

# Magnetotransport in a 2D System with Strong Scatterers: Renormalization of Hall Coefficient Caused by non-Markovian Effects<sup>1</sup>

A. P. Dmitriev and V. Yu. Kachorovskii<sup>^</sup>

*Ioffe Physicotechnical Institute, Russian Academy of Sciences, St. Petersburg, 194021 Russia*

*^e-mail: Kachor@vip1.ioffe.ru*

Received February 6, 2008; accepted for publication February 11, 2008

**Abstract**—We show that a sharp dependence of the Hall coefficient  $R$  on the magnetic field  $B$  arises in two-dimensional electron systems with strong scatterers. The phenomenon is due to classical memory effects. We calculate analytically the dependence of  $R$  on  $B$  for the case of scattering by antidots (modeled by hard disks of radius  $a$ ), randomly distributed with concentration  $n_0 \ll 1/a^2$ . We demonstrate that in very weak magnetic fields ( $\omega_c \tau_{tr} \lesssim n_0 a^2$ ), memory effects lead to a considerable renormalization of the Boltzmann value of the Hall coefficient:  $\delta R/R \sim 1$ . With increasing magnetic field, the relative correction to  $R$  decreases, then changes sign, and saturates at the value  $\delta R/R \sim -n_0 a^2$ . We also discuss the effect of the smooth disorder on the dependence of  $R$  on  $B$ .

PACS numbers: 05.60.+w, 73.43.Qt, 73.50.Bk, 73.50.Jt, 73.63.Hs

DOI: 10.1134/S1063782608080113

The problem of magnetoresistance (MR) in metal and semiconductor structures has been intensively discussed in the literature over the past three decades. A large number of both theoretical and experimental papers on this subject have been published. Most of these works were devoted to the case of the degenerate two dimensional electron gas where electrons move in the plane perpendicular to the magnetic field and scatter on a random impurity potential, so that only electrons with energy close to the Fermi energy participate in conductance. The simplest theoretical description of such a situation is based on the Boltzmann equation, which yields the well-known expressions for the components of the conductivity tensor

$$\sigma_{xx} = \frac{\sigma_0}{(1 + \omega_c^2 \tau_{tr}^2)}, \quad \sigma_{xy} = \frac{\sigma_0(\omega_c \tau_{tr})}{(1 + \omega_c^2 \tau_{tr}^2)}. \quad (1)$$

Here,  $\sigma_0 = e^2 n \tau_{tr} / m$  is the Drude conductivity at  $B = 0$ ,  $\omega_c = |e|B/mc$  is the cyclotron frequency,  $\tau_{tr}$  is the transport scattering time, and  $n$  is the electron concentration. The resistivity tensor, which can be obtained by inverting the conductivity tensor, has an even simpler form:

$$\rho_{xx} = \frac{m}{e^2 n \tau_{tr}}, \quad \rho_{xy} = \frac{m \omega_c}{e^2 n} = -RB, \quad (2)$$

where  $R = 1/enc < 0$  is the Hall coefficient. Thus, in the frame of the Boltzmann approach,  $\rho_{xx}$  and  $R$  do not depend on magnetic field  $B$ . Experimental measurements of  $\rho_{xx}$  and  $R$  are widely used to find  $\tau_{tr}$  and  $n$ .

It is known that Eqs. (2) may become invalid due to a number of effects of both quantum and classical nature. The most remarkable of them is the quantum Hall effect. Another quantum effect, weak localization, leads to the decrease of  $\rho_{xx}$  with  $B$ , concentrated in the region of weak magnetic fields [1]. Furthermore, the dependence of  $\rho_{xx}$  on  $B$  appears due to quantum effects related to electron–electron interaction [2] (see also [3] for review). At the same time, both weak localization and electron–electron interaction (in standard Altshuler–Aronov theory) do not result in any dependence of  $R$  on  $B$  (see [4] and [2, 4–6], respectively), though such a dependence arises in the regime of strong localization [7].

The dependence of  $\rho_{xx}$  on  $B$  may also be caused by classical memory effects (ME) which are neglected in the Boltzmann approach. Such effects arise as a manifestation of the non-Markovian nature of electron dynamics in a static random potential. Physically, a diffusive electron returning to a certain region of space “remembers” the random potential landscape in this region, so its motion is not purely chaotic, as is assumed in the Boltzmann picture. For  $B = 0$ , non-Markovian corrections to kinetic coefficients are usually small. In particular, in the case of hard-core scatterers of radius  $a$  randomly distributed with concentration  $n_0$ , the ME-induced relative correction to the resistivity is proportional to the gas parameter  $\beta_0 = all = 2n_0 a^2 \ll 1$

<sup>1</sup> The text was submitted by the authors in English.

