_r^2-1}\coth k_{br} b, \quad \xi_r = \left(\frac{m_b}{m}\frac{\varepsilon_r}{V-\varepsilon_r}\right)^{1/2}, \quad k_{br} = \sqrt{2m_b(V-\varepsilon_r)}/\hbar. \tag{53}$$

From Eqs. (50) and (53) (in the region of terahertz frequencies $n\omega T_r/k_r a = n\hbar\omega/2\varepsilon_r \ll 1$ at $n = 10$–100), the resonance-level energies $\varepsilon_r = \hbar^2k_r^2/2m$ are determined. Expanding the quantities entering Δ_n in series (see Eqs. (32), (33), (43)) in $(\varepsilon - n\hbar\omega - \varepsilon_r)/\varepsilon_r \ll 1$, retaining the first nonzero expansion terms, we obtain

$$\frac{4}{\Delta_n} = 4\left\{\operatorname{Re}\Delta_n(\varepsilon_r) + i(\varepsilon - n\hbar\omega - \varepsilon_r)\left[\frac{d}{d\varepsilon}\operatorname{Im}\Delta_n\right]_{\varepsilon=\varepsilon_r}\right\}^{-1} = \frac{\Gamma_0}{i(\delta - n\hbar\omega) - \Gamma} = \Gamma_0\int_{-z}^{-\infty} dz'\exp\{(\Gamma - i\delta + in\hbar\omega)(z+z')\}, \tag{54}$$

where $\delta = \varepsilon - \varepsilon_r$,

$$\Gamma_0 = \frac{4}{\left\{\frac{d}{d\varepsilon}\operatorname{Im}\Delta_n\right\}_{\varepsilon=\varepsilon_r}} = \frac{4\Gamma}{\{|K_1||K_2| - |Z_1||Z_2|\}_{\varepsilon=\varepsilon_r}}, \tag{55}$$

$$K_n = 2\cosh k_n b_n + i\left(\xi_n - \frac{1}{\xi_n}\right)\sinh k_n b_n,$$
$$Z_n = i\left(\xi_n + \frac{1}{\xi_n}\right)\sinh k_n b_n, \quad (56)$$

$$\Gamma = -\frac{\mathrm{Re}\,\Delta_n(\varepsilon_r)}{\left(\frac{d}{d\varepsilon}\mathrm{Im}\,\Delta_n\right)_{\varepsilon=\varepsilon_r}} = \varepsilon_r \frac{\{|K_1||K_2| - |Z_1||Z_2|\}_{\varepsilon=\varepsilon_r}}{4R_{1r}R_{2r}\cosh k_{1r}b_1\cosh k_{2r}b_2}$$
$$\times\left\{k_r a + \frac{1}{4}\sum_{n=1,2} R_{nr}^{-2}\left[\frac{V_n}{V_n - \varepsilon_r}\left(\frac{1}{\xi_{nr}} + \xi_{nr}\right)\tanh k_{nr}b_n \right.\right. \quad (57)$$
$$\left.\left. + k_{nr}b_n\frac{\varepsilon_r}{V_n - \varepsilon_r}\left(\frac{1}{\xi_{nr}} - \xi_{nr}\right)(2 - \tanh^2 k_{nr}b_n)\right]\right\}^{-1},$$

$$R_{nr} = \left\{1 + \frac{1}{4}\left(\frac{1}{\xi_{nr}} - \xi_{nr}\right)^2\tanh^2 k_{nr}b_n\right\}^{1/2},$$
$$|K_n|^2 - |Z_n|^2 = 4. \quad (58)$$

Substituting (54) into (49) and using [11], we find $F(z)$ and $|F(z)|^2$ in the integral representation ($\tau_1 = z' - z$),

$$F(z) = -q\Gamma_0\Phi_0(z)e^{-ika}\int_0^\infty d\tau_1\exp[-(\Gamma - i\delta)\tau_1]$$
$$\times\exp\left[-i\frac{W}{\hbar\omega}\sin\omega(z+\tau_1)\right], \quad (59)$$
$$|F(z)|^2 = q^2\Gamma_0^2\int_0^\infty d\tau_1\exp\left[-(\Gamma - i\delta)\tau_1 - i\frac{W}{\hbar\omega}\sin\omega(z+\tau_1)\right]$$
$$\times\int_0^\infty d\tau_2\exp\left[-(\Gamma + i\delta)\tau_2 + i\frac{W}{\hbar\omega}\sin\omega(z+\tau_3)\right].$$

