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OSCILLATIONS AND WAVES IN PLASMA

Influence of Electron Collisions on the Breaking of Plasma Oscillations

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Abstract—The influence of electron collisions on the breaking of plane nonlinear plasma oscillations is analyzed. Numerical calculations by the particle method and analytical consideration in the weakly nonlinear regime show that the breaking time of plasma oscillations increases with increasing electron collision frequency. The threshold value of the electron collision frequency above which no singularity in the electron density arises is found. In this case, the density maximum formed outside the symmetry plane of oscillations, the growth of which in the weakly collisional regime leads to the breaking effect, begins to decrease after some growth because of oscillation damping.

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1. INTRODUCTION

Plasma is a highly nonlinear medium in which even relatively small initial collective displacements of particles lead to the excitation of oscillations and waves with a rather large amplitude. The time evolution of highly nonlinear oscillations and waves in dissipationless plasma leads to their breaking due to the appearance of a singularity in the electron density [1]. The limiting amplitude of the electric field up to which a one-dimensional plane nonlinear plasma wave in cold plasma can exist and, on approaching which, electron density perturbations become infinitely large was found in [2]. At the same time, it was shown in [3, 4] that plasma oscillations can also break after a certain time even if the field amplitude is below the limiting value. It was established that the breaking time is inversely proportional to the third power of the electric field [3, 4], which leads to the rapid increase in the breaking time with decreasing oscillation amplitude. In one-dimensional planar geometry, the breaking of plasma oscillations considered in [4] is related to the dependence of the frequency on the amplitude due to relativistic effects. In [4], the dependence of the breaking time on the oscillation amplitude was obtained, but the proportionality coefficient was not found, which does not allow one to accurately determine the time at which the singularity in the electron density arises. The breaking of cylindrical and spherical plasma oscillations considered in [3] was explained by the intersection of the electron trajectories, which arises due to the frequency shift caused by electron nonlinearities. It should be noted that, in [5], it was shown that the appearance of a singularity in the electron density or, which is the same, the breaking of oscillations is caused by the intersection of electron trajectories. In [6], where the time evolution of nonlinear cylindrical and plane plasma oscillations was studied numerically and analytically, the results obtained in [3] were refined. It was shown in [6] that nonlinear cylindrical oscillations break almost 1.5 times faster than it was predicted in [3], because of the intersection of the trajectories of neighboring particles, rather than particles spaced along the radius by a distance equal to the doubled oscillation amplitude, as was claimed in [3]. Moreover, it was found in [6] that breaking of nonlinear cylindrical plasma oscillations is associated with the formation of an off-axis peak of the electron density, the growth of which leads to a singularity. In the case of plane relativistic plasma oscillations, a numerical and analytical study carried out in [7] also demonstrates the breaking effect due to the appearance of a singularity in the density outside of the plane relative to which plasma electrons oscillate.

In this paper, the results previously obtained for the breaking of plane nonlinear oscillations in collisionless plasma [6, 7] are generalized to the case of a nonzero electron collision frequency. The paper is arranged as follows. In Section 2, we present a system of one-dimensional hydrodynamic equations and Maxwell's equations in the Eulerian and Lagrangian variables, which are then used for numerical calculations and analytical consideration in the weakly nonlinear regime. We also formulate the initial and boundary conditions required to describe the time evolution of localized plane plasma oscillations. In Section 3, plane plasma oscillations are simulated numerically by the particle method with the use of the so-called leapfrog scheme [8]. The calculated results are presented in the form of the time dependences of the maximum electron density. It is shown that breaking of plasma oscillations is associated with the formation of a maximum in the electron density outside the symmetry plane of oscillations, which grows with time and, after a few periods, turns to infinity. It is established that, in the presence of electron collisions, the breaking time of plasma oscillations increases with increasing collision frequency. It is established numerically that, for each initial amplitude of the electric field, there is a certain threshold value of the collision frequency above which no singularity in the density appears. The calculations show that, at collision frequencies above the threshold value, the peak of the density formed outside the symmetry plane of oscillations grows for some time, reaches its maximum value, and then decreases due to oscillation damping. Section 4 presents results of an analytic study of the time evolution of plane plasma oscillations in a weakly nonlinear regime. Based on the equations of motion in Lagrangian variables, an expression for the particle displacement as a function of time and the initial coordinate is obtained. From the condition of turning the electron density to infinity, the breaking time is found and shown to increase with increasing collision frequency. It is established that the breaking effect occurs only in plasma with relatively rare collisions. An analytical expression is obtained for the threshold value of the collision frequency above which no singularity in the density appears. Good agreement with the results of the numerical simulation is noted. In the Conclusions, the results obtained are summarized and the numerical values of the breaking time for some typical plasma parameters are presented.