(here,  $l = v_F \tau$ ,  $v_F$  is the Fermi velocity, and  $\tau$  is the mean free time). However, for  $B \neq 0$ , the role of ME is dramatically increased due to a strong dependence of the return probability on  $B$ . In particular, two years prior to [1] there appeared a publication [8] where a classical mechanism of strong negative magnetoresistance was discussed. The mechanism was investigated by the example of a gas of non-interacting electrons scattering on hard disks (antidots). It was shown that with increasing magnetic field there is an increasing number of closed electron orbits which avoid scatterers and therefore are not diffusive (see also recent discussions [9–11] of this mechanism). Electrons occupying these orbits do not participate in diffusion (so-called “circling electrons”). As a result, the longitudinal resistance turns out to be proportional to the factor  $1 - P$ , where  $P = \exp(-2\pi/\omega_c \tau)$  is the probability of the existence of the circular closed orbit (here  $\omega_c$  is the cyclotron frequency,  $\tau$  is the scattering time). Another classical mechanism was presented in [12], where the MR due to non-Markovian dynamics of electrons trapped in some region of space was discussed.

Notwithstanding these developments, the role of classical effects in magnetotransport was underappreciated for a long time. A new boost to the research in this direction was given by [13], where it was shown that, if electrons move in a smooth disorder potential and in a sufficiently strong magnetic field, a phenomenon called “classical localization” occurs. This phenomenon leads to the exponential suppression of the longitudinal resistance: most electrons are trapped in localized equipotential trajectories and do not participate in diffusion. This work was followed by a series of works [14–21] discussing different aspects of classical magnetotransport in 2D systems. It was shown [15] that for lower magnetic fields near the onset of the classical localization, the magnetoresistance is positive; i.e., the longitudinal resistance grows with increasing magnetic field. In [16, 17] the combination of smooth disorder and strong scatterers (antidots) was considered. It was shown that in this system under certain conditions there are several regimes of the behavior of magnetoresistance depending on the strength of the magnetic field: first the longitudinal resistance decreases with growing field, then it saturates and then begins to grow. The role of non-Markovian effects in the cyclotron resonance was also discussed [18].

In [8–18] magnetoresistance was studied in a situation where the magnetic field is classically strong, that is where, the parameter  $\beta = \omega_c \tau$  is large. Recently, the region of classically small magnetic fields  $\beta \ll 1$  was investigated numerically [19, 20] for the case of electrons scattering on strong scatterers. It was shown [19] that memory effects due to double scattering of an electron on the same disk lead to a negative parabolic magnetoresistance (in [8], where these processes were not taken into account, exponentially small MR was predicted). The numerical simulations [20] discovered a low-field classical anomaly of the MR. The anomaly

was attributed to the memory effects specific for back-scattering events. The simulations were performed for the 2D Lorenz gas, which is a system of 2D electrons scattering on hard disks randomly distributed in the plane with average concentration  $n$ . Magnetotransport in this system is characterized by two dimensionless parameters:  $\beta = \omega_c \tau$ , and the gas parameter  $\beta_0$ . The anomaly was observed in the case  $\beta \ll 1$ ,  $\beta_0 \ll 1$ . Both the numerical simulations and the qualitative considerations [20] indicated that at zero temperature the MR can be expressed in terms of a dimensionless function  $f(z)$  via

$$\frac{\delta \rho_{xx}}{\rho} = -\beta_0 f\left(\frac{\beta}{\beta_0}\right), \quad (3)$$

where  $\rho$  is the resistivity for  $B = 0$ . The analytical theory of the effect was developed in [21], where it was shown that the function  $f(z)$  has the asymptotics

$$f(z) = \begin{cases} 0.32z^2 & \text{for } z \rightarrow 0 \\ 0.39 - 1.3/\sqrt{z} & \text{for } z \rightarrow \infty, \end{cases} \quad (4)$$

and can be well approximated by the linear function

$$f(z) \approx 0.032(z - 0.04), \quad (5)$$

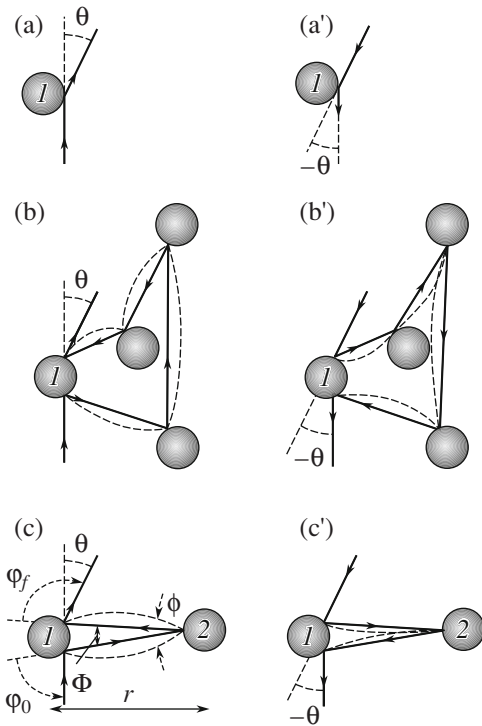
in the interval  $0.05 \leq z \leq 2$ .

