

# Resonant Spin Amplification in Nanostructures with Anisotropic Spin Relaxation and Spread of the Electronic $g$ Factor

M. M. Glazov<sup>^</sup> and E. L. Ivchenko

*Ioffe Physicotechnical Institute, Russian Academy of Sciences, Politekhnicheskaya ul. 26, St. Petersburg, 194021 Russia*

<sup>^</sup>*e-mail: glazov@coherent.ioffe.ru*

Submitted February 6, 2008; accepted for publication February 11, 2008

**Abstract**—Spin dynamics of electrons in semiconductors and structures with quantum wells under conditions where pumping and probing are performed in the form of a periodical series of pulses is theoretically studied. It is shown that, at a fixed delay between the pump and probe pulses, the signal of spin amplification as a function of the magnetic field consists of a series of narrow peaks conditioned by commensurability of the period of spin precession and pulse repetition interval. In the case of anisotropic spin relaxation, the peak centered at the zero magnetic field is suppressed compared with the neighboring peaks. The role of inhomogeneous broadening of the frequency of the Larmor precession in the formation of the spin amplification signal is analyzed.

PACS numbers: 73.30.Pk, 78.47.+p, 78.66.Fd, 78.67.De

DOI: 10.1134/S1063782608080137

## 1. INTRODUCTION

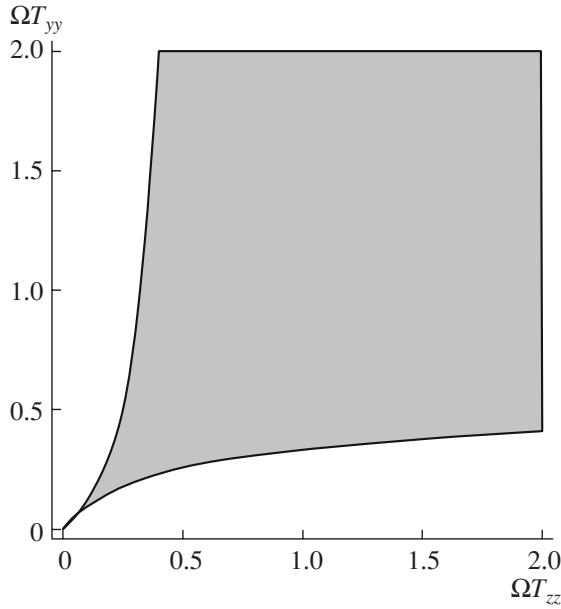
The double-pulsed pumping–probing method is a widespread method of examination of the spin coherency of charge carriers in semiconductor structures [1–5]. The method is based on the use of a periodic sequence of two pulses that are incident on the sample. These are a high-power circularly polarized pump pulse orienting spins of electrons and holes and a weak linearly polarized probe pulse probing the spin dynamics of electrons. The dependence of the rotation angle of the polarization plane of the probe beam on the delay between the pump and probe pulses during reflection of this beam (Kerr signal) or during its passage through the sample (Faraday signal) is studied. From this dependence, the time of spin relaxation of charge carriers and their complexes can be found. In such experiments, spin relaxation can be observed in time intervals shorter than the pulse repetition period, which is usually about 10 ns.

In bulk GaAs semiconductor [1], CdTe/(Cd,Mg)Te quantum wells [3, 5], and quantum dot arrays [6], the relaxation time of spin beats in a transverse magnetic field attains several nanoseconds. Therefore, to the point in time of the arrival of the sequential pulse, the spin polarization in the system does not decay completely. In this case, to determine the times of spin relaxation, the procedure of resonant spin amplification is used. It involves the following. At a fixed value of the interpulse time delay, the dependence of the Kerr signal or of the Faraday signal on the value of the transverse magnetic field is studied. If, at a certain value of the field, the repetition time of the pulses becomes multiple to the period of the spin precession, the rotation angle of the polarization plane of the probe pulse increases

abruptly compared with its value for incommensurable frequencies; i.e., the resonant spin amplification is observed. From such magnetic field dependences, one can find the times of spin relaxation [1].

The dominant mechanism of spin relaxation in semiconductor quantum wells in a broad range of temperatures and carrier concentrations is the precession mechanism of spin relaxation (D'yakonov–Perel' mechanism). It is conditioned by spin splitting of the dispersion curve of electrons, which plays the role of the effective magnetic field rigidly associated with the wave vector of the electron. During the free motion of carriers, the electron spin precesses around the effective magnetic field, while scattering leads to random changes in the direction of the spin precession frequency [7, 8]. In quantum wells, the electron spin relaxation is substantially anisotropic. For example, in symmetric structures with the zinc-blende lattice grown along the [001] axis, there are two linearly independent components of the tensor of inverse spin-relaxation times describing damping of the spin oriented along the growth axis and normal to it. The asymmetry of the structure leads to the emergence of anisotropy of spin relaxation in the plane; i.e., the components of the tensor of the inverse spin-relaxation times corresponding to axes  $[1\bar{1}0]$  and  $[110]$  are different [9].