2. BASIC RELATIONSHIPS

We will consider highly nonlinear plasma oscillations in plane geometry, when all physical quantities depend only on the coordinate x and time t and the velocity and electric field are directed along the x axis. Then, the system of the hydrodynamic equation for the dimensionless velocity $V = v_x/c$, momentum $P = p_x/(m_e c)$, and electron density $N = n_e/N_{0e}$ with allowance for collisions with a frequency ν_e, as well as of Maxwell's equations for the electric field $E = -eE_x/(m_e\omega_p c)$, has the form

$$\frac{\partial}{\partial\theta}P(\rho,\theta) + V(\rho,\theta)\frac{\partial}{\partial\rho}P(\rho,\theta) = -E(\rho,\theta) - \nu P(\rho,\theta), \tag{2.1}$$

$$\frac{\partial}{\partial\theta}E(\rho,\theta) - N(\rho,\theta)V(\rho,\theta) = 0, \tag{2.2}$$

$$N(\rho,\theta) = 1 - \frac{\partial}{\partial\rho}E(\rho,\theta), \tag{2.3}$$

where $\theta = \omega_p t$, $\rho = k_p x$, $\nu = \nu_e/\omega_p$, ω_p is the plasma frequency, $k_p = \omega_p/c$, N_{0e} is the electron density in the equilibrium state, e and m_e are the charge and mass of an electron, and c is the speed of light. In this case, the electron velocity is related to the electron momentum as

$$V(\rho,\theta) = \frac{P(\rho,\theta)}{\sqrt{1 + P^2(\rho,\theta)}}. \tag{2.4}$$

In what follows, we will assume that the inequality $\nu \ll 1$ is satisfied, which means that the electron collision frequency is much lower than the plasma frequency. To solve Eqs. (2.1)–(2.4), we must complement them with initial and boundary conditions. We will consider plasma oscillations localized in space near the plane $\rho = 0$. We also assume that the electron velocity and momentum at the initial time ($\theta = 0$) are zero,

$$V(\rho,\theta = 0) = 0, \quad P(\rho,\theta = 0) = 0. \tag{2.5}$$

We will assume that the oscillations are excited at the initial instant of time by an electric field of the form [7]

$$E(\rho,\theta = 0) = \left(\frac{a_*}{\rho_*}\right)^2 \rho\exp\left(-\frac{2\rho^2}{\rho_*^2}\right), \tag{2.6}$$

where the parameters ρ_* and a_* characterize the scale length of the localization region and the maximum value of the electric field (2.6), $E_{\max} = a_*^2/(\rho_* 2\sqrt{e}) \approx 0.3 a_*^2/\rho_*$, respectively.

In accordance with the form of electric field (2.6), the electrons at the initial instant of time are displaced from the plane $\rho = 0$ in different directions, which leads to their subsequently oscillations about this plane. The form of function (2.6) is chosen so that such oscillations can be excited in an underdense plasma $(\omega_0 \gg \omega_p)$ by a laser pulse with a frequency ω_0 when it is focused into a line (this can be achieved by means of a cylindrical lens).

If the laser electric field has a Gaussian spatiotemporal distribution,

$$E_L(\rho,z,t) = E_{0L}\exp\left[-\frac{\rho^2}{\rho_*^2} - \frac{\omega_p^2}{\tau_*^2}\left(t - \frac{z}{c}\right)^2\right] \times \cos\left[\omega_0\left(t - \frac{z}{c}\right)\right], \tag{2.7}$$

where $\tau_* = \omega_p\tau_p$ and $\rho_* = k_p L_x$ are the normalized values of the duration τ_p and transverse size L_x of the laser pulse, then, at a certain point z separated from the trailing edge of the pulse by a distance exceeding the plasma wavelength $(k_p|z| \gg 1)$, the quantity a_* is related to the laser pulse parameters as [6, 9]

$$a_*^2 = a_0^2\sqrt{\pi/2}\,\tau_* \exp\left(-\tau_*^2/8\right), \tag{2.8}$$

where $a_0 = eE_{0L}/(m_e\omega_0 c)$ is the normalized amplitude of the laser field. Under the conditions of the optimal excitation of the wakefield wave $(\tau_* = 2)$, when its amplitude is maximal, relationship (2.8) takes the form $a_*^2 = \sqrt{2\pi/e}\,a_0^2 \approx 1.52a_0^2$.