In spite of large number of publications devoted to the study of the influence of non-Markovian effects on the  $\rho_{xx}$ , the dependence of  $R$  on  $B$  induced by such effects was investigated (to the best of our knowledge) only in the context of “circling electrons” [8]. It was found that though the existence of circling orbits leads to a strong dependence of  $\rho_{xx}$  on  $B$  in the region of classically strong  $B$  ( $\omega_c \tau_{tr} \gg 1$ ), the corresponding dependence of  $R$  on  $B$  is very weak in the whole range of  $B$  [8].

In this paper, we propose another mechanism of dependence of  $R$  on  $B$ . It does not rely upon the existence of non-colliding electrons but, in contrast, assumes that transport properties of colliding electrons are modified by classical ME. The mechanism turns out to be especially effective in the region of very weak fields,  $\omega_c \tau_{tr} \lesssim \beta_0$ .

We will study dependence of  $R$  on  $B$  in 2D degenerated electron gas in a system of randomly located classical scatterers modeled by impenetrable disks of radius  $a \gg \lambda_F$ , where  $\lambda_F = \hbar/mv_F$  is the Fermi wavelength. The simplest realization of such a system is a quantum well with random array of antidots. We restrict ourselves to the case  $\omega_c \tau_{tr} \ll 1$ . The electron dynamics is studied classically. The role of quantum effects is briefly discussed at the end of the paper.

We start with recalling that in the frame of the Boltzmann approach, the collision with a single scatterer is described by a differential scattering cross-section  $\sigma(\theta)$  (see Fig. 1a) and the collisions with different scatterers are independent. Inverting in time the process shown in Fig. 1a, we get a process shown in Fig. 1a' correspond-



**Fig. 1.** Processes of single scattering by angle  $\theta$  (a) and  $-\theta$  (a') characterized by a scattering cross-section  $\sigma(\theta)$  ( $\sigma(\theta) = \sigma(-\theta)$ ) both for  $B = 0$  and for  $B \neq 0$ , and processes of scattering on complexes of scatterers (b, b', c, c'), including double scattering on the scatterer 1. Correction to the cross-section due to multi-scattering processes remains symmetric for  $B = 0$ . Magnetic field bends trajectories as shown in (b, b', c, c') by dashed lines. As a result, the symmetry with respect to inversion of  $\theta$  is broken, so that  $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$  for  $B \neq 0$ .

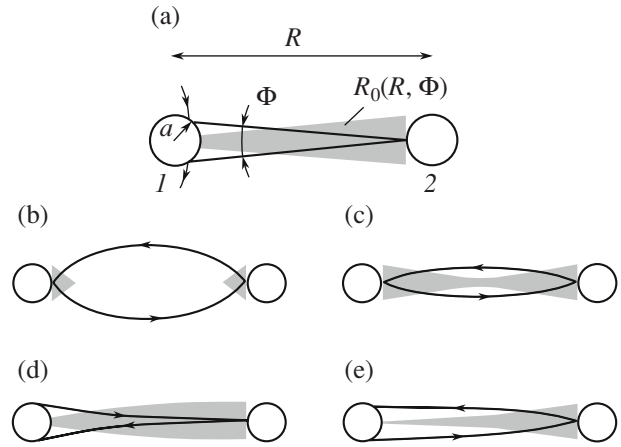
ing to scattering by the angle  $-\theta$ . This implies an important property of a single scattering, symmetry with respect to replacement of  $\theta$  by  $-\theta$  (reciprocity theorem):  $\sigma(\theta) = \sigma(-\theta)$  [22]. This is the property which provides that  $R$  does not depend on  $B$ . If, for any reason, scattering cross-section acquires an asymmetric correction  $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$ , the expression for  $\rho_{xy}$  becomes

$$\rho_{xy} = \frac{m(\omega_c + \Omega)}{e^2 n} = -B(R + \delta R), \quad (6)$$

where

$$\Omega = -n_0 v_F \int d\theta \delta\sigma(\theta) \sin \theta, \quad \frac{\delta R}{R} = \frac{\Omega}{\omega_c}. \quad (7)$$