Formulas (59) are similar to those obtained in [5, 9]; however, in our RTD model with asymmetric rectangular barriers of finite height and width, which also considers the difference of electron effective masses in the quantum well and barriers, the quantities Γ, ε, δ and their dependences on the structural parameters are defined by other expressions (see (50)–(58)). In particular, the terms $\exp(-k_{nr}b_n)$ enter (50)–(58) (see also (78)–(80)), which define the probability of electron tunneling through barriers.

3. CALCULATION OF RTD CURRENTS IN A STRONG ac ELECTRIC FIELD

The electron current I through the wavefunction Ψ is given by the formula

$$I = -i\frac{e\hbar}{2m(x)}\{\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\}, \quad (60)$$

where i is the imaginary unit, e is the electron-charge magnitude, $\hbar$ is Planck's constant, and $m(x)$ is the electron mass.

Substituting wavefunctions (23), (21), (22), (24), (25) into (60) and considering relations (35)–(40), we obtain, respectively, the currents in the quantum well $I_W(x, t)$, in the emitter $I_e(t)$, in the first barrier $I_1(t)$, in the second barrier $I_2(t)$, and in the collector $I_c(t)$,

$$I_W(x,t) = ev\left\{|F(z)|^2 + \frac{1}{4}\left(\xi_2 + \frac{1}{\xi_2}\right)^2\right.$$
$$\left.\times\sinh^2 k_2b_2[|F(z)|^2 - |F(\tilde{z})|^2]\right\}, \quad (61)$$

$$z = \frac{x-a-b_1}{v} - t, \quad \tilde{z} = -\frac{x-a-b_1}{v} - t,$$

$$I_e(t) = I_1(t) = I_W(b_1, t), \quad (62)$$

$$I_2(t) = I_c(t) = I_W(a + b_1, t). \quad (63)$$

It follows from (61)–(63) that the problem is reduced to calculation of the quantum-well current $I_W(x, t)$ using formulas (59) which, following the calculations in [9], we write in the form

$$|F(z)|^2 = q^2\Gamma_0^2\int_0^\infty d\tau_1\exp[-(\Gamma - i\delta)\tau_1] \quad (64)$$
$$\times\int_0^\infty d\tau_2\exp[-(\Gamma + i\delta)\tau_2]\exp[iA\sin\omega z + iB\cos\omega z],$$

$$A = \frac{W}{\hbar\omega}[\cos\omega\tau_2 - \cos\omega\tau_1],$$
$$B = \frac{W}{\hbar\omega}[\sin\omega\tau_2 - \sin\omega\tau_1]. \quad (65)$$

The active current component proportional to $\cos\omega t$ is calculated using the Fourier transform

$$I_{CW}(x) = \frac{\omega}{\pi}\int_0^{2\pi/\omega} dt\cos\omega t\, I_W(x,t). \quad (66)$$

Substituting (61), (64), (65) into (66) and using the formula

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega} dt\exp(in\omega t)\exp(a^+e^{i\omega t}+a^-e^{-i\omega t})$$
$$=\frac{1}{2\pi}\int_0^{2\pi} d\xi(e^{i\xi})^n\exp[2(a^+a^-)^{1/2}\cos(\xi+\lambda)] \quad (67)$$
$$= i^n\left(\frac{a^-}{a^+}\right)^{n/2}J_n[-2i(a^+a^-)^{1/2}],$$

in which (see (65))

$$a^+ = \frac{1}{2}(iB-A), \quad a^- = \frac{1}{2}(iB+A),$$
$$\xi = \omega t, \quad e^{i\lambda} = \left(\frac{a^+}{a^-}\right)^{1/2}, \quad (68)$$