In addition, it should be noted that, in view of initial condition (2.6) plasma oscillations are not excited at large distances from the plane $\rho = 0$. Therefore, we will assume that

$$P(\rho\to\infty,\theta) = 0, \quad V(\rho\to\infty,\theta) = 0, \quad E(\rho\to\infty,\theta) = 0. \tag{2.9}$$

Thus, analysis of nonlinear plane localized plasma oscillations is reduced to solving the system of equations (2.1)–(2.4) with initial and boundary conditions (2.5), (2.6), and (2.9).

If we pass to the consideration of particle trajectories by using the formula

$$\rho(\rho_0,\theta) = \rho_0 + R(\rho_0,\theta), \tag{2.10}$$

where $R(\rho_0,\theta)$ is the particle displacement from the initial position ρ_0, then we obtain from Eqs. (2.1)–(2.4) the following equations in the Lagrangian variables ρ_0 and θ:

$$\frac{d}{d\theta}R(\rho_0,\theta) = V(\rho_0,\theta), \tag{2.11}$$

$$\frac{d}{d\theta}P(\rho_0,\theta) = -R(\rho_0,\theta) - \nu P(\rho_0,\theta), \tag{2.12}$$

$$E(\rho_0 + R(\rho_0,\theta),\theta) = R(\rho_0,\theta), \tag{2.13}$$

$$V(\rho_0,\theta) = \frac{P(\rho_0,\theta)}{\sqrt{1+P^2(\rho_0,\theta)}}, \tag{2.14}$$

where $d/d\theta = \partial/\partial\theta + V\,\partial/\partial\rho$ is the total time derivative. The relationship between the electric field and particle displacement (2.13) obviously follows from Eq. (2.2) for the electric field after substituting into it expression (2.3) for the electron density. System of equations (2.11)–(2.14) allows one to analyze particle trajectories numerically, as well as analytically in the case of weak nonlinearity. According to Eqs. (2.3) and (2.13), the electron density in plasma oscillations is expressed through the particle displacements by the formula

$$N(\rho_0 + R(\rho_0,\theta),\theta) = \frac{1}{1+\dfrac{\partial}{\partial\rho_0}R(\rho_0,\theta)}. \tag{2.15}$$

From system of equations (2.11)–(2.14), we obtain the following equation describing the particle trajectories in phase space $\{P, R\}$:

$$\sqrt{1+P^2} - 1 = \frac{R_m^2 - R^2}{2} - \frac{\nu}{2}\left[R\sqrt{R_m^2 - R^2} - R_m^2 \arccos\left(\frac{R}{R_m}\right)\right], \tag{2.16}$$

where R_m is the particle displacement at the turning point, at which the particle momentum is zero $(P = 0)$. Since we assume that the condition $\nu \ll 1$ is satisfied, it follows from formula (2.16) that collisions between particles insignificantly affect their trajectories in phase space.

3. NUMERICAL SIMULATION OF PLANE PLASMA OSCILLATIONS

In this section, we will numerically integrate Eqs. (2.11)–(2.14) with initial conditions (2.5) and (2.6). Let, at the initial instant of time $\theta = 0$, the kth particle be characterized by an initial radial position $\rho_0(k)$ and initial displacement $R(k,0)$, $1 \le k \le M$, where M is the total number of particles. The initial positions of all particles correspond to electric field (2.6). On the other hand, the electric field created by displaced particles at the initial time $\theta = 0$ at a point with the coordinate $\rho_k = \rho_0(k) + R(k,0)$ is described by formula (2.13). Comparing expressions (2.6) and (2.13), we can find the sought-for quantities $\rho_0(k)$ and $R(k,0)$. To this end, we define the initial spatial grid $\rho_k = kh$, where h is the parameter of discretization over the spatial coordinate, characterizing the proximity of neighboring particles. The electric field $E_0(\rho_k)$ created by displaced particles at the grid nodes is described by formula (2.6). Therefore, from Eq. (2.13), we obtain the following equations for the initial positions $\rho_0(k)$:

$$\rho_k = \rho_0(k) + R(k,0) = \rho_0(k) + E_0(\rho_k), \tag{3.1}$$

where $E_0(\rho_k)$ is the distribution of the electric field at the initial time $\theta = 0$. Thus, in order to calculate the trajectory of each particle, we obtain the initial values $\rho_0(k)$ and $R(k,0)$, to which we should add the condition of particles being at rest at the initial time, $P(k,0) = 0$, which follows from Eq. (2.5).