At this point, a theory of resonant spin amplification in systems with isotropic spin relaxation has been constructed (see, for example, [1, 10] and references given in these publications). In this paper, we develop a spin amplification theory for the case of anisotropic spin relaxation.



**Fig. 1.** Region in the plane  $(\Omega T_{zz}, \Omega T_{yy})$  in which spin beats are observed (shaded area).

## 2. SPIN BEATS DURING ANISOTROPIC RELAXATION

Let us consider a quantum well grown along the axis  $z \parallel [001]$  and select the axes  $x$  and  $y$  in directions  $[1\bar{1}0]$  and  $[110]$ , respectively. The magnetic field is applied in the structure plane along one of the main axes, for example, along the  $x$  axis. Kinetic equations that describe the dynamics of the total spin of the electron gas  $\mathbf{s}$  with the components  $s_x$ ,  $s_y$ , and  $s_z$  have the form

$$\begin{aligned} \frac{ds_x}{dt} + \frac{s_x}{T_{xx}} &= 0, & \frac{ds_y}{dt} + \frac{s_y}{T_{yy}} + \Omega s_z &= 0, \\ \frac{ds_z}{dt} + \frac{s_z}{T_{zz}} - \Omega s_y &= 0. \end{aligned} \quad (1)$$

Here,  $\Omega = g_e \mu_B B / \hbar$  is the frequency of the Larmor spin precession in the external field  $\mathbf{B}$ ;  $g_e$  is the electron  $g$  factor, which is assumed to be isotropic for the sake of simplicity; and  $T_{jj}$  is the spin relaxation time for the spin component  $s_j$  ( $j = x, y, z$ ). The spin component  $s_x$  is not mixed by the field with two other components, and the solution of the first of Eqs. (1) has a simple form:  $s_x(t) = s_{0,x} \exp(-t/T_{xx})$ , where  $s_{0,x}$  is its initial value. We further suppose the conditions of optical excitation, under which  $s_{0,x} = 0$ , and discuss exclusively the behavior of components  $s_y$  and  $s_z$ .

In the case of spin relaxation conditioned by the D'yakonov–Perel' mechanism, which is important for

comparison with the experiments, the relations for the inverse spin relaxation times have the form [11]

$$\frac{1}{T_{zz}} = \frac{2}{T_{xx}} = \frac{2}{T_{yy}} = 4 \left( \frac{\beta}{\hbar} \right)^2 \langle k^2 \tau_1 \rangle. \quad (2)$$

Here,  $\beta$  is the constant at spin-dependent terms in the effective Hamiltonian. These terms are linear by the wave vector  $\mathbf{k}$  and have the form of the Dresselhaus term  $\beta(\sigma_x k_y + \sigma_y k_x)$ , or the Rashba term  $\beta(\sigma_x k_y - \sigma_y k_x)$ ,  $\tau_1$  is the relaxation time of the momentum of one electron [12], and angular brackets denote averaging over the carrier ensemble.

Let us assume that at the initial point in time  $t = 0$ , a short circularly polarized optical pulse had generated the spin oriented along the  $z$  axis. The initial condition for system (1) in this case will be  $s_z = s_0$ ,  $s_x = 0$ ,  $s_y = 0$ , where the initial spin polarization  $s_0$  depends on the selection rules for excitation of photoelectrons. The time dependence  $s_z(t)$  is written in the form [13]

$$s_z(t) = s_1 e^{-\lambda_1 t} + s_2 e^{-\lambda_2 t}, \quad (3)$$

where

$$\lambda_{1,2} = \frac{1}{\bar{T}} \pm i\tilde{\Omega}, \quad (4)$$

$$\frac{1}{\bar{T}} = \frac{1}{2} \left( \frac{1}{T_{zz}} + \frac{1}{T_{yy}} \right), \quad (5)$$

$$\tilde{\Omega} = \sqrt{\Omega^2 - \frac{1}{4} \left( \frac{1}{T_{zz}} - \frac{1}{T_{yy}} \right)^2},$$

$$s_1 = \frac{T_{zz}^{-1} - \lambda_2}{\lambda_1 - \lambda_2} s_0, \quad s_2 = \frac{T_{zz}^{-1} - \lambda_1}{\lambda_1 - \lambda_2} s_0. \quad (6)$$

Note that, depending on the ratio between the frequency  $\Omega$  and inverse time

$$\frac{1}{T'} = \frac{1}{2} \left| \frac{1}{T_{zz}} - \frac{1}{T_{yy}} \right|,$$

the quantity  $\Omega$  is either purely real or purely imaginary.