$J_n[-2i(a^+a^-)^{1/2}]$ is the Bessel function (n is the natural number), we find the expression for the active current (hereafter, we set $evq^2 = 1$),

$$I_{CW}(x) = 2i\Gamma_0^2\int_0^\infty d\tau_2\exp[-(\Gamma+i\delta)\tau_2]$$
$$\times\int_0^\infty d\tau_1\exp[-(\Gamma-i\delta)\tau_1]J_1\left(2\frac{W}{\hbar\omega}\sin\omega\frac{\tau_2-\tau_1}{2}\right) \quad (69)$$
$$\times\left\{\cos\omega\frac{\tau_2+\tau_1}{2}+\frac{a+b_1-x}{a}\frac{\omega}{\Gamma_2}\sin\omega\frac{\tau_2+\tau_1}{2}\right\},$$

$$\frac{1}{\Gamma_2} = \frac{1}{2}\frac{a}{v}\left(\xi_2+\frac{1}{\xi_2}\right)^2\sinh^2 k_2b_2. \quad (70)$$

Introducing the variable $t = \tau_2 - \tau_1$ into (69) and integrating over τ_1, we finally obtain

$$I_{CW}(x) = \frac{4\Gamma_0^2}{\omega^2+4\Gamma^2}\left\{2\Gamma S_1-\omega S_2\right.$$
$$\left.+\frac{a+b_1-x}{a}\frac{\omega}{\Gamma_2}(\omega S_1+2\Gamma S_2)\right\}, \quad (71)$$

where

$$S_1 = \sum_{n=0}^{\infty}\int_{n\pi/\omega}^{(n+1)\pi/\omega} dte^{-\Gamma t}J_1\left(\beta\sin\frac{\omega t}{2}\right)\sin\delta t\cos\frac{\omega t}{2}, \quad (72)$$

$$S_2 = \sum_{n=0}^{\infty}\int_{n\pi/\omega}^{(n+1)\pi/\omega} dte^{-\Gamma t}J_1\left(\beta\sin\frac{\omega t}{2}\right)\sin\delta t\sin\frac{\omega t}{2}, \quad (73)$$

$$\beta = -\frac{eEa}{\hbar\omega}\left(1+2\frac{b_1}{a}\right), \quad e>0. \quad (74)$$

The expression for the reduced current takes the form

$$I_{CW} = \frac{4\Gamma_0^2}{\omega^2+4\Gamma^2}$$
$$\times\left\{2\Gamma S_1-\omega S_2+\frac{1}{2}\frac{\omega}{\Gamma_2}(\omega S_1+2\Gamma S_2)\right\}. \quad (75)$$

Using the above method, we also find the reactive current $I_{SW}(x)$ and the direct current $I_0(\beta)$ in a strong ac field,

$$I_{SW}(x) = \frac{4\Gamma_0^2}{\omega^2+4\Gamma^2}\left\{\omega S_1+2\Gamma S_2\right.$$
$$\left.-\frac{a+b_1-x}{a}\frac{\omega}{\Gamma_2}(2\Gamma S_1-\omega S_2)\right\}, \quad (76)$$

$$I_0 = \frac{\Gamma_0^2}{\Gamma}\sum_{n=0}^{\infty}\int_{n\pi/\omega}^{(n+1)\pi/\omega} dte^{-\Gamma t}J_0\left(\beta\sin\frac{\omega t}{2}\right)\cos\delta t. \quad (77)$$

Let us consider some limiting cases.

3.1. Resonant Tunneling in a Strong ac Electric Field

At $\omega = \delta \gg \Gamma$ (quantum mode); $\xi_{nr} \ll 1$, $k_{nr}b_n \gg 1$ (extremely strong barriers), from (55)–(58), (70), (72), (73), using [12], we obtain (see (10) and (27)),

$$\Gamma = \frac{8\varepsilon_r}{k_ra}[\xi_{1r}^2\exp(-2k_{1r}b_1)+\xi_{2r}^2\exp(-2k_{2r}b_2)], \quad (78)$$

$$\Gamma/\Gamma_0 = \frac{1}{2}\left[\frac{\xi_{1r}}{\xi_{2r}}\exp(k_{2r}b_2-k_{1r}b_1)\right.$$
$$\left.+\frac{\xi_{2r}}{\xi_{1r}}\exp(k_{1r}b_1-k_{2r}b_2)\right], \quad (79)$$

$$\Gamma/\Gamma_2 = \frac{1}{2}\left\{1+\left(\frac{\xi_{1r}}{\xi_{2r}}\right)^2\exp[2(k_{2r}b_2-k_{1r}b_1)]\right\}, \quad (80)$$

$$S_1 = \frac{2}{\beta\Gamma}J_1^2\left(\frac{\beta}{2}\right), \quad S_2 = \frac{4}{\pi\beta\Gamma}J_2(\beta). \quad (81)$$