Equations (2.11)–(2.12) can be integrated numerically by the second-order difference method (the so-called leapfrog scheme) [8], traditionally used to solve the equations of motion. Let τ be the parameter of

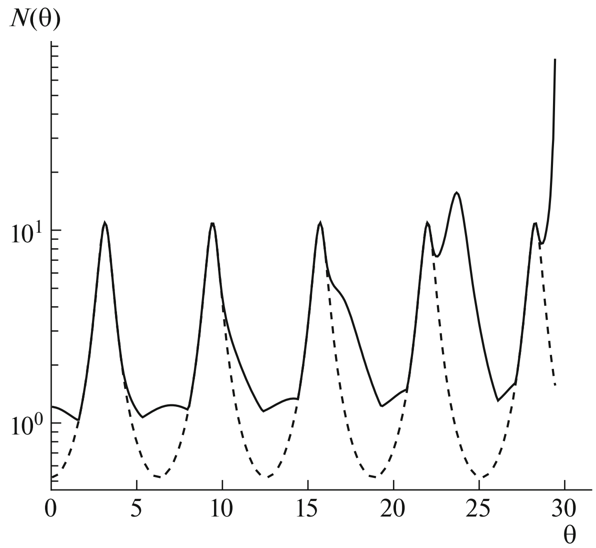

Fig. 1. Time dependence of the electron density in nonlinear oscillations in collisionless plasma $(\nu\theta_{\rm wb} = 0)$. The solid and dashed lines shows the time evolution of the maximum density in the entire computational domain and in the symmetry plane $\rho = 0$, respectively.

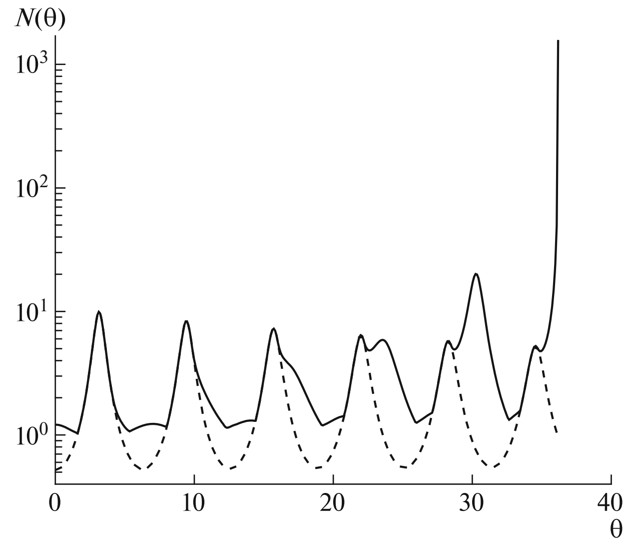

Fig. 2. Time dependence of the electron density in nonlinear plasma oscillations in weakly collisional plasma $(\nu\theta_{\rm wb} = 0.2)$. The solid and dashed lines shows the time evolution of the maximum density in the entire computational domain and in the symmetry plane $\rho = 0$, respectively.

time discretization, i.e., $\theta_j = j\tau,\ j \ge 0$. Then, the computational formulas will have the form

$$\frac{P(k,\theta_{j+1/2}) - P(k,\theta_{j-1/2})}{\tau} = -R(k,\theta_j) - \nu\frac{P(k,\theta_{j+1/2}) + P(k,\theta_{j-1/2})}{2}, \tag{3.2}$$

$$\frac{R(k,\theta_{j+1}) - R(k,\theta_j)}{\tau} = \frac{P(k,\theta_{j+1/2})}{\sqrt{1+P^2(k,\theta_{j+1/2})}}. \tag{3.3}$$

In this case, at an arbitrary instant of time θ_j, the variable Eulerian grid at the nodes of which the values of the electric field are determined by the formula $E(\rho_k,\theta_j) = R(k,\theta_j)$ can be calculated by the formula

$$\rho_k = \rho_0(k) + R(k,\theta_j), \quad 1 \le k \le M. \tag{3.4}$$

This is used to graphically represent the electron density, because, in the calculations, we used the following formula of numerical differentiation of the second order of accuracy at the midpoints of subintervals:

$$N\left(\frac{\rho_{k+1}+\rho_k}{2},\theta_j\right) = 1 - \frac{E(\rho_{k+1},\theta_j) - E(\rho_k,\theta_j)}{\rho_{k+1}-\rho_k}. \tag{3.5}$$

For definiteness, we set, $a_* = 3.105$ and $\rho_* = 4.5$ in Eq. (2.6). This version was considered in [7] when calculating the relativistic breaking of undamped oscillations. In the boundary conditions, all physical quantities can be assumed to vanish at the boundary of the computational domain $-d \le \rho \le d$. In our simulations, the artificial boundary was set at $d = 4.5\rho_*$ and the total number of particles was $M = 51841$. Accordingly, the quantity h, which determines the initial distance between particles, was $h = 2d/(M-1)$. The time step τ was taken equal to h. To control the integration accuracy, we regularly performed calculations with the grid parameters two times smaller than those in the main calculation version.