For further analysis, it is convenient to rewrite Eq. (3) in the form

$$s_z(t) = s_0 e^{-t/\bar{T}} \left[ \cos \tilde{\Omega} t + \left( \frac{1}{T_{zz}} - \frac{1}{T_{yy}} \right) \frac{\sin \tilde{\Omega} t}{2\tilde{\Omega}} \right], \quad (7)$$

which is valid at an arbitrary ratio between  $\Omega$  and inverse time  $1/T'$ . Hence it follows that the spin polarization in the system can both demonstrate damping beats and drop without oscillations [14]. The first case is realized at real  $\tilde{\Omega}$ , where  $\Omega T' > 1$ , and the second case is realized at a sufficiently large anisotropy of relaxation times, where  $\Omega T' < 1$ , i.e.,  $|T_{zz}^{-1} - T_{yy}^{-1}| \geq 2\Omega$ , and the quantity  $\tilde{\Omega}$  is imaginary. In the second case, the

ratio  $\sin(\tilde{\Omega}t)/\tilde{\Omega}$  can be replaced by  $\sinh(|\tilde{\Omega}|t)/|\tilde{\Omega}|$ . Figure 1 schematically represents the region of parameters  $\Omega T_{zz}$  and  $\Omega T_{yy}$  where oscillations of spin polarization are observed. At the region boundary, the quantity  $\tilde{\Omega} = 0$ ; we will refer to this magnetic field as boundary. It is evident that at small  $\Omega T_{zz}$  and  $\Omega T_{yy}$ , this region is concentrated near the diagonal  $T_{zz} = T_{yy}$ , and as the values of  $\Omega T_{zz}$  and  $\Omega T_{yy}$  increase, the region of spin beats is broadened. The existence of two modes of spin dynamics and renormalization of the spin precession frequency in comparison with the Larmor frequency are the differences of the situation under consideration with respect to the isotropic spin relaxation mode.

### 3. RESONANT SPIN AMPLIFICATION

Resonant spin amplification appears during the injection of electron spins by the sequence of polarized optical pulses passing with the interval  $T_{\text{rep}}$ . Taking into account that the number of these pulses in the experiment is large, we assume that their sequence is infinite. According to (3), the steady-state value of spin polarization of carriers observed at a fixed delay  $\Delta t$  between the pump and probe pulses can be written in this case as

$$s_z(\Delta t) = \sum_{n=1}^{\infty} [s_1 e^{-\lambda_1(\Delta t + nT_{\text{rep}})} + s_2 e^{-\lambda_2(\Delta t + nT_{\text{rep}})}]. \quad (8)$$

Here,  $\Delta t$  is the delay between the probe pulse and nearest subsequent pump pulse. It can take any negative values between  $-T_{\text{rep}}$  and 0.

Summing the geometric series in (8), we obtain the following compact expression describing the resonant spin amplification in anisotropic systems:

$$s_z(\Delta t) = \frac{s_0}{2} e^{-(T_{\text{rep}} + \Delta t)/\bar{T}} \times \frac{e^{T_{\text{rep}}/\bar{T}} C[\tilde{\Omega}(T_{\text{rep}} + \Delta t)] - C(\tilde{\Omega}\Delta t)}{\cosh(T_{\text{rep}}/\bar{T}) - \cos(\tilde{\Omega}T_{\text{rep}})}, \quad (9)$$

where

$$C(x) = \cos x - \frac{1}{2\tilde{\Omega}} \left( \frac{1}{T_{zz}} - \frac{1}{T_{yy}} \right) \sin x.$$

Equation (9) is the central result of this study. It describes the spin polarization in the pumping–probing experiments at arbitrary anisotropy of spin relaxation and arbitrary ratio between the frequency of the Larmor precession of electron spins and repetition rate of pulses.

Analysis of formula (9) shows that the dependence of the electron spin on the magnetic field  $B$  at a fixed delay  $\Delta t$  consists of the sequence of peaks corresponding to the condition  $\cos(\tilde{\Omega}T_{\text{rep}}) \approx 2\pi N$ , where  $N$  is an integer number. Indeed, if the period of the spin preces-

sion in the magnetic field  $2\pi/\tilde{\Omega}$  and the pulse repetition interval are multiple, then the spin injected by the sequential pump pulse is in phase with the precessing electron spin. As a result, the spin polarization in the system increases.