Having substituted (81) into (75), we find the reduced current in the quantum well,

$$I_{CW} = \frac{(\Gamma_0/\Gamma)^2}{\omega^2+4\Gamma^2}\left(4\Gamma^2+\omega^2\frac{\Gamma}{\Gamma_2}\right)\frac{4}{\beta}J_1^2\left(\frac{\beta}{2}\right). \quad (82)$$

For the symmetric barriers ($k_{1r}b_1 = k_{2r}b_2$),

$$I_{CW} = \frac{4}{\beta}J_1^2\left(\frac{\beta}{2}\right). \quad (83)$$

Formula (83) is identical to formula (113) in [9]. At $k_{2r}b_2 \gg k_{1r}b_1$ (the collector barrier is stronger than the emitter barrier; see (78)–(80)),

$$I_{CW} = \frac{8}{\beta}J_1^2\left(\frac{\beta}{2}\right). \quad (84)$$

At $k_{1r}b_1 \gg k_{2r}b_2$ (the emitter barrier is stronger than the collector barrier),

$$I_{CW} = \frac{8}{\beta}J_1^2\left(\frac{b}{2}\right)Z_{12};$$
$$Z_{12} = \frac{V_2 - \varepsilon_r}{V_1 - \varepsilon_r}\exp[-2(k_{1r}b_1 - k_{2r}b_2)]. \quad (85)$$

It follows from (83)–(85) that the reduced current in the case of a stronger collector barrier is higher than the reduced current in the case of a stronger emitter barrier by one–two orders of magnitude (as estimations show) and is two times higher than the reduced current in the case of strong symmetric barriers.

In all three cases, the current decreases at $\beta \longrightarrow 0$; as β increases, the current runs over a number of maxima with decreasing height. The maxima are separated by minima in which the current vanishes. The optimum ac field amplitude at which the first current maximum is reached is determined from the equation

$$\frac{dI_{CW}(z_0)}{dz_0} = 0, \quad J_1(z_0) = 2z_0J_2(z_0), \quad z_0 = \frac{\beta}{2}. \quad (86)$$

From (83) and (86), we obtain $z_0 = 1.36$ and a maximum current

$$I_{CW}(z_0) = \frac{4J_1^2(z_0)}{z_0} = -0.84 \quad (87)$$

almost equal (in magnitude) to the constant resonant current in the case of symmetric potential barriers, i.e. (see (77)),

$$I_0(\beta = 0, \delta = 0, \Gamma_0 = \Gamma) = 1. \quad (88)$$

As shown below (see (107)), the current $I_{CW}(z_0)$ significantly exceeds the reduced current in the classical mode, the collector current, and somewhat exceeds the reduced low-frequency current. Hence, the interference of electrons in the RTD causes significant amplification and considerable electromagnetic-radiation generation powers in the terahertz frequency range $\omega \gg \Gamma$.

Substituting (78)–(81) into (76) and (77), we find the reduced reactive current I_{SW} and direct current $I_0(\beta)$,

$$I_{SW} = \frac{8\Gamma_0^2}{\beta\Gamma(\omega^2 + 4\Gamma^2)}\left\{\omega\left(1 - \frac{\Gamma}{\Gamma_2}\right)J_1^2\left(\frac{\beta}{2}\right) + \frac{4\Gamma^2 + \omega^2\Gamma/\Gamma_2}{\pi\Gamma}J_2(\beta)\right\}, \quad (89)$$

$$I_0(\beta) = \frac{\Gamma_0^2}{\Gamma^2}J_1^2\left(\frac{\beta}{2}\right). \quad (90)$$

At symmetric barriers (see (78)–(80)),

$$I_{SW} = \frac{8}{\pi}\frac{J_2(\beta)}{\beta}, \quad I_0(\beta) = J_1^2\left(\frac{\beta}{2}\right). \quad (91)$$