The results of calculations of the electron density as a function of time for different values of the parameter ν, characterizing the collisional damping of electron oscillations, are shown in Figs. 1–3. Figure 1 shows the time dependence of the maximum electron density in collisionless plasma at $\nu = 0$, i.e., in the absence of oscillation damping. It follows from Fig. 1 that, at the given calculation parameters, a peak of the density forms in the fourth period of oscillations outside the $\rho = 0$ plane. This maximum density tends to infinity already in the next period at $\theta_{\rm wb}^{(0)} \approx 29.49$. In the presence of electron collisions, the time at which the singularity in the electron density appears increases. For example, for $\nu\theta_{\rm wb}^{(0)} = 0.2$, the maximum density tends to infinity only in the third period after its formation (see Fig. 2), rather than in the second period as it was in collisionless plasma. The calculations show that, for the given initial parameters, the singularity in the density appears only at relatively rare collisions, such that $\nu\theta_{\rm wb}^{(0)} \le 0.422$. When the equality $\nu\theta_{\rm wb}^{(0)} = 0.422$ holds, the singularity in the density appears at $\theta_{\rm wb} \approx 75.22$,

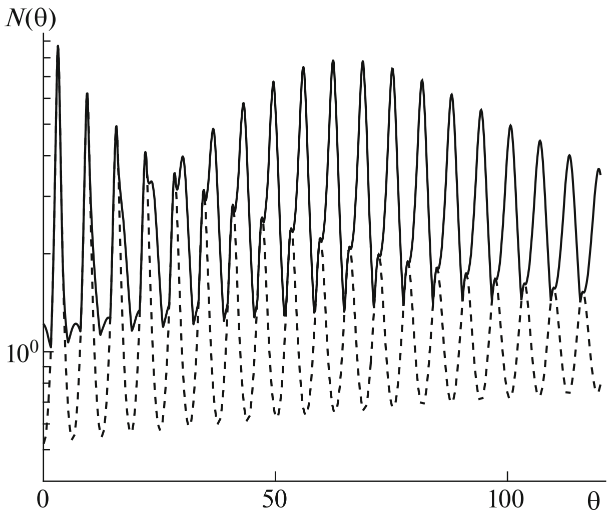

Fig. 3. Time dependence of the electron density in nonlinear plasma oscillations in plasma with frequent collisions ($\nu\theta_{wb} = 0.5$). The solid and dashed lines shows the time evolution of the maximum density in the entire computational domain and in the symmetry plane $\rho = 0$, respectively.

which is about 2.5 times greater than the breaking time in collisionless plasma. At $\nu\theta_{wb}^{(0)} > 0.422$, no singularity in the density arises. This scenario of the time evolution of the maximum electron density is illustrated in Fig. 3 for $\nu\theta_{wb}^{(0)} = 0.5$. In this case, the density maximum formed outside the $\rho = 0$ plane first increases, but then it decreases due to strong oscillation damping.

It should be noted that the results of calculations by the particle method presented above were fully reproduced in additional computations by the splitting scheme in the Eulerian variables [7] and the classical fourth-order Runge–Kutta method for integrating a system of ordinary differential equations.

4. ANALYTICAL THEORY IN THE WEAKLY NONLINEAR REGIME

In the weakly nonlinear regime, where the expansion $P \approx V\left(1 + V^2/2\right)$ can be used, the system of equations (2.11) and (2.12) is reduced to one equation for a small deviation of a particle from its initial position ($|R(\rho_0,\theta)| \ll 1$),

$$\left(\frac{d^2}{d\theta^2} + \nu\frac{d}{d\theta} + 1\right)R(\rho_0,\theta) + \frac{1}{2}\left(\frac{d}{d\theta} + \nu\right)\left[\frac{d}{d\theta}R(\rho_0,\theta)\right]^3 = 0. \quad (4.1)$$

The solution to Eq. (4.1) is sought in the form

$$R(\rho_0,\theta) = \frac{1}{2}R_1(\rho_0,\theta) \times \exp\left\{-i\int_0^{\theta} d\theta'\Omega(\rho_0,\theta')\right\} + \text{c.c.}, \quad (4.2)$$

where $R_1(\rho_0,\theta)$ is the amplitude of the electron displacement, which varies slowly in time over the oscillation period, and $\Omega(\rho_0,\theta)$ is the dimensionless oscillation frequency (the frequency normalized to the plasma frequency ω_p). Then, separating the terms in Eq. (4.1) in time scales and taking into account representation (4.2), we find from Eq. (4.1) the reduced (without a small second time derivative) equation for the displacement amplitude,

$$2i\left[\frac{d}{d\theta}R_1(\rho_0,\theta) + \frac{\nu}{2}R_1(\rho_0,\theta)\right] + \left[\Omega^2(\rho_0,\theta) - 1 + \frac{3}{8}|R_1(\rho_0,\theta)|^2\right]R_1(\rho_0,\theta) = 0, \quad (4.3)$$

which is valid under the conditions $\nu \ll 1$ and $|R(\rho_0,\theta)| \ll 1$.