In the case of isotropic spin relaxation, when  $T_{zz} = T_{yy} \equiv T$  and  $\tilde{\Omega} = \Omega$ , the function  $C(x) \rightarrow \cos x$  and (9) is reduced to the relation known in the literature [10]

$$s_z(\Delta t) = \frac{s_0}{2} e^{-(T_{\text{rep}} + \Delta t)/T} \times \frac{\cos(\Omega\Delta t) - e^{T_{\text{rep}}/T} \cos[\Omega(T_{\text{rep}} + \Delta t)]}{\cos(\Omega T_{\text{rep}}) - \cosh(T_{\text{rep}}/T)}. \quad (10)$$

Let us analyze three important limiting cases for Eq. (9) describing the spin amplification during the anisotropic relaxation.

(i) The magnetic field is zero, and after a single pump pulse,  $s_z(t) = s_0 e^{-t/T_{zz}}$ . Therefore, the signal of the resonant spin amplification is

$$s_z(\Delta t; \Omega = 0) = s_0 \sum_{n=1}^{\infty} e^{-(\Delta t + nT_{\text{rep}}/T_{zz})} = \frac{e^{-\Delta t/T_{zz}}}{e^{T_{\text{rep}}/T_{zz}} - 1} s_0. \quad (11)$$

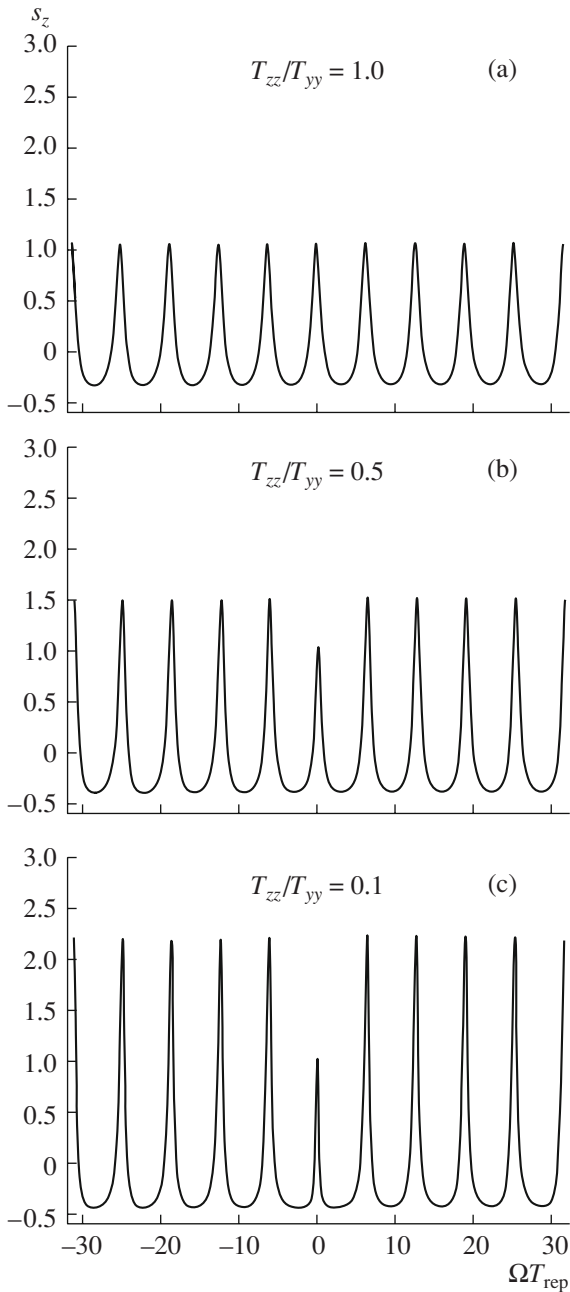
(ii) In the boundary magnetic field,  $\tilde{\Omega} = 0$ , disregarding  $|\Delta t|$  in comparison with the pulse repetition interval, we have

$$s_z(0; \tilde{\Omega} = 0) = s_0 \left[ \frac{1}{e^{T_{\text{rep}}/\bar{T}} - 1} - \frac{1}{4} \left( \frac{1}{T_{zz}} - \frac{1}{T_{yy}} \right) \frac{T_{\text{rep}}}{\cosh(T_{\text{rep}}/\bar{T}) - 1} \right]. \quad (12)$$

(iii) The magnetic field corresponding to the maximum:  $\tilde{\Omega} = 2\pi N$ ,  $N \neq 0$  is an integer number ( $2\pi N|\Delta t/T_{\text{rep}}| \ll 1$ ). In this case, we have

$$s_z(\Delta t; \tilde{\Omega}T_{\text{rep}} = 2\pi N) = \frac{e^{-\Delta t/\bar{T}}}{e^{T_{\text{rep}}/\bar{T}} - 1} s_0. \quad (13)$$

It is evident from comparison of Eqs. (11) and (13) that  $s_z(\Delta t; \tilde{\Omega} = 2\pi N) > s_z(\Delta t; \Omega = 0)$  since, in the D'yakonov–Perel' mechanism,  $T_{zz} < T_{yy}$  (see Eq. (2) and [9]).