In particular, such an asymmetric correction arises due to ME specific to processes of double scattering on a scatterer after return to it (see Figs. 1b, 1b', 1c, 1c'). Though such processes are beyond the Boltzmann picture, they can be formally included into the kinetic equation by a slight modification of the Boltzmann collision integral. Specifically, one can introduce a small change of the scattering cross-section  $\sigma(\theta) \rightarrow \sigma(\theta) + \delta\sigma(\theta)$  on the disk where double scattering takes place



**Fig. 2.** (a) Backscattering process responsible for leading nonanalytic contribution to the resistivity at  $B = 0$ . (b) For  $B \neq 0$ , the overlap area,  $S_B$ , between two corridors is small at large  $B$ . (c) For  $\Phi = 0$ ,  $S_B$  decreases with  $B$ . (d) For  $\Phi \neq 0$  and small  $B$ , the values of  $S_B - S_0$  for time reversed trajectories have opposite signs.

(disk 1 in Figs. 1b, 1b', 1c, 1c') [21, 23]. For  $B = 0$ , the cross-section remains symmetric:  $\delta\sigma(\theta) = \delta\sigma(-\theta)$ . However, for  $B \neq 0$ , the time inversion symmetry is broken, so that the cross-section becomes asymmetric:  $\delta\sigma(\theta) \neq \delta\sigma(-\theta)$ . The point is that the influence of the magnetic field is different for the processes where the closed return path is passed counterclockwise (Figs. 1b, 1c) and clockwise (Figs. 1b', 1c').

The return after one scattering (see Figs. 1c, 1c') needs special attention, because the probability of such a process very sharply depends on  $B$  due to the “empty corridor effect” (ECE) [20, 21]. The mechanism of this phenomenon proposed in [20] is linked to the memory effects arising in backscattering events. It has a close relation to the well known non-analyticity of the virial expansion of transport coefficients [24–28] which we briefly recall. For  $B = 0$ , the leading nonanalytic correction to resistivity,  $\delta\rho$ , is due to the processes of return to a scatterer after a single collision on another scatterer (see Fig. 2a). The relative correction,  $\delta\rho/\rho$ , is proportional to the corresponding backscattering probability, given by the product of  $e^{-r/l} d\Phi dr/l$  (which is the probability of reaching scatterer 2 without collision and scattering in the angle  $d\Phi$ ) and the probability  $p$  of returning without collisions from 2 to 1 (here  $l$  is the mean free pass). Assuming  $p = \exp(-r/l)$  and integrating over intervals  $0 < \Phi < a/r$ ,  $a < r < \infty$ , one obtains [24–28]

$$\delta\rho/\rho \sim \int_a^\infty \frac{dr}{l} \int_0^{a/r} d\Phi e^{-2r/l} \sim \beta_0 \ln(1/2\beta_0). \quad (8)$$

In [20] it was shown that the probability  $p$  is actually larger than  $\exp(-r/l)$ . Indeed, the exponent  $\exp(-r/l)$  can be written as  $\exp(-nS)$ , where  $S = 2ar$ . This represents the probability of the existence of an empty corri-

dor (free of the centers of the disk) of width  $2a$  around the electron trajectory from 2 to 1. However, the passage of a particle from 1 to 2 ensures the absence of the disk centers in the region of width  $2a$  around this part of trajectory (from 1 to 2). This reduces the scattering probability on the way back. The correct value of  $p$  can be estimated as

$$\begin{aligned} p(R, \Phi) &= \exp[-n(S - S_0)] \\ &= \exp[-r/l + nS_0(r, \Phi)], \end{aligned} \quad (9)$$

where

$$S_0(r, \Phi) = 2ar - r^2|\Phi|/2 \quad (10)$$

is the area of the overlap of the two corridors (see Fig. 2a). For example, for  $\Phi = 0$ , we have  $S_0 = 2ar$  and  $p = 1$ , which reflects the obvious fact that the particle cannot scatter if it travels back along the same path. Taking into account the effect of ‘‘empty corridor,’’ we get

$$\frac{\delta\rho}{\rho} \sim \int_a^{\infty} \frac{dr}{l} \int_0^{alr} d\Phi e^{-(2r/l) + nS_0} \approx \beta_0 \ln\left(\frac{C}{2\beta_0}\right), \quad (11)$$

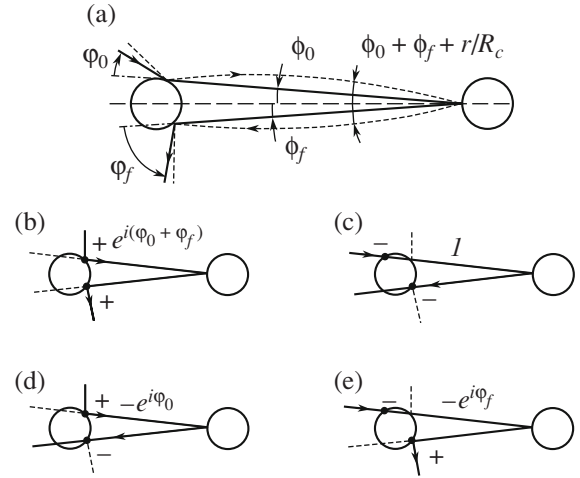
where  $C$  is a constant of the order of unity. Thus, for  $B = 0$ , the ‘‘empty corridor’’ effect simply changes the constant in the argument of the logarithm.