We will assume that the equality

$$\Omega(\rho_0,\theta) = 1 - \frac{3}{16}|R_1(\rho_0,\theta)|^2, \quad (4.4)$$

which determines the relativistic correction for the plasma oscillation frequency, is satisfied. Then the solution to Eq. (4.3) takes the form

$$R_1(\rho_0,\theta) = R_1(\rho_0,\theta = 0)\exp\left(-\frac{1}{2}\nu\theta\right). \quad (4.5)$$

Taking into account relationships (4.4) and (4.5), expression (4.2) for the electron displacement is reduced to

$$R(\rho_0,\theta) = R_0(\rho_0)\exp\left(-\frac{1}{2}\nu\theta\right)\cos[\varphi(\rho_0,\theta)], \quad (4.6)$$

where $R_0(\rho_0) = R(\rho_0,\theta = 0)$ is the particle displacement at the initial time and the phase $\varphi(\rho_0,\theta)$ has the form

$$\varphi(\rho_0,\theta) = \theta - \frac{3}{16}R_0^2(\rho_0)\frac{1 - \exp(-\nu\theta)}{\nu}. \quad (4.7)$$

From formula (4.7), in the limit $\nu \to 0$, we obtain the well-known result $\varphi(\rho_0,\theta) = \left[1 - (3/16)R_0^2(\rho_0)\right]\theta$ for the phase of plane electron oscillations in collisionless plasma with allowance for relativistic nonlinearity [2].

It follows from Eq. (2.15) that a singularity in the electron density arises when the condition

$$1 + \frac{\partial}{\partial\rho_0}R(\rho_0,\theta) = 0 \quad (4.8)$$

is satisfied. Taking into account expression (4.6) for the electron displacement, we obtain from condition (4.8) the following equation determining the breaking time of plasma oscillations:

$$1+\frac{3}{8}\sin[\varphi(\rho_0,\theta)]G(\theta)R_0^2(\rho_0)\frac{\partial}{\partial\rho_0}R_0(\rho_0)=0, \quad (4.9)$$

where

$$G(\theta)=\frac{1-\exp(-\nu\theta)}{\nu}\exp\left(-\frac{1}{2}\nu\theta\right). \quad (4.10)$$

The function $G(\theta)$ increases with time at small θ, reaches its maximum value $G_{\max}=2/(3\sqrt{3}\nu)$ at $\theta_{\max}=(1/\nu)\ln 3$, and then decreases and vanishes as $\theta\to\infty$. When solving Eq. (4.9), we take into account that, according to formula (2.13), the initial displacements of particles in the weakly nonlinear regime are

$$R_0(\rho_0)=\left(\frac{a_*}{\rho_*}\right)^2\rho_0\exp\left(-2\frac{\rho_0^2}{\rho_*^2}\right). \quad (4.11)$$

Then, Eq. (4.9) under the condition $R_0(\rho_0)\ll 1$ takes the form

$$1-\frac{3}{8}\frac{a_*^6}{\rho_*^4}G(\theta)F(\rho_0)=0, \quad (4.12)$$

where the function $F(\rho_0)$ is defined as

$$F(\rho_0)=\frac{\rho_0^2}{\rho_*^2}\left|1-4\frac{\rho_0^2}{\rho_*^2}\right|\exp\left(-6\frac{\rho_0^2}{\rho_*^2}\right). \quad (4.13)$$

It follows from Eq. (4.12) that the minimum time for the appearance of a singularity in the electron density is realized when $F(\rho_0)$ as a function of ρ_0 reaches its maximum value. Formula (4.13) implies that the maximum value of the function $F(\rho_0)$, $F_{\max}=1/(18\sqrt{e})$ (where $e=\exp(1)\approx 2.718\ldots$) is reached at a distance $\rho_0=\rho_*/(2\sqrt{3})$ from the symmetry plane. Therefore, Eq. (4.12) can be written in the form

$$[1-\exp(-\nu\theta)]\exp\left(-\frac{1}{2}\nu\theta\right)=\nu\theta_{\rm wb}^{(0)}, \quad (4.14)$$

where

$$\theta_{\rm wb}^{(0)}=48\sqrt{e}\frac{\rho_*^4}{a_*^6} \quad (4.15)$$

is the breaking time of weakly nonlinear oscillations in collisionless plasma ($\nu=0$) [6, 7].