**Fig. 2.** Signal of resonant spin amplification  $s_z(\Omega T_{\text{rep}})$ . (a) The case of isotropic spin relaxation ( $T_{zz}/T_{yy} = 1$ ); (b)  $T_{zz}/T_{yy} = 1/2$ , which corresponds to the axially symmetric quantum well; and (c)  $T_{zz}/T_{yy} = 1/10$ . The curves are normalized by the value  $s_z(t=0)$ . Other calculation parameters:  $T_{\text{rep}}/T_{zz} = 2/3$ ,  $-\Delta t/T_{\text{rep}} = 1/250$ .

Near the maximum, where  $|\tilde{\Omega}T_{\text{rep}} - 2\pi N| \ll 1$ , and  $|\Delta t/T_{\text{rep}}| \ll 1$ , relation (9) can be rewritten in the form of the Lorentzian function

$$s_z(0, \tilde{\Omega}) = s_0 \frac{1 - e^{-T_{\text{rep}}/\bar{T}}}{(\tilde{\Omega}T_{\text{rep}} - 2\pi N)^2 + 2[\cosh(T_{\text{rep}}/\bar{T}) - 1]} \quad (14)$$

In this approximation, the peak width is determined by the quantity

$$\Delta = \sqrt{2[\cosh(T_{\text{rep}}/\bar{T}) - 1]}, \quad (15)$$

which, in the limit of large times of spin relaxation  $T_{\text{rep}}/\bar{T} \ll 1$ , is reduced to  $\Delta \approx T_{\text{rep}}/\bar{T}$ ; the smaller this quantity is, the longer the spin relaxation time  $\bar{T}$ .

Dependences  $s_z$  at a fixed delay  $\Delta t$  on the magnetic field (expressed in units  $\Omega T_{\text{rep}}$ ) are represented in Fig. 2. Figure 2a corresponds to the case of isotropic spin relaxation; Fig. 2b corresponds to the case of moderate anisotropy,  $T_{zz}/T_{yy} = 1/2$ ; and Fig. 2c corresponds to the case of pronounced anisotropy,  $T_{zz}/T_{yy} = 1/10$ . According to (9), the signal of the resonant spin amplification as a function of the magnetic field consists of the peaks corresponding to  $\tilde{\Omega}T_{\text{rep}} = 2\pi N$ , where  $N$  is an integer.

The structure of the peaks substantially differs in the cases of isotropic and anisotropic spin relaxation. In the first case (Fig. 2a), all peaks have the same height, while during the anisotropic spin relaxation (Figs. 2b, 2c), the peak height at  $\Omega \neq 0$  is larger. This is a consequence of the anisotropy of spin relaxation. In the zero magnetic field, the signal of spin amplification is determined only by the value of longitudinal time of spin relaxation  $T_{zz}$  (see Eq. (11)). As the magnetic field increases, the spin precession leads to the fact that the damping of spin beats, according to (5), is determined by the longer time  $\bar{T}$ , since relaxation of the  $y$  component of the spin proceeds more slowly.

The ratio of the heights of the zero peak and subsequent ones with the numbers satisfying the condition  $2\pi N\Delta t/T_{\text{rep}} \ll 1$ , according to (11), (13), is defined as

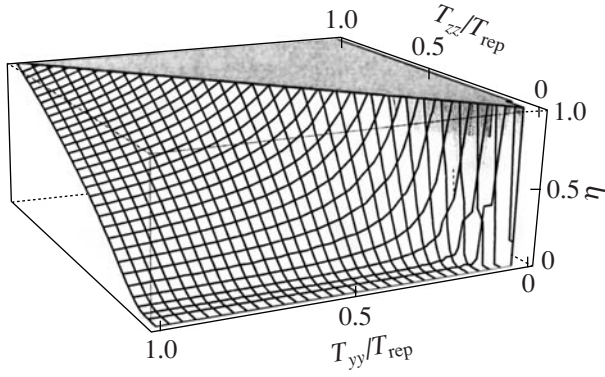
$$\eta = \frac{e^{T_{\text{rep}}/\bar{T}} - 1}{e^{T_{\text{rep}}/T_{zz}} - 1} \leq 1. \quad (16)$$

The corresponding dependence  $\eta(T_{zz}, T_{yy})$  is shown in Fig. 3. It is clearly seen from Fig. 3 that as the anisotropy of spin relaxation increases, the amplitude of the central peak decreases. Note that if the pulse repetition interval is short,  $T_{\text{rep}} \ll T_{zz}, T_{yy}$ , then  $\eta = \bar{T}/T_{zz}$  and is independent of the pulse repetition interval. In the opposite limiting case,  $T_{\text{rep}} \gg T_{zz}, T_{yy}$ ,  $\eta \rightarrow \exp[T_{\text{rep}}(T_{yy} - T_{zz})/(2T_{yy}T_{zz})]$ , and the central peak is exponentially suppressed, while the dependence of  $\eta$  on  $T_{zz}, T_{yy}$  is especially sharp.