The key idea suggested in [20] was that for  $B \neq 0$  the area of the overlap of the two corridors,  $S_B$ , sharply depends on  $B$ , resulting in the observed MR. Indeed, it is seen from Fig. 2b that for  $\beta \gtrsim \beta_0 S_B \rightarrow 0$ , resulting in a sharp negative MR

$$\frac{\delta\rho_{xx}}{\rho} \sim \int_0^{\infty} \frac{dr}{l} \int_0^{alr} d\phi e^{-2r/l} (e^{nS_B} - e^{nS_0}). \quad (12)$$

The following qualitative explanation of the observed linear MR was presented in [20]. The value  $n(S_B - S_0)$  was estimated for  $\phi = 0$  (see Fig. 2c) to the first order in  $B$  as  $-nr^2/R_c = -r^3/2alR_c$ , where  $R_c$  is the cyclotron radius. Assuming that this estimate also works at  $\phi \neq 0$  and expanding  $e^{nS_B} - e^{nS_0}$  to the first order in  $B$ , one gets  $\delta\rho_{xx}/\rho \sim -l/R_c = -\omega_c\tau$ .

In fact, the physical picture of the phenomenon is more subtle. The contribution of any trajectory with  $\phi \neq 0$  is cancelled to the first order in  $B$  by the contribution of the time-reversed trajectory, since the values of  $S_B - S_0$  are opposite for these paths (see Figs. 2d, 2e). The cancellation does not occur only at very small  $\phi \sim \beta$ . The integration in Eq. (12) over  $\phi < \beta$  yields  $\delta\rho_{xx}/\rho \sim -\beta^2/\beta_0$ . Larger values of  $\phi$  also give a quadratic in  $\beta$  contribution to the MR. This contribution is positive and comes from the second order term in the expansion of  $e^{nS_B} - e^{nS_0}$  in  $B$ . A more rigorous approach [21] demonstrated that the contribution of small angles is dominant, result-



**Fig. 3.** Backscattering process is parameterized by the angles  $\phi_0, \phi_f$ . The magnetic field changes the backscattering angle  $\phi = \phi_0 + \phi_f + r/R_c$ . (a) The solid (dashed) line represents electron trajectory for  $B = 0$  ( $B \neq 0$ ). (b–e) Different processes contributing to cross-section renormalization.

ing in a negative parabolic MR, and that the parabolic MR crosses over to linear at very small  $\beta \approx 0.05\beta_0$ , which explains why the parabolic MR was not seen in numerical simulations [20] (see Eqs. (4), (5)).

The calculation of  $R$  is quite analogous to the calculation of  $\rho_{xx}$  presented in [21]. As was shown in this paper, the correction to cross-section arises due to four scattering processes (see Fig. 3). In the process (+, +) (Fig. 3b) an electron has two real scatterings on a disk placed at point  $\mathbf{r}$ . The process (–, –) (Fig. 3c) does not correspond to any real scattering at point  $\mathbf{r}$ . It just allows us to calculate correctly the probability of an electron passing twice a region of size  $a$  around point  $\mathbf{r}$  without scattering. To interpret the process (+, –), note that in the Boltzmann picture, which neglects correlations, electrons can scatter on a disk and later passes through the region occupied by this disk without scattering (Fig. 3d). The (+, –) correction to the cross-section modifies the Boltzmann result by subtracting the contribution of such unphysical process. Analogous consideration is valid for the process (–, +) shown in Fig. 3e. A rigorous method of calculation  $\delta\sigma(\theta)$  accounting for all four processes was developed in [21]. The calculations yield

$$\begin{aligned} \delta\sigma(\theta) &= \frac{1}{4l} \int_a^{\infty} \frac{dr}{r} e^{-2r/l} \int_0^{2\pi} d\phi_0 \int_0^{2\pi} d\phi_f \sigma(\phi_0) \sigma(\phi_f) e^{n_0 S_B} \\ &\quad \times [\delta(\theta - \phi_{\phi_0, \phi_f}) + \delta(\theta - \pi) \\ &\quad - \delta(\theta - \phi_{\phi_0, 0}) - \delta(\theta - \phi_{0, \phi_f})]. \end{aligned} \quad (13)$$