At $k_{2r}b_2 \gg k_{1r}b_1$,

$$I_{SW} = \frac{16}{\pi}\frac{J_2(\beta)}{\beta}, \quad I_0(\beta) = 4Z_{21}J_1^2\left(\frac{\beta}{2}\right),$$
$$Z_{21} = \frac{V_1 - \varepsilon_r}{V_2 - \varepsilon_r}\exp[-2(k_{2r}b_2 - k_{1r}b_1)]. \quad (92)$$

At $k_{1r}b_1 \ll k_{2r}b_2$ (see (85)),

$$I_{SW} = \frac{16}{\pi}Z_{12}\frac{J_2(\beta)}{\beta}, \quad I_0(\beta) = 4Z_{12}J_1^2\left(\frac{\beta}{2}\right). \quad (93)$$

Finally, let us calculate the dependence of the emitter currents I_{ce}, I_{se} and collector currents I_{cc}, I_{sc} on the field amplitude E and frequency ω. At $\omega = \delta \gg \Gamma$, it follows from (62), (63), (76), (78)–(81) that

$$I_{ce} = I_{cw}(b_1) = \frac{\Gamma}{\Gamma_2}\left(\frac{\Gamma_0}{\Gamma}\right)^2\frac{8}{\beta}J_1^2\left(\frac{\beta}{2}\right), \quad (94)$$

$$I_{se} = \frac{16}{\pi}\frac{\Gamma}{\Gamma_2}\left(\frac{\Gamma_0}{\Gamma}\right)^2\frac{J_2(\beta)}{\beta} = I_{sw}(b_1), \quad (95)$$

$$I_{cc} = I_{cw}(a + b_1) = 16\left(\frac{\Gamma_0}{\omega}\right)^2 \times \left\{\frac{J_1^2\left(\frac{\beta}{2}\right)}{\beta} - \int_0^1 x^2dx\left(1 - \frac{2}{\pi}\arcsin x\right)J_1(\beta x)\right\}, \quad (96)$$

$$I_{sc} = 8\left(\frac{\Gamma_0}{\Gamma}\right)^2\frac{\Gamma}{\omega}\frac{J_1^2(\beta/2)}{\beta}. \quad (97)$$

At $\beta \ll 1$ (weak field), the emitter and collector currents tend to zero; at $\beta \gg 1$ (strong field), the currents decrease in proportion to W^2 (see (47)); e.g.,

$$I_{cc} = -\frac{1}{\pi}\left(\frac{4\Gamma_0}{W}\right)^2. \quad (98)$$

The frequency dependence of the emitter and collector current maxima ($\beta \approx 2.8$) is described by the following relations

$$I_{ce} = \text{const}, \quad I_{se} = \text{const}, \\ I_{cc} \propto \left(\frac{\Gamma}{\omega}\right)^2, \quad I_{sc} \propto \frac{\Gamma}{\omega}. \quad (99)$$

3.2. *Non-Resonant Classical Mode* ($\omega \gg \Gamma, \delta; \delta \approx \Gamma$)

In this case (see (72)–(75)), the reduced active current in the quantum well is given by

$$I_{CW} = -\frac{2i\Gamma_0^2}{\omega^2 + 4\Gamma^2}\left\{\left(2\Gamma + \frac{\omega^2}{2\Gamma_2}\right)(S_3 - S_3^*) + \omega\left(\frac{\Gamma}{\Gamma_2} - 1\right)(S_4 - S_4^*)\right\}, \quad (100)$$

$$S_3 = \sum_{n=0}^{\infty} \int_{n\pi/\omega}^{(n+1)\pi/\omega} dt e^{-(\Gamma - i\delta)t}\cos\frac{\omega t}{2} J_1\left(\beta\sin\frac{\omega t}{2}\right), \quad (101)$$

$$S_4 = \sum_{n=0}^{\infty} \int_{n\pi/\omega}^{(n+1)\pi/\omega} dt e^{-(\Gamma - i\delta)t}\sin\frac{\omega t}{2} J_1\left(\beta\sin\frac{\omega t}{2}\right). \quad (102)$$