#### 4. ROLE OF INHOMOGENEOUS BROADENING

We assumed above that the damping time of spin beats is independent of the magnetic field. This condition is apparently violated in the pumping-probing experiments, for example, in bulk GaAs [1] and in the CdTe/(Cd,Mg)Te quantum wells [3], where the main





**Fig. 3.** Ratio of the height of the central peak of the spin amplification signal to the first one as a function of  $T_{zz}/T_{\text{rep}}$  and  $T_{yy}/T_{\text{rep}}$ . The dependence at  $T_{zz} \leq T_{yy}$  is shown in part.

mechanism of spin decoherence is the spread of  $g$  factors. As the magnetic field increases, the observed times of spin relaxation shorten, while the peaks of resonant spin amplification increase.

To describe this effect, let us average expression (9) over the distribution of the Larmor frequencies  $f(\Omega)$ , which is caused by the spread of the electronic  $g$  factor. A simple analytical expression describing the peaks of resonant spin amplification can be obtained using approximation (14) for the peak shape in the absence of inhomogeneous broadening and assuming that the spread of the Larmor frequencies is also described by the Lorentzian

$$f(\Omega) = \frac{\sigma}{\pi[(\Omega - \Omega_0)^2 + \sigma^2]},$$

where  $\Omega_0 = \bar{g}_e \mu_B B / \hbar$ ,  $\bar{g}_e$  is the average value of the electronic  $g$  factor, and the frequency variance  $\sigma$  is associated with the spread of the  $g$  factor by the relation

$$\sigma = \frac{\Delta g_e}{\bar{g}_e} \Omega_0.$$

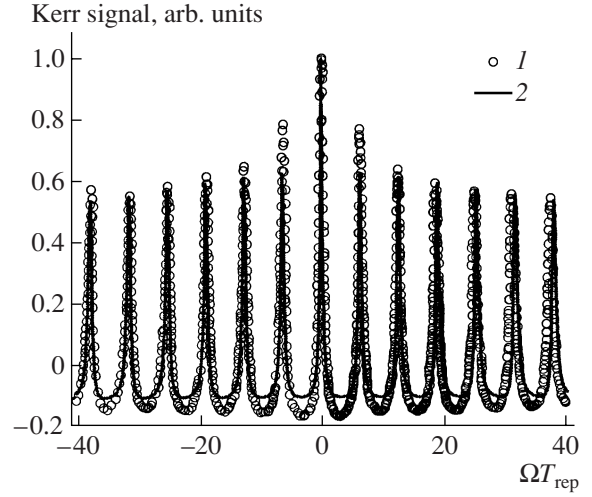
In this case, we have

$$\begin{aligned} \langle s_z(0, \tilde{\Omega} T_{\text{rep}}) \rangle &= s_0 \frac{\Delta + \sigma}{\Delta} \\ &\times \frac{1 - e^{-T_{\text{rep}}/\bar{T}}}{(\tilde{\Omega} T_{\text{rep}} - 2\pi N)^2 + (\Delta + \sigma)^2}. \end{aligned} \quad (18)$$

As inhomogeneous broadening  $\sigma$  increases, the height of peaks described by Eq. (18) decreases, while the width increases.

In the experiments, as a rule, a Gaussian distribution of the frequencies of the spin precession is realized,

$$f(\Omega) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(\Omega - \Omega_0)^2}{2\sigma^2}\right].$$