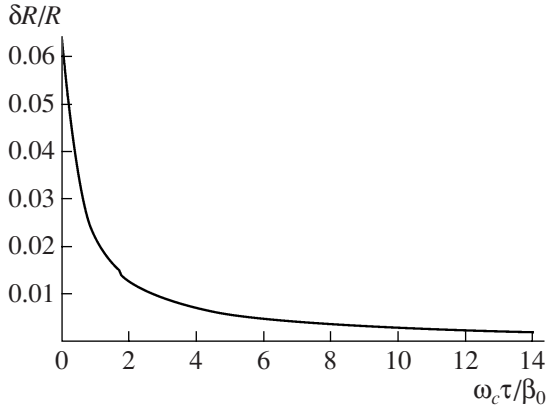


Fig. 4. Magnetic field dependence of the relative correction to the Hall coefficient caused by ECE.

Here,  $\varphi_{\varphi_0, \varphi_f} = (\pi + \varphi_0 + \varphi_f) \pmod{2\pi}$ ,  $\sigma(\varphi) = (a/2)|\sin(\varphi/2)|$  is the single scattering cross-section,

$$S_B = \int_0^r dr' (2a - |\varphi r' - r'^2/R_c|) \Theta[2a - |\varphi r' - r'^2/R_c|],$$

$\Theta[\dots]$  is the Heaviside step function,  $\phi = \Phi + r/R_c$ ,  $R_c$  is the cyclotron radius, and  $\Phi \approx (alr)[\cos(\varphi_0/2) + \cos(\varphi_f/2)]$  (see Fig. 1c). Four terms  $[\delta(\theta - \varphi_{\varphi_0, \varphi_f}) + \delta(\theta - \pi) - \delta(\theta - \varphi_{\varphi_0, 0}) - \delta(\theta - \varphi_{0, \varphi_f})]$  in Eq. (13) correspond to four types of non-Markovian processes shown in Fig. 3. Introducing dimensionless variables  $T = r/l$ ,  $z = \omega_c \tau / \beta_0$  and using Eq. (7), we get

$$\frac{\delta R}{R} = g(z) = - \int_0^\infty \frac{dT}{T} e^{-2T} \int_0^\pi d\alpha \int_0^\pi d\gamma \sin(\alpha + \gamma) \times \sin^2 \alpha \sin^2 \gamma \frac{e^{s_z} - e^{s_0}}{2z}. \quad (14)$$

Here,

$$s_z = \int_0^T dt \left( 1 - \left| \zeta t - \frac{z t^2}{2} \right| \right) \Theta \left( 1 - \left| \zeta t - \frac{z t^2}{2} \right| \right),$$

$\zeta = (\cos \alpha + \cos \gamma)/2T + zT/2$ ,  $s_0 = s_z \rightarrow 0$ . Function  $g(z)$  calculated numerically with the use of Eq. (14) is plotted in Fig. 4. For  $z \ll 1$ ,  $g(z) \ll 0.064 - 4z^2$ . For  $z \gg 1$ ,  $g(z)$  decreases as  $0.35/z^{3/2}$ . It worth emphasizing that  $\delta R/R \sim 1$  for  $z \lesssim 1$ . This means that the correction is not parametrically small in a gas parameter  $\beta_0$ , which is usually considered an expansion parameter for ME-induced corrections.

Next we calculate  $\delta R$  for stronger fields,  $\beta_0 \ll \omega_c \tau_r \ll 1$ . At such fields, the empty corridor effect is suppressed and returns after one scattering (Figs. 1c, 1c'), and after a number of scatterings (Figs. 1b, 1b') equally

contributes to  $\delta R$ . In this case, one can also introduce the effective scattering cross-section [23], which turns out to be frequency-dependent and for  $\omega = 0$  reads<sup>2</sup>

$$\delta \sigma(\theta - \theta') = v_F \int [\sigma(\theta - \varphi) - \sigma_0 \delta(\theta - \varphi)] \tilde{G}(0, \varphi - \varphi') \times [\sigma(\varphi' - \theta') - \varphi_0 \delta(\varphi' - \theta')] d\varphi d\varphi'. \quad (15)$$