Summation of series (101) and (102) under the condition $\omega \gg \Gamma \approx \delta$, $\pi(\Gamma - i\delta)/\omega \ll 1$ results in

$$S_3 - S_3^* = \frac{4}{3}i\frac{\pi^2\Gamma\delta}{\omega^3}\int_0^1 dx J_1(\beta x)\varphi(x)[1 - \varphi^2(x)], \quad (103)$$

$$S_4 - S_4^* = \frac{4}{3}i\frac{\pi^2\Gamma\delta}{\omega^3}\int_0^1 dx x(1 - x^2)^{-1/2} \times J_1(\beta x)\varphi(x)[1 - \varphi^2(x)], \quad (104)$$

$$\varphi(x) = 1 - \frac{2}{\pi}\arcsin x. \quad (105)$$

With sufficient accuracy [9], we can write

$$\varphi(x)[1 - \varphi^2(x)] \approx x(1 - x^2)^{1/2}. \quad (106)$$

Substituting integrals (103) and (104) calculated with such accuracy (see [12]) into (100), we obtain ($\omega \gg \Gamma \approx \delta$)

$$I_{CW} = \frac{2}{3}\frac{\pi^3\delta\Gamma^2}{\omega^3}\frac{\Gamma}{\Gamma_2}\left(\frac{\Gamma_0}{\Gamma}\right)^2\frac{J_1^2\left(\frac{\beta}{2}\right)}{\beta}. \quad (107)$$

Thus, the reduced current in the classical mode has a small factor $(\Gamma/\omega)^3$ in comparison with the current in the quantum mode. At symmetric barriers, the factor $\Gamma/\Gamma_2(\Gamma_0/\Gamma)^2 = 1$ (see (79), (80)), and formula (107) is identical to formula (119) in [9]; at $k_{2r}b_2 \gg k_{1r}b_1$, the factor $\Gamma/\Gamma_2(\Gamma_0/\Gamma)^2 = 2$; at $k_{1r}b_1 \gg k_{2r}b_2$, the additional small factor

$$\frac{\Gamma}{\Gamma_2}\left(\frac{\Gamma_0}{\Gamma}\right)^2 = 2\frac{V_2 - \varepsilon_r}{V_1 - \varepsilon_r}\exp[-2(k_{1r}b_1 - k_{2r}b_2)] \ll 1 \quad (108)$$

appears in (107). The emitter and collector currents in the nonresonant classical mode are

$$I_{ce} = \frac{\pi^3\Gamma^2\delta}{3\omega^3}\frac{\Gamma_0^2}{\Gamma\Gamma_2}\frac{4}{\beta}J_1^2\left(\frac{\beta}{2}\right), \quad (109)$$

$$I_{cc} = \frac{8}{3}\frac{\pi^3\Gamma^2\delta}{\omega^3}\frac{\Gamma_0^2}{\beta\omega^2}\left\{J_1^2\left(\frac{\beta}{2}\right) - \frac{\omega}{\pi\Gamma}J_2(\beta)\right\}. \quad (110)$$

As follows from (110), the active collector current changes sign at high frequencies. The reactive current in the classical mode can be calculated using formulas (76), (72), (73), (101)–(106).

3.3. *Calculation of Currents at the Low-Frequency Limit* ($\omega \ll \Gamma$)

In this case, as follows from (71)–(75),

$$S_2 = \frac{\omega}{2\Gamma}S_1 \ll S_1, \quad I_{cw} = 2\Gamma\left(\frac{\Gamma_0}{\Gamma}\right)^2 S_1, \quad (111)$$

where (see [12], p. 721)

$$S_1 = \int_0^{\infty} dt e^{-\Gamma t}J_1(Wt)\sin\delta t = \frac{g_0^{-1/4}}{\sqrt{2}W}\left\{\delta\left(1 + \frac{1}{\sqrt{1+\alpha^2}}\right)^{1/2} - \Gamma\left(1 - \frac{1}{\sqrt{1+\alpha^2}}\right)^{1/2}\right\}, \quad (112)$$

$$g_0 = (\Gamma^2 - \delta^2 + W^2)^2 + 4\Gamma^2\delta^2, \quad \alpha = \frac{2\Gamma\delta}{\Gamma^2 - \delta^2 + W^2}.$$

At $W \ll \Gamma$,

$$S_1 = \frac{\Gamma\delta W}{(\Gamma^2 + \delta^2)^2}, \quad I_{cw} = \frac{2\Gamma_0^2\delta W}{(\Gamma^2 + \delta^2)^2}. \quad (113)$$

At $W \gg \Gamma$,

$$S_1 = -\frac{\delta}{W^2}, \quad I_{cw} = -2\left(\frac{\Gamma_0}{\Gamma}\right)^2\frac{\Gamma\delta}{W^2}. \quad (114)$$

It follows from (111)–(114), (78), (79) that the current I_{cw} does not change, if the asymmetric emitter and collector potential barriers are interchanged. The maximum current I_{cw} at the low-frequency limit for the case of symmetric barriers is about one third of the direct resonant current; for the case of asymmetric barriers, it is a significantly smaller fraction of the direct resonant current.