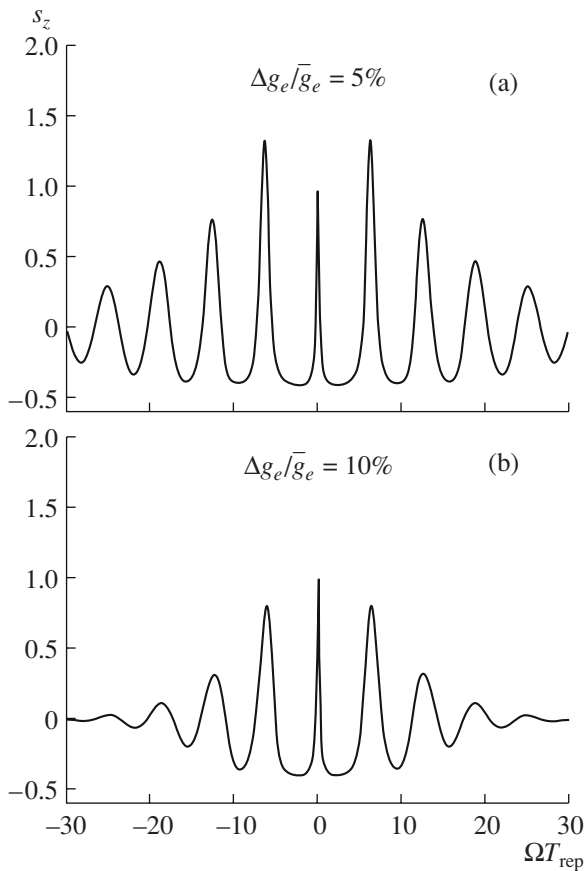


**Fig. 4.** Resonant spin-amplification signal as a function of  $\Omega T_{\text{rep}}$ . (1) Angles of the Kerr rotation, the data of study [10] for the sample of bulk GaAs at 6 K. (2) Fitting, calculation using the following parameters: spread of the values of the  $g$  factor  $\Delta g_e/\bar{g}_e = 0.35\%$ , pulse repetition period  $T_{\text{rep}} = 12.5$  ns, times of spin relaxation  $T_{zz} = T_{yy} = 40$  ns, and delay  $\Delta t = -50$  ps.

We will further use this distribution for numerical calculations of the signals of spin amplification.

Figure 4 represents the comparison of the theory developed here with the experimental data on the resonant spin amplification in bulk  $n$ -GaAs (see Fig. 18 from [10]). Points (1) show the experimental data on the Kerr rotation, and curve 2 is calculated using the following parameters:  $\Delta g_e/\bar{g}_e = 0.35\%$ ,  $T_{\text{rep}} = 12.5$  ns,  $T_{zz} = T_{yy} = 40$  ns, and  $\Delta t = -50$  ps (values of  $\Delta t$  and  $T_{\text{rep}}$  are taken from [10]). The values of the times of spin relaxation and spread of values of the electron  $g$  factor served as the adjustable parameters and were selected from the best agreement with the experiment. Note that the spread of  $g$  factors is very close to the value obtained for bulk GaAs crystals in [1]. Some disagreement of the minima of the signal could be eliminated by correction of the selection of the signal zero, i.e., by the total vertical shift of experimental points.

The results of calculation of the signals of spin amplification during anisotropic relaxation are represented in Fig. 5. Figure 5a corresponds to  $\Delta g_e/\bar{g}_e = 0.05$ , while Fig. 5b corresponds to  $\Delta g_e/\bar{g}_e = 0.1$ . Amplitudes of all peaks excluding the central one decrease monotonically as their number decreases, and the peaks themselves are broadened since, according to (17), the variance  $\sigma$  is proportional to the magnetic field  $B$  and, consequently, the peak width increases linearly as  $|N|$  increases. The ratio of the heights of the central peak and the peaks with the numbers  $\pm 1$  can be arbitrary. At a sufficiently small spread of  $g$  factors, the central peak is lower than the neighboring ones (see Fig. 5a), which



**Fig. 5.** Resonant spin amplification signal  $s_z(t)$  as a function of  $\Omega T_{\text{rep}}$ . The spin relaxation is anisotropic,  $T_{zz}/T_{yy} = 1/10$ . (a) 5 and (b) 10% spread of the values of the  $g$  factor. The curves are normalized by the value  $s_z(t=0)$ . Other parameters are the same as for the calculation of the curves in Fig. 2:  $T_{\text{rep}}/T_{zz} = 2/3$ ,  $-\Delta/T_{\text{rep}} = 1/250$ .

corresponds to the case where the dephasing rate due to inhomogeneous broadening at  $\Omega T_{\text{rep}} = \pm 2\pi$  is small compared with  $\bar{T}$ . As inhomogeneous broadening increases, the peaks with the numbers  $\pm 1$  become increasingly lower, and, if dephasing caused by the spread of  $g$  factors is dominant even at  $\Omega T_{\text{rep}} = \pm 2\pi$ , the central peak is higher than the neighboring ones, similar to the case of isotropic spin relaxation (Fig. 5b).

Note that, in the course of pumping and probing in the same direction  $z$  and applying the magnetic field at a right angle to the initial spin of photocarriers, the average spin  $\langle s_z(0, \tilde{\Omega} T_{\text{rep}}) \rangle$  is an even function of the field, and the peaks with the numbers  $|N|$  and  $-|N|$  have identical width and height.