Here,  $\sigma_0 = \int d\varphi \sigma(\varphi)$  is the total cross-section for single scattering,  $\tilde{G}(0, \varphi - \varphi') = \tilde{G}(\mathbf{r}, \varphi, \varphi')|_{r \rightarrow 0}$ ,  $\tilde{G}(\mathbf{r}, \varphi, \varphi') = G(\mathbf{r}, \varphi, \varphi') - G^{\text{ball}}(\mathbf{r}, \varphi, \varphi')$ ,  $G(\mathbf{r}, \varphi, \varphi')$  is the Green function of the stationary Boltzmann equation,

$$G^{\text{ball}}(\mathbf{r}, \varphi, \varphi') = \frac{\exp(\theta_r/\beta)}{v_F r \cos(\theta_r/2)}$$

$$\times \delta(\varphi - \varphi_r + \theta_r/2) \delta(\varphi' - \varphi_r - \theta_r/2)$$

is the Green function of the Boltzmann equation without the in-scattering term,  $\varphi_r$  is the angle of vector  $\mathbf{r}$ , and  $\theta_r = 2 \arcsin(\beta r/2l)$ . Substituting Eq. (15) into Eq. (7) and using the property  $\int d\varphi d\varphi_0 G(0, \varphi, \varphi') \sin(\varphi - \varphi') = 0$ ,<sup>3</sup> we get after some algebra

$$\frac{\delta R}{R} = - \frac{n_0 \sigma_{\text{tr}}^2}{2\pi} \ll 1, \quad (16)$$

where  $\sigma_{\text{tr}} = \int d\theta \sigma(\theta)(1 - \cos \theta) = 8a/3$ . Hence, with increasing  $B$  the relative correction decreases according to Eq. (14), then changes sign and saturates at small negative values. It is noteworthy that, as follows from the above derivation, Eq. (16) is valid not only for the case of impenetrable disks but also for any type of well-separated scatterers.

Above we discussed an idealized system where only strong scatterers are present. Let us now assume (see [29–31]) that in addition to strong scatterers there is a weak smooth random potential  $U(\mathbf{r})$  with the rms amplitude  $U$  and the correlation length  $d$  ( $a \ll d \ll l$ ). The presence of such a potential does not influence the ECE provided that  $\Lambda \gg l$ , where  $\Lambda \sim d(E_F/U)^{2/3}$  is the Lyapunov length characterizing the divergence of the electron trajectories in the potential  $U(\mathbf{r})$ . In the opposite limit,  $\Lambda \ll l$ , one should restrict integration over  $r$

<sup>2</sup> Equation (15) is obtained by integration over time Eq. (13) of [23] and extracting the contribution of the ballistic term. The latter one represents propagation without collision and, evidently, can not give any contribution to return processes.

<sup>3</sup> To obtain this property we first integrate the Boltzmann equation (in  $q$ -space) over  $\varphi$ , which yields  $\int G_{\mathbf{q}}(\varphi, \varphi') i\mathbf{q}v d\varphi = 1$ . One can show that integration over  $\varphi'$  yields the same result  $\int G_{\mathbf{q}}(\varphi, \varphi') i\mathbf{q}v' d\varphi' = 1$ . Using these identities we get  $\int d\varphi d\varphi' d\mathbf{q} G_{\mathbf{q}}(\varphi, \varphi') \sin(\varphi - \varphi') = \int d\varphi d\varphi' d\mathbf{q} G_{\mathbf{q}}(\varphi, \varphi') [\sin(\varphi - \varphi_q) \cos(\varphi_q - \varphi') + \cos(\varphi - \varphi_q) \sin(\varphi_q - \varphi')] = 0$ .

in Eq. (13) by  $\Lambda$ . In this case, relative correction to  $R$  decreases:  $\delta R/R \sim \Lambda/l$ . On the other hand, the field needed for suppression of the ECE increases and can be found from the estimate  $\omega_c \tau_{tr} \sim \beta_0(l/\Lambda)^2$  (at such a field two corridors corresponding to passage  $1 \rightarrow 2$  and  $2 \rightarrow 1$  (see Fig. 2a) between disks 1 and 2 separated by a distance  $r \sim \Lambda$  cease to overlap). One can also show, that at stronger fields the effect of smooth disorder leads to appearing of a very weak parabolic dependence of  $R$  on  $B$ :  $\delta R/R \sim -(\omega_c \tau)^2 (d/l)^2 (U/E_F)^2$ .

Finally, we briefly discuss the role of quantum effects. As was mentioned in [21], for  $\lambda_F > a^2/l$  the corridor effect is suppressed by diffraction on the disk's edges. In this case, the integration over  $r$  in Eq. (13) is limited by  $\Lambda' = a^2/\lambda_F$ . Hence,  $\delta R/R \sim \Lambda'/l < 1$ , and the field needed for suppression of the ECE can be found from the estimate  $\omega_c \tau_{tr} \sim \beta_0(l/\Lambda')^2$ .