Equation (4.14) is cubic with respect to $\exp(-\nu\theta/2)$ and has real solutions under the condition

$$\nu\theta_{\rm wb}^{(0)}\le\frac{2}{3\sqrt{3}}\approx 0.385. \quad (4.16)$$

An approximate solution to Eq. (4.14) under the condition $\nu\theta_{wb}^{(0)}<1$ has the form

$$\theta_{\rm wb}\approx\theta_{\rm wb}^{(0)}\left[1+\nu\theta_{\rm wb}^{(0)}\right]. \quad (4.17)$$

It follows from formula (4.17) that the collisions lead to an increase in the breaking time. In this case, breaking of plasma oscillations occurs only under condition (4.16) and the density singularity appears as a result of an increase in the maximum of the electron density formed outside the symmetry plane. When the dimensionless electron collision frequency ν exceeds the threshold value $\nu_{\rm th}=2/(3\sqrt{3}\theta_{\rm wb}^{(0)})$ and the inequality

$$\nu\theta_{\rm wb}^{(0)}>\frac{2}{3\sqrt{3}}\approx 0.385, \quad (4.18)$$

inverse to condition (4.16), is satisfied, then the singularity in the density does not arise because of the damping of plasma oscillations. Note that, at $\nu\theta_{\rm wb}^{(0)}\to 2/(3\sqrt{3})$, the breaking time is $\theta_{\rm wb}\approx 2.854\theta_{\rm wb}^{(0)}$, as it follows from analysis of Eq. (4.14).

The expression for the threshold value of the collision frequency $\nu_{\rm th}=2/(3\sqrt{3}\theta_{\rm wb}^{(0)})$ with allowance for relationship (4.15) takes the form

$$\nu_{\rm th}=\frac{1}{72\sqrt{3e}}\frac{a_*^6}{\rho_*^4}\approx 4.86\times 10^{-3}\frac{a_*^6}{\rho_*^4}. \quad (4.19)$$

It follows from this formula that, for a fixed size ρ_* of the localization region of oscillations, the threshold value of the collision frequency is proportional to the third power of the initial amplitude of the electric field (2.6).

Let us compare the results of the weakly nonlinear theory with the results of numerical calculations by the particle method. For $a_*=3.105$ and $\rho_*=4.5$, the numerically calculated breaking time in collisionless plasma is $\theta_{\rm wb}^{(0)}\approx 29.49$. In this case, the singularity in the density appears at $\nu\theta_{\rm wb}^{(0)}\le 0.422$. As the threshold value $\nu\theta_{\rm wb}^{(0)}=0.422$ is approached, the breaking time tends to $\theta_{\rm wb}\approx 75.22$. For large values of the collision frequency, when the inequality $\nu\theta_{\rm wb}^{(0)}>0.422$ is satisfied, there is no singularity in the density. In this case, the density maximum formed outside the $\rho=0$ plane reaches its peak value at a certain time, after which it decreases (see Fig. 3). According to the weakly nonlinear theory (see formula (4.15) and inequality (4.16)), for the above parameters of the initial electric field, we have $\theta_{\rm wb}^{(0)}\approx 36.2$ and $\nu\theta_{\rm wb}^{(0)}\le 0.385$, which satisfactorily agrees with the numerical results. In this case, the breaking time at $\nu\theta_{\rm wb}^{(0)}\to 0.385$ is equal to $\theta_{\rm wb}\approx 2.854\theta_{\rm wb}^{(0)}$, which also correlates well with the results of calculations by the particle method.

5. CONCLUSIONS

In this paper, the influence of electron collisions on the breaking of plane nonlinear plasma oscillations has been studied numerically and analytically. In the absence of electron collisions, breaking of plasma oscillations is caused by the formation of a maximum in the electron density outside the symmetry plane of oscillations, which grows with time and, after a few oscillation periods, tends to infinity. It is shown that, in the presence of electron collisions, breaking of oscillations is also associated with the growth of the density maximum formed outside the symmetry plane; however, in collisional plasma, its growth is slower and the breaking time increases with increasing collision frequency. It is established numerically and analytically that there is a certain threshold value of the electron collision frequency above which the singularity in the density does not arise. It is shown that, at a collision frequency above the threshold value, the density maximum formed outside the symmetry plane, grows only for some time after its formation, but then it decreases due to strong oscillation damping.