3.4. Calculation of Currents in a Weak Electric Field ($eEa \ll \hbar\omega$, $|\beta| \ll 1$)

The active current in the quantum well is given by formula (71), in which in the case at hand

$$J_1\left(\beta\sin\frac{\omega t}{2}\right) \approx \frac{\beta}{2}\sin\frac{\omega t}{2}, \quad (115)$$

$$S_1 = \frac{\beta}{2}\frac{\Gamma\delta\omega}{[\Gamma^2 + (\delta-\omega)^2][\Gamma^2 + (\delta+\omega)^2]}, \quad (116)$$

$$S_2 = \frac{\beta\delta\omega^2}{4} \times \frac{3\Gamma^2 + \omega^2 - \delta^2}{(\Gamma^2+\delta^2)[\Gamma^2 + (\delta-\omega)^2][\Gamma^2 + (\delta+\omega)^2]}. \quad (117)$$

At symmetric barriers ($k_{2r}b_2 = k_{1r}b_1$), the active currents in the quantum well, emitter, and collector are given by the expressions

$$I_{CW}(x) = I_{CW}\frac{\Gamma^2 + \delta^2 + \omega^2 - 2\omega^2 x/a}{\Gamma^2 + \delta^2}, \quad (118)$$

$$I_{ce} = I_{CW}\frac{\Gamma^2 + \delta^2 + \omega^2}{\Gamma^2 + \delta^2}, \quad (119)$$

$$I_{cc} = I_{CW}\frac{\Gamma^2 + \delta^2 - \omega^2}{\Gamma^2 + \delta^2}, \quad (120)$$

respectively, in which the reduced active current in the quantum well is given by

$$I_{CW} = -\frac{\Gamma^2\delta\left(1 + 2\frac{b_1}{a}\right)eaE}{[\Gamma^2 + (\delta-\omega)^2][\Gamma^2 + (\delta+\omega)^2]}. \quad (121)$$

At $k_{2r}b_2 \gg k_{1r}b_1$,

$$I_{CW}(x) = I_{CW}\frac{(\Gamma^2 + \delta^2 - \omega^2)\Gamma_0^2/\Gamma^2 + 4\omega^2(a + b_1 - x)/a}{\Gamma^2 + \delta^2}, \quad (122)$$

$$I_{ce} = I_{CW}\frac{(\Gamma^2 + \delta^2 - \omega^2)\Gamma_0^2/\Gamma^2 + 4\omega^2}{\Gamma^2 + \delta^2}, \quad (123)$$

$$I_{cc} = I_{CW}\frac{(\Gamma^2 + \delta^2 - \omega^2)\Gamma_0^2/\Gamma^2}{\Gamma^2 + \delta^2}. \quad (124)$$

At $k_{1r}b_1 \gg k_{2r}b_2$ (see (85)),

$$I_{CW}(x) = 4I_{CW}\frac{\Gamma^2 + \delta^2 + \omega^2(b_1 - x)/a}{\Gamma^2 + \delta^2}Z_{12}, \quad (125)$$

$$I_{ce} = 4I_{CW}Z_{12}, \quad I_{cc} = 4I_{CW}\frac{\Gamma^2 + \delta^2 - \omega^2}{\Gamma^2 + \delta^2}Z_{12}. \quad (126)$$

The reactive current in the quantum well is given by formula (76) in which S_1 and S_2 are defined by expressions (116) and (117).