To conclude, we have shown that the classical memory effects might strongly renormalize the Hall coefficient. The most interesting phenomena arise due to the empty corridor effect, which leads to a sharp field dependence of return probability and, in turn, results in a sharp dependence of  $R$  on  $B$ . The analytical calculation of dependence  $R(B)$  was presented for a 2D system with a random array of antidots modeled by hard-core spherical scatterers. It was demonstrated that the empty corridor effect leads to a very sharp dependence of  $R$  on  $B$  concentrated in the region of very weak fields ( $\omega_c \tau \lesssim a/l$ ). The total variation of  $R$  in this region of fields is on the order of the Boltzmann value of  $R$ . At larger fields, where  $a/l \ll \omega_c \tau_{tr} \ll 1$ , the ME lead to a small field-independent correction to  $R$  and (in a presence of smooth disorder) to a very weak parabolic dependence.

We acknowledge I. Gornyi and G. Minkov for fruitful discussion. The work was supported by RFBR, by grant of Russian Scientific School, and by programs of the RAS. V.Yu.K. was supported by Dynasty foundation.

## REFERENCES

1. B. L. Altshuler, D. Khmel'nitskii, A. I. Larkin, and P. A. Lee, Phys. Rev. B **22**, 5142 (1980).
2. B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems*, Ed. by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
3. I. V. Gornyi and A. D. Mirlin, Phys. Rev. B **69**, 045313 (2004).
4. H. Fukuyama, J. Phys. Soc. Jpn. **49**, 644 (1980).
5. G. Zala, B. N. Naroshny, and I. L. Aleiner, Phys. Rev. B **64**, 201201(R) (2001).
6. I. Gornyi, private communication.
7. O. Entin-Wohlman, A. G. Aronov, Y. Levinson, and Y. Imry, Phys. Rev. Lett. **75**, 4094 (1995).
8. E. M. Baskin, L. N. Magarill, and M. V. Entin, Sov. Phys. JETP **48**, 365 (1978).
9. A. V. Bobylev, F. A. Maaø, A. Hansen, and E. H. Hauge, Phys. Rev. Lett. **75**, 197 (1995).
10. A. V. Bobylev, F. A. Maaø, A. Hansen, and E. H. Hauge, J. Stat. Phys. **87**, 1205 (1997).
11. E. M. Baskin and M. V. Entin, Physica B **249**, 805 (1998).
12. D. Polyakov, Sov. Phys. JETP **63**, 317 (1986).
13. M. Fogler, A. Dobin, V. Perel, and B. Shklovskii, Phys. Rev. B **56**, 6823 (1997).
14. M. M. Fogler and B. I. Shklovskii, Phys. Rev. Lett. **80**, 4749 (1998).
15. A. D. Mirlin, J. Wilke, F. Evers, et al., Phys. Rev. Lett. **83**, 2801 (1999).
16. A. D. Mirlin, D. G. Polyakov, F. Evers, and P. Wolfle, Phys. Rev. Lett. **87**, 126805 (2001).
17. D. G. Polyakov, F. Evers, A. D. Mirlin, and P. Wolfle, Phys. Rev. B **64**, 205306 (2001).
18. M. M. Fogler and B. I. Shklovskii, Phys. Rev. Lett. **80**, 4749 (1998); D. G. Polyakov, F. Evers, and I. V. Gornyi, Phys. Rev. B **65**, 125326 (2002).
19. A. Dmitriev, M. Dyakonov, and R. Jullien, Phys. Rev. B **64**, 233321 (2001).
20. A. Dmitriev, M. Dyakonov, and R. Jullien, Phys. Rev. Lett. **89**, 266804 (2002).
21. V. V. Cheianov, A. P. Dmitriev, and V. Yu. Kachrovskii, Phys. Rev. B **68**, 201304(R) (2003); Phys. Rev. B **70**, 245307 (2004).
22. Here we neglect the effect of magnetic field on a single scattering event.
23. I. S. Lyubinskiy and V. Yu. Kachorovskii, Phys. Rev. B **73**, 041301(R) (2006).
24. J. Dorfman and E. Cohen, Phys. Lett. **16**, 124 (1965).
25. J. Van Leeuwen and A. Weiland, Physica **36**, 457 (1967).
26. J. Van Leeuwen and A. Weiland, Physica **38**, 35 (1968).
27. E. H. Hauge, in *Transport Phenomena*, Ed. by G. Kirczenow (Springer, New York, 1974), J. Marro Lect. Notes Phys., No. 31, p. 337.
28. R. Peierls, *Surprises in Theoretical Physics* (Princeton Univ. Press, Princeton, 1979).
29. G. Gusev, P. Basmaji, Z. Kvon, et al., Surf. Sci. **305**, 443 (1994).
30. H.-I. Cho, G. M. Gusev, Z. D. Kvon, et al., Phys. Rev. B **71**, 245323 (2005).
31. N. M. Sotomayor, G. M. Gusev, J. R. Leite, et al., Phys. Rev. B **70**, 235326 (2004).