Let us estimate the breaking time of nonlinear oscillations for certain typical plasma parameters. If plasma oscillations are driven by electric field (2.6) with the parameters $a_* = 3.105$ and $\rho_* = 4.5$, then it follows from the numerical calculations that the breaking time in collisionless plasma is $\theta_{\mathrm{wb}}^{(0)} \approx 29.49$. Hence, according to the numerical result for the threshold value of the dimensionless collision frequency, $\nu\theta_{\mathrm{wb}}^{(0)} \le 0.422$, breaking of plasma oscillations occurs under the condition $\nu \le 1.43\times10^{-2}$. In fully ionized plasma, the dimensionless electron–ion collision frequency is defined by the formula [10]

$$\nu = Z\frac{\sqrt{8}}{3}\eta^{3/2}\ln\Lambda, \tag{5.1}$$

where Z is the ion charge number; $\ln\Lambda$ is the Coulomb logarithm; and η is the ratio of the electron interaction energy $e^2N_{0e}^{1/3}$ to the electron kinetic energy T_e,

$$\eta = \frac{e^2N_{0e}^{1/3}}{T_e}. \tag{5.2}$$

Suppose that a laser pulse with the wavelength $\lambda_0 = 1.24$ μm (frequency $\omega_0 \approx 1.5\times10^{15}$ s^{-1}), duration $\tau \approx 36$ fs, and dimensionless amplitude of the electric field $a_0 \approx 2.5$ propagates in a fully ionized underdense plasma with the ion charge number $Z = 5$, electron density $N_{0e} = 10^{18}$ cm^{-3}, and electron temperature $T_e = 50$ eV. If the laser pulse is focused by a cylindrical lens into a line with a transverse size $L_x \approx 24$ μm, then plane one-dimensional electron oscillations with the parameters $a_* \approx 3.1$ and $\rho_* \approx 4.5$, which are close to those used in the above calculations, are excited in the wakefield wave generated behind the pulse. It should be noted that, if the laser pulse propagates in an underdense plasma with a moderately relativistic intensity $a_0 \approx 1$–3, then the condition for the optimal excitation of plasma waves ($\tau_* = 2$) is approximately preserved and the amplitude of plasma oscillations is related to the laser field by the same relationship $a_*^2 \approx 1.52a_0^2$ as in the nonrelativistic limit. For the above plasma parameters, we find from formulas (5.1) and (5.2) that the dimensionless collision frequency $\nu \approx 0.5\times10^{-2}$ is below the threshold value of 1.43×10^{-2}. Therefore, in this case, breaking of electron oscillations takes place and the breaking time is $\theta_{\mathrm{wb}} \approx 33$, because $\nu\theta_{\mathrm{wb}}^{(0)} \approx 0.15$. If we consider the propagation of a laser pulse with the above parameters in plasma with the same density but with the temperature $T_e = 20$ eV, then the calculations by formulas (5.1) and (5.2) yield the dimensionless collision frequency $\nu \approx 1.8\times10^{-2}$ ($\nu\theta_{\mathrm{wb}}^{(0)} \approx 0.52$), which exceeds the threshold value. Therefore, in this case, no breaking of plasma oscillations in the laser pulse wakefield occurs because of strong oscillation damping.

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REFERENCES

1. R. C. Davidson, *Methods in Nonlinear Plasma Theory* (Academic, New York, 1972).
2. A. I. Akhiezer and R. V. Polovin, Sov. Phys. JETP **3**, 696 (1956).
3. J. M. Dawson, Phys. Rev. **113**, 383 (1959).
4. G. Lehmann, E. W. Laedke, and K. H. Spatschek, Phys. Plasmas **14**, 103109 (2007).
5. Ya. B. Zel'dovich and A. D. Myshkis, *Elements of Mathematical Physics* (Nauka, Moscow, 1973) [in Russian].
6. L. M. Gorbunov, A. A. Frolov, E. V. Chizhonkov, and N. E. Andreev, Plasma Phys. Rep. **36**, 345 (2010).
7. A. A. Frolov and E. V. Chizhonkov, Vychisl. Metody Program. **15**, 537 (2014).
8. R. W. Hockney and J. W. Eastwood, *Computer Simulation Using Particles* (McGraw-Hill, New York, 1981).
9. V. E. Grishkov and S. A. Uryupin, Plasma Phys. Rep. **43**, 315 (2017).
10. V. P. Silin and A. A. Rukhadze, *Electromagnetic Properties of Plasmas and Plasma-Like Media* (Gosatomizdat, Moscow, 1961; Gordon & Breach, New York, 1965).

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