The above-suggested method of making allowance for inhomogeneous broadening of the Zeeman effect implies that the electrons are localized at impurities or inhomogeneities of the crystal lattice, the values of the electron  $g$  factor are different, and the time of possible jump of the electron from one localization center to

another exceeds the pulse repetition period  $T_{\text{rep}}$ . For free three- or two-dimensional electrons, the  $g$  factor depends on their kinetic energy:  $g_e = g_e(E)$ . This dependence also leads to dephasing of the resonant spin amplification with the effective rate of spin relaxation  $T_{\text{eff}}^{-1} \sim (\Delta\Omega)^2 \tau_e$ , where  $\Delta\Omega$  is the characteristic deviation of the frequency of the spin precession from its average value at the specified energy distribution of the electron gas and  $\tau_e$  is the electron energy relaxation time. The given evaluation for  $T_{\text{eff}}$  is in agreement with the results [15], if we equate the time  $\tilde{\tau}_{00}^{-1}$  introduced in that study with  $\tau_e$ . An important difference in the considered mechanisms of inhomogeneous broadening is the dependence of the effective dephasing time of the beats on the magnetic field. If the electrons are localized, then, it is not difficult to make sure from (17) and (18) that  $T_{\text{eff}}^{-1} \propto B$ . For free carriers, the effective rate of spin relaxation  $T_{\text{eff}}^{-1} \propto B^2$ .

## 5. CONCLUSIONS

Therefore, we theoretically studied the resonant spin amplification in semiconductor structures with the anisotropic spin relaxation. The observed angle of rotation of the polarization plane in the pumping–probing experiments is proportional to spin polarization of charge carriers. At a fixed delay between the test and probe beams, the dependence of the Kerr rotation angle of the probe pulse on the magnetic field consists of a series of abrupt peaks. The peak corresponding to the magnetic field is lower than the subsequent ones. The ratio of the peak heights characterizes the degree of anisotropy of the times of spin relaxation for the spin components along the structure axis and in the structure plane in the direction normal to the magnetic field.

The effect of inhomogeneous broadening associated with the spread of the electronic  $g$  factor is studied. It is shown that, depending on broadening and anisotropy of spin relaxation, the peak can be lower or higher than the neighboring ones, and the heights of the subsequent peaks can decrease monotonically as the magnetic field increases.

## ACKNOWLEDGMENTS

This study was supported in part by the Russian Foundation for Basic Research, by the Programs of the Russian Academy of Sciences, and by the Foundation for Non-Profit Programs Dynasty–Moscow International Center for Fundamental Physics.

We thank D.R. Yakovlev and I.A. Yugova for helpful discussions.

## REFERENCES

1. J. M. Kikkawa and D. D. Awschalom, *Phys. Rev. Lett.* **80**, 4313 (1998).
2. M. A. Brand, A. Malinowski, O. Z. Karimov, et al., *Phys. Rev. Lett.* **89**, 236601 (2002).
3. E. A. Zhukov, D. R. Yakovlev, M. Bayer, et al., *Phys. Status Solidi B* **243**, 878 (2006).
4. G. V. Astakhov, T. Kiessling, D. R. Yakovlev, et al., *Phys. Status Solidi B* **243**, 858 (2006).
5. E. A. Zhukov, D. R. Yakovlev, M. Bayer, et al., *Phys. Rev. B* **76**, 205310 (2007).
6. A. Greilich, R. Oulton, E. A. Zhukov, et al., *Phys. Rev. Lett.* **96**, 227401 (2006).
7. M. I. D'yakonov and V. I. Perel', *Fiz. Tverd. Tela* **13**, 3581 (1972) [*Sov. Phys. Solid State* **13**, 3023 (1972)].
8. M. I. D'yakonov and V. Yu. Kachorovskii, *Fiz. Tekh. Poluprovodn.* **20**, 178 (1986) [*Sov. Phys. Solid State* **20**, 110 (1986)].
9. N. Averkiev, L. Golub, and M. Willander, *J. Phys.: Condens. Matter* **14**, 271 (2002).
10. B. Beschoten, in *Lecture Manuscripts of the 36th Spring School of the Institute of Solid State Research "Magnetism Goes Nano,"* Ed. by S. Bluegel, Th. Brueckel, and C. M. Schneider, Reihe Matter and Materials (Forschungszentrum Juelich GmbH, 2005), Vol. 26, p. E7.1.
11. E. L. Ivchenko, *Optical Spectroscopy of Semiconductor Nanostructures* (Alpha Science, Harrow UK, 2005).
12. W. J. H. Leyland, G. H. John, R. T. Harley, et al., *Phys. Rev. B* **75**, 165309 (2007).
13. S. Döhrmann, D. Hägele, J. Rudolph, et al., *Phys. Rev. Lett.* **93**, 147405 (2004).
14. K. Morita, H. Sanada, S. Matsuzaka, et al., *Appl. Phys. Lett.* **87**, 171905 (2005).
15. F. X. Bronolt, I. Martin, A. Saxena, and D. L. Smith, *Phys. Rev. B* **66**, 233206 (2002).

*Translated by N. Korovin*