For the case of symmetric barriers, reactive currents in the quantum well, emitter, and collector are defined by the expressions (see (121))

$$I_{SW}(x) = I_{CW}\frac{\omega}{\Gamma}\frac{2\Gamma^2 + (\omega^2 - \Gamma^2 - \delta^2)(a + b_1 - x)/a}{\Gamma^2 + \delta^2}, \quad (127)$$

$$I_{se} = I_{CW}\frac{\omega}{\Gamma}\frac{\omega^2 + \Gamma^2 - \delta^2}{\Gamma^2 + \delta^2}, \quad I_{sc} = I_{CW}\frac{2\Gamma\omega}{\Gamma^2 + \delta^2}. \quad (128)$$

At $k_{2r}b_2 \gg k_{1r}b_1$ (see (92)),

$$I_{SW}(x) = I_{CW}\frac{2\omega}{\Gamma} \times \frac{4\Gamma^2 Z_{21} + (\omega^2 - \Gamma^2 - \delta^2)(a + b_1 - x)/a}{\Gamma^2 + \delta^2}, \quad (129)$$

$$I_{se} = I_{CW}\frac{2\omega}{\Gamma}\frac{4\Gamma^2 Z_{21} + (\omega^2 - \Gamma^2 - \delta^2)}{\Gamma^2 + \delta^2}, \quad (130)$$

$$I_{sc} = I_{CW}\frac{8\Gamma\omega}{\Gamma^2 + \delta^2}Z_{21}. \quad (131)$$

At $k_{1r}b_1 \gg k_{2r}b_2$ (see (84)),

$$I_{SW}(x) = I_{CW}\frac{2\omega}{\Gamma}Z_{12} \times \frac{4\Gamma^2 + (\omega^2 - \Gamma^2 - \delta^2)(a + b_1 - x)/a}{\Gamma^2 + \delta^2}, \quad (132)$$

$$I_{se} = I_{CW}\frac{2\omega}{\Gamma}Z_{12}\frac{\omega^2 + 3\Gamma^2 - \delta^2}{\Gamma^2 + \delta^2}, \quad (133)$$

$$I_{sc} = I_{CW}\frac{8\Gamma\omega}{\Gamma^2 + \delta^2}Z_{12}. \quad (134)$$

4. CONCLUSIONS

The electron wavefunctions and tunneling function were determined in a resonant-tunneling diode with asymmetric barriers of finite height and width in a strong electromagnetic field by solving the Schrödinger equations with boundary conditions expressing the equality of the wavefunctions and electron fluxes at the barrier boundaries. Using the tunneling functions, analytical expressions were obtained for the active and reactive polarization currents in the resonant-tunneling diode structures in a strong electromagnetic field in both the general case and a number of limiting cases (resonant tunneling at the high-frequency ($\omega = \delta \gg \Gamma$) and low-frequency ($\omega = \delta \ll \Gamma$) limits and the nonresonant classical mode ($\omega = \delta \approx \Gamma$).

It was shown that the dependences of the polarization currents, energy, and width of the resonance levels on the structural parameters in the theory of a RTD with barriers of finite height and width quantitatively and qualitatively differ from the corresponding dependences obtained in the theory of a RTD with δ-functional barriers [6–10].

It was shown that in the quantum mode where $\delta = \varepsilon - \varepsilon_r = \hbar\omega \gg \Gamma$, the active alternating current in a RTD with asymmetric barriers is negative and, at $E \approx 2.8\hbar\omega/ea$, reaches a maximum equal to 84% (in magnitude) of the direct resonant current, $I_0(E = \delta = 0) = 1$, if the probability of electron tunneling through the emitter barrier is much higher than that of electron tunneling through the collector barrier. Negative differential resistance appears, which makes it possible to amplify and generate electromagnetic waves. The response power at the frequency $\omega = 10^{13}$ s^{-1} reaches in this case ($v = 5 \times 10^7$ cm/s, $n = 10^{17}$ cm^{-3}) $P_c = 2.8\hbar\omega v n \approx 10^4$ W/cm^2, the generation power reaches 10^6 W/cm^2. For the case of symmetric barriers, the active alternating current in the quantum mode reaches a value equal (in magnitude) to 42% of the direct resonant current.

If the probability of electron tunneling through the emitter barrier is much lower than that of electron tunneling through the collector barrier, the active alternating current becomes lower than the direct resonant current by 1–2 orders of magnitude; the radiation-generation power decreases to a similar extent.

At low frequencies ($\omega \ll \Gamma$), polarization currents in the RTD and the radiation-generation power are independent of the ratio of the probabilities of electron tunneling through the emitter and collector barriers